IOP Conference Series: Materials Science and Engineering

PAPER • OPEN ACCESS

A cooperative game in Asian international electric power integration

To cite this article: Ilya Minarchenko 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **537** 062003

IOP Conf. Series: Materials Science and Engineering **537** (2019) 062003 doi:10.1088/1757-899X/537/6/062003

A cooperative game in Asian international electric power integration

Ilya Minarchenko

Melentiev Energy Systems Institute of Siberian Branch of the Russian Academy of Sciences, Lermontov str., 130, Irkutsk, Russia

E-mail: eq.progr@gmail.com

Abstract. The paper examines effects of long-term international integration in the field of electric power industry. An advantage of integration is provoked by the fact that the cost of introducing new generation capacities significantly exceeds the cost of new power lines and transmission of the energy from existing power station of another country. When countries form a coalition, the problem is to allocate the coalition's surplus over its participants. It can be solved by notions of cooperative game theory. The present investigation is based on the real data on six countries of the Northeast Asian region: Russia, Mongolia, China, North Korea, South Korea and Japan. Electric power system is described by the ORIRES model. This model optimizes power generation, power flows, and the development of generation capacities and power lines of the system. We formulate corresponding cooperative game in characteristic function form, and specify the Core and the Shapley value. Effects of international integration are discussed.

1. Introduction

The present study is concerned with a special model of long-term development and operation of electrical power systems. This model is called the ORIRES model [1]. ORIRES is a Russian abbreviation that means Optimization of Development and Operating Conditions of Electric Power Systems. One of its main features is that it takes development of a power system into consideration. This way, the model can provide not only the amount of hourly generated electricity and power flows, but also the capacities of power generators and transmission lines installed in the system during a particular target year. In the original statement, the ORIRES model is a linear programming problem, in which the objective function is the total annual system cost. That function is to be minimized subject to balance equations and other system-specific constraints. In [2], this model was reformulated for analysis of an imperfect electric energy market, where generation companies compete in accordance with the Cournot oligopoly model. A solution of such a problem is a Nash equilibrium situation rather than a point that minimizes a single objective function.

This paper focuses on the international power grid interconnections in the Northeast Asian region. One of the main issues that rises in this field is an estimation of the economic benefit of such a cooperation, possibly with increased attention to usage of renewable energy resources and emissions reduction. There have been some papers on grid interconnections in Northeast Asia, see for example [3-7] and the references therein. The investigations in [5-7] use the ORIRES model as a basic tool for computing the integration effect. Note that in [7], the ORIRES is called the ExOMPS model.

IOP Conf. Series: Materials Science and Engineering **537** (2019) 062003 doi:10.1088/1757-899X/537/6/062003

In addition to estimation of the benefit for the cooperation as a whole, there is a concurrent problem of how to divide this benefit among the participants. One approach is based on dual analysis of the corresponding linear optimization problem [7]. It suggests to consider the values of the corresponding Lagrange multipliers as nodal prices. More detailed description of the dual ORIRES model is presented in [8]. However, this approach does not provide any strict way to divide the cost of international lines, and this type of costs has to be allocated over the countries according to some additional assumptions.

In our investigation, we propose to consider the electric power cooperation in the framework of the game theory. The cooperative game theory [9] seems to be a natural technique for modelling international cooperation. One of the problems that the cooperative game theory can handle is to allocate the total benefit of the cooperation over the participants with respect to some optimality condition. We examine two classical concepts of the "optimal" allocations: the Core and the Shapley value. An advantage of the game theory approach is that it takes interests of the countries into consideration, in contrast to optimizing some single aggregate criterion. Moreover, it allows us to divide all the costs of the integrated power system including the cost of the lines. In order to define a cooperative game (in characteristic function form) we need to establish the minimal total cost of every possible cooperation between the given countries. It means that all possible coalitions of a smaller size than the maximal cooperation should also be considered. We use the ORIRES model for minimizing the total cost of every coalition.

Our study is based on real data for six countries of Northeast Asia: Russia, Mongolia, China, North Korea, South Korea, and Japan. In Russia, we consider two local energy systems in the eastern part of the territory: Siberia and Far East. China is also represented by two nodes: North China and Northeast China. For other countries, we do not separate nodes in their national energy systems. Analysis is performed for the target year 2030.

The paper is structured as follows. In Section 2 we provide a mathematical description of the ORIRES model. Section 3 provides some basic definitions and relations of the cooperative game theory. Section 4 contains the main results of the paper. Here we give a procedure that defines the characteristic function and, consequently, the corresponding cooperative game. Also here we describe and discuss the Core and the Shapley value allocations. In Section 5, we highlight some basic effects of the whole cooperation. The final Section 6 summarizes conclusions and directions for future work.

2. The ORIRES model

ORIRES represents a static multi-node linear model that optimizes generation capacities, operating powers, capacities of power lines, and power flows. The model distinguishes between the four seasons and working days and holidays, and every day is separated into 24 equal periods of time (hours). The variables in the model are as follows: the capacities and operating powers of the generation stations, the capacities of the transmission lines, and the amounts of electricity transmitted between the nodes via the lines. The total annual discounted cost of the system plays the role of the objective function. The cost consists of the following components: the power generation cost, the capital cost, and the maintenance cost of both the power generators and transmission lines. A node represents a local energy system, which contains possibly various types of electrical power plants such as thermal, hydroelectric, pumped storage, or nuclear. Every node belongs to a certain country presented in the system. For mathematical formulation of the ORIRES model, let us introduce notations for the parameters and variables.

The parameters defined by exogenous input data are as follows. J is the set of the nodes in the system (Russia – Siberia, Russia – Far East, Mongolia, North China, Northeast China, North Korea, South Korea, Japan), I is the set of the generation types (hydro, thermal on gas, thermal on coal, pumped storage, nuclear, and others), S is the set of the seasons (winter, spring, summer, fall), T is the set of the hours ($T = \{1, \ldots, 24\}$), $T^*(s)$ is the set of the peak hours in season s (for winter and summer only) such that

$$T^*(\text{'winter'}) = \{8, 9, 11\},\ T^*(\text{'}summer\text{'}) = \{4, 5, 6\}$$

τ_s^w is the number of working days in season s, τ_s^h is the number of holidays in season s, c_{ji} is the unit cost of generation type i in node j, f is the rate of return, γ_{ji} is the unit cost of capacities of type i in node j, z_{ji}^0 is the already installed capacities of type i in node j, $\bar{z}_{ji}$ is the maximal capacity of type i that can be installed in node j, k_{ji} is the unit costs for maintenance of capacities of type i in node j, $\rho_{jj'}$ is the unit cost of installing a line between nodes j and j', $v_{jj'}^0$ is the existing line capacity between nodes j and j', $\bar{v}_{jj'}$ is the maximal capacity of the line that can be installed between nodes j and j', $b_{jj'}$ is the unit cost of the line maintenance between nodes j and j', $A = \left(a_{jj'}\right)_{j,j' \in J}$ is the symmetric matrix of connections ($a_{jj'} = 1$ if the line between nodes j and j' exists or can be potentially installed and $a_{jj'} = 0$ otherwise), $\delta_{jj'}$ is the loss rate of power transmitting from j to j', d_{jst}^w is the demand in node j in season s at hour t on a working day, d_{jst}^h is the demand in node j in season s at hour t on a holiday, r_{jst} is the power reserve required in node j in season s at hour t, α_{jis} is the rate of the lower operating limit of generation type i in node j in season s, β_{jis} is the rate of the upper operating limit of generation type i in node j in season s, J_S is the set of the nodes with a constraint on season output of hydro power plants, J_Y is the set of the nodes with a constraint on annual output of hydro power plants, H_{js}^S is the maximal seasonal number of operating hours of hydro power plants in node $j \in J_S$ in season s, H_j^Y is the maximal annual number of operating hours of hydro power plants in node $j \in J_Y$, G_{js} is the maximal charge rate of the pumped storage power plants in node j in season s, q_j is the efficiency rate of the pumped storage power plants in node j, H_j^D is the maximal daily number of operating hours of the pumped storage power plants in node j.

The variables of the model are the following. x_{jist}^w is the amount of generated electricity of type i in node j in season s at hour t on a working day, x_{jist}^h is the amount of generated electricity of type i in node j in season s at hour t on a holiday, z_{ji} is the capacity of type i in node j, $v_{jj'}$ is the capacity of the line between nodes j and j', $y_{jj'st}^w$ is the power flow from node j to node j' in season s at hour t on a working day, $y_{jj'st}^h$ is the power flow from node j to node j' in season s at hour t on a holiday, $\tilde{y}_{jj'st}$ is the power flow under emergency conditions in the case of a failure at a peak hour $t \in T^*(s)$ from node j to node j' in season s, u_{jst}^w is the amount of charge of the pumped storage plants in node j in season s at hour t on a working day, u_{jst}^h is the amount of charge of the pumped storage plants in node j in season s at hour t on a holiday. For convenience, let us gather all the variables into a single vector:

$$X = \left(x_{jist}^w, x_{jist}^h, z_{ji}, v_{jj'}, y_{jj'st}^w, y_{jj'st}^h, \tilde{y}_{jj'st}, u_{jst}^w, u_{jst}^h\right).$$

For a particular set of countries, let set J be the set of all nodes located in these countries. For a given J, the ORIRES model is presented by the following linear programming problem:

$$\sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\tau_s^w c_{ji} x_{jist}^w + \sum_{j\in J}\sum_{i\in I}\sum_{s\in S}\sum_{t\in T}\tau_s^h c_{ji} x_{jist}^h \tag{1}$$

$$+ f\sum_{j\in J}\sum_{i\in I}\gamma_{ji}\left(z_{ji} - z_{ji}^0\right) + \sum_{j\in J}\sum_{i\in I}k_{ji}\gamma_{ji}z_{ji} \tag{2}$$

$$+ f\sum_{j\in J}\sum_{j'\in J}\rho_{jj'}\left(v_{jj'} - v_{jj'}^0\right) + \sum_{j\in J}\sum_{j'\in J}b_{jj'}\rho_{jj'}v_{jj'} \to \min_X, \tag{3}$$

$$X \in \Omega, \tag{4}$$

where set Ω will be specified further. In the objective function, (1) represents the total annual operating cost, while (2) is the cost of the development and maintenance of the generation capacities, and the summation in (3) provides the cost of the development and maintenance of the power lines.

The feasible set Ω is determined by the following linear constraints.

IOP Conf. Series: Materials Science and Engineering **537** (2019) 062003 doi:10.1088/1757-899X/537/6/062003

The balance equations for every node at every hour on working days and holidays:

$$\sum_{i\in I} x^{w}_{jist} + \sum_{j'\in J} a_{jj'}\left(y^{w}_{j'jst}\left(1-\delta_{j'j}\right) - y^{w}_{jj'st}\right) - u^{w}_{jst} = d^{w}_{jst}, \quad j\in J, s\in S, t\in T, \tag{5}$$

$$\sum_{i\in I} x^{h}_{jist} + \sum_{j'\in J} a_{jj'}\left(y^{h}_{j'jst}\left(1-\delta_{j'j}\right) - y^{h}_{jj'st}\right) - u^{h}_{jst} = d^{h}_{jst}, \quad j\in J, s\in S, t\in T. \tag{6}$$

The constraints on operating power for every type of generation:

$$\alpha_{jis} z_{ji} \le x^{w}_{jist} \le \beta_{jis} z_{ji}, \quad j\in J, i\in I, s\in S, t\in T, \tag{7}$$

$$\alpha_{jis} z_{ji} \le x^{h}_{jist} \le \beta_{jis} z_{ji}, \quad j\in J, i\in I, s\in S, t\in T. \tag{8}$$

The feasibility constraints on development of generation capacities and power lines

$$z^{0}_{ji} \le z_{ji} \le \overline{z}_{ji}, \quad j\in J, i\in I,$$

$$v^{0}_{jj'} \le v_{jj'} \le \overline{v}_{jj'}, \quad j\in J, j'\in J.$$

The constraints on generation capacities with respect to emergency situations in the case of failure during peak hours

$$\sum_{i\in I} z_{ji} + \sum_{j'\in J} a_{jj'}\left(\tilde{y}_{j'jst}\left(1-\delta_{j'j}\right) - \tilde{y}_{jj'st}\right) = d^{w}_{jst}\left(1+r_{jst}\right), \quad j\in J, s\in S, t\in T^{*}(s). \tag{9}$$

The feasibility constraints on flows and emergency flows

$$y^{w}_{jj'st} \le v_{jj'}, \quad j\in J, j'\in J, s\in S, t\in T, \tag{10}$$

$$y^{h}_{jj'st} \le v_{jj'}, \quad j\in J, j'\in J, s\in S, t\in T, \tag{11}$$

$$\tilde{y}_{jj'st} \le v_{jj'}, \quad j\in J, j'\in J, s\in S, t\in T^{*}(s). \tag{12}$$

The constraints on the annual and seasonal outputs of hydroelectric plants:

$$\sum_{t\in T}\left(\tau^{w}_{jst} x^{w}_{jist} + \tau^{h}_{jst} x^{h}_{jist}\right) \le H^{S}_{js} z_{ji}, \quad j\in J, s\in S,$$

$$\sum_{s\in S}\sum_{t\in T}\left(\tau^{w}_{jst} x^{w}_{jist} + \tau^{h}_{jst} x^{h}_{jist}\right) \le H^{Y}_{j} z_{ji}, \quad j\in J,$$

where index i indicates individual hydroelectric plants only.

The constraints on the maximal charge of pumped storage power plants:

$$u^{w}_{jst} \le G_{js} z_{ji}, \quad j\in J, s\in S, t\in T,$$

$$u^{h}_{jst} \le G_{js} z_{ji}, \quad j\in J, s\in S, t\in T.$$

The constraints on the output of pumped storage power plants with respect to the efficiency of discharge:

$$\sum_{t\in T} x^{w}_{jist} \le q_{j} \sum_{t\in T} u^{w}_{jst}, \quad j\in J, s\in S,$$

IOP Conf. Series: Materials Science and Engineering **537** (2019) 062003 doi:10.1088/1757-899X/537/6/062003

$$\sum_{t\in T} x_{jist}^{h} \le q_j \sum_{t\in T} u_{jst}^{h}, \quad j\in J, s\in S,$$

and with respect to the maximal number of operating hours in a day:

$$\sum_{t\in T} x_{jist}^{w} \le H_j^D z_{ji}, \quad j\in J, s\in S,$$

$$\sum_{t\in T} x_{jist}^{h} \le H_j^D z_{ji}, \quad j\in J, s\in S,$$

where i indicates pumped storage power plants only.

Balance equations (5), (6) equalize the total amount of electricity available in a node with the demand in that node. Available power is formed by the generation in the node plus incoming flows from the adjacent nodes minus outgoing power transmitted to other nodes and minus the power spent for charging the pumped storage plants of this node. Incoming flows always enter with some power loss defined by the coefficients $\delta_{jj'}$. Values of $\delta_{jj'}$ are usually from 2 to 7%. The feasibility constraints (7), (8) on operating powers set a limit for the amount of electricity that can be generated depending on the installed capacity and the operating limit rates. It is assumed that plants are always in operational condition. For a majority of generation types, it implies that the lower power output may not be equal to zero. The balance equations (9) ensure the reliability of the system. These relations claim that, with some reserve r_{jst}, the capacities of the system are able to meet the demand in the peak hours. Coefficients r_{jst} are usually set to be 20-22%. According to (10)-(12), the power flow at every hour is bounded from above by the corresponding power line capacity.

With the given data on the six countries of the Northeast Asian region, the problem (1)-(4) has 42000 variables and 56000 constraints in the case of the whole cooperation, when J consists of all nodes of all countries under consideration.

3. Some basic notions of the cooperative game theory

In a cooperative game [9], players may form coalitions. It would be natural to expect that such integrations benefit their participants. We would think that if there is a lack of energy, grid development and transmission of electricity from existing power plants of another country would be less expensive than expansion of the country's own capacities. Indeed, real data indicate that costs of installing new generation capacities significantly exceed costs of installing new power lines and transmitting energy. More precisely, power line investments lie in the interval of \$180-950 per kW, whereas investments to generation capacities reach \$800-8500 per kW. The most costly investments to both generation and lines relate to Japan due to geographical reasons. Thus, any power grid coalition results in non-increasing of the total cost. Since a coalition acts as a single player, the coalition's surplus should be allocated over its participants. Let us discuss reasonable methods of allocating the surplus over a coalition.

Let $N = \{1, 2, \dots, n\}$ be the set of players. Every nonempty subset $K \subseteq N$ is called a coalition. Set N is referred to as the grand coalition. The characteristic function v: $\{K \mid K \subseteq N\} \to \mathbb{R}$ is a function that assigns to every coalition $K \subseteq N$ an attainable payoff $v(K)$ such that $v(\emptyset) = 0$. Normally, v satisfies a superadditivity condition

$$v(K)+v(T)\le v(K\cup T) \text{ for every } K\subseteq N, T\subseteq N, K\cap T=\varnothing. \tag{13}$$

Condition (13) is met for an electric power integration due to the properties discussed above. The pair (N, v) is called a cooperative game (in characteristic function form). An imputation is a vector $x \in \mathbb{R}^n$ satisfying the efficiency condition

$$\sum_{i\in N} x_i = v(N) \tag{14}$$

and the individual rationality condition

$$x_i \geq v(\{i\}) \text{ for every } i \in N. \tag{15}$$

Imputation x dominates another imputation, not equal to x, if there exists a nonempty coalition $K \subset N$ such that

$$\sum_{i \in K} x_i \leq v(K),$$

$$x_i > y_i \quad \text{for every } i \in K.$$

If v is superadditive, then the Core is the set of undominated imputations [9]. An imputation x is an element of the Core if and only if

$$\sum_{i \in K} x_i \geq v(K) \text{ for every } K \subset N. \tag{16}$$

Obviously, the Core is a closed convex set determined by linear inequalities. The feature of a Core allocation is that it discourages any participant or a group of participants to leave the grand coalition and form smaller coalitions. For a Core allocation, smaller coalitions do not provide any additional benefit. Note that in general, a Core may turn out to be an empty set.

The Shapley value is an imputation defined as

$$x_i = \sum_{K: i \in K \subseteq N} \frac{(|K|-1)!(n-|K|)!}{n!} \left(v(K) - v(K \setminus \{i\}) \right), \ i \in N. \tag{17}$$

Here $|K|$ denotes the number of elements in set K. The Shapley value is a unique imputation that always exists for every cooperative game [9]. Informally, the Shapley value may be referred to as a "fair" allocation, since it considers the marginal contribution of a participant to every feasible coalition.

Both the Core and the Shapley values have a combinatorial nature since one needs to enumerate all possible coalitions in order to define these imputations.

4. Main results

In order to define a cooperative game, we need to specify the characteristic function value $v(K)$ for every coalition K. Value $v(K)$ represents the maximal payoff that may be guaranteed for K. As we mentioned before, building up capacities is significantly more expensive than generating and transmitting electricity. Hence, the guaranteed payoff of a coalition is defined by the coalition's minimal cost in the case of its isolated functioning. In that way, countries within a coalition can transmit energy to each other, but they cannot receive electricity from countries that are not in the coalition. It is assumed that every coalition is feasible. Consequently, we need to solve $(2^{|N|} - 1)$ linear programming problems (1)-(4) of various dimensions to specify $v(K)$ for every coalition $K \subseteq N$. Since our basic model is formulated as a cost minimization problem, we consider $v(K)$ as the loss of coalition K, and imputation x allocates the coalition's cost rather than its profit or surplus. In this case, the corresponding changes in the inequalities (13), (15), (16) should be made.

So, $v(K)$ is the optimal value of the objective (1)-(3), where J is the set of all nodes belonging to the countries of coalition K. On the other hand, for convenience, we can solve the problem (1)-(4) with all nodes included in J along with the assumption that coalitions K and $N \setminus K$ are isolated from each other. Isolation of coalitions is provided by the corresponding changes in the matrix A. A solution of this problem implies values $v(K)$ and $v(N \setminus K)$ simultaneously. Since $|N| = 6$, the game has 63 coalitions including the grand coalition N and the singleton coalitions. Thus, one needs to solve $\frac{2^{|N|}}{2} = 32$ full-dimensional problems (1)-(4) providing the following 32 pairs:

$$\left(v(N), v(\varnothing)\right), \left(v(N \setminus \{1\}), v(\{1\})\right), \left(v(N \setminus \{1,2\}), v(\{1,2\})\right), \ldots$$

IOP Conf. Series: Materials Science and Engineering **537** (2019) 062003 doi:10.1088/1757-899X/537/6/062003

We have solved these 32 linear programming problems by IBM ILOG CPLEX solver [10].

For our cooperative game, the Core (14)-(16) is a non-empty set with infinite number of elements. This case gives us an opportunity to optimize over the Core imputations with some additional criterion, what may be used in further investigation of the power integration problem. Assign the numbers 1 to 6 to Russia, Mongolia, China, North Korea, South Korea, and Japan respectively. The Core has the following form:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 333001,$$
$$x_1 + x_2 + x_3 + x_4 + x_5 \le 203847,$$
$$x_1 + x_2 + x_3 + x_4 + x_6 \le 286887,$$
$$x_1 + x_2 + x_3 + x_5 + x_6 \le 338509,$$
$$\ldots$$
$$x_4 \le 5046,$$
$$x_5 \le 57153,$$
$$x_6 \le 138441.$$

The right hand sides in these relations are exactly the values of characteristic function expressed in millions of dollars.

Let us give some examples of imputations belonging to the Core. Since the Core is a polyhedron, we suggest to compute the Chebyshev point of the Core first. The Chebyshev point is the center of the maximal sphere that can be inscribed into the polyhedron. Informally, this center is equidistant from the linear constraints. Table 1 describes the Chebyshev center of the Core. The column “Isolated” represents the minimal costs for a country when it does not cooperate with any of the other countries. The last two columns reflect the change in costs when countries cooperate in the grand coalition with the given imputation, as compared to isolated functioning.

Table 1. The Chebyshev center of the Core.

	Imputation (costs)		Isolated	Integration effect	
	$ million	%	$ million	%	$ million
Russia	7353	2.21	7591	-3.14	-239
Mongolia	670	0.20	909	-26.26	-239
China	135301	40.63	147757	-8.43	-12455
North Korea	0	0	5047	-100.00	-5047
South Korea	54239	16.29	57153	-5.10	-2915
Japan	135438	40.67	138441	-2.17	-3004
Total	333001	100.00	356899	-6.70	-23898

It is worth to note that North Korea bears no cost in the Chebyshev point. Moreover, the relative integration effects (the 5th column) differ dramatically. Instead, we have generated 30 random points from the Core and have chosen an imputation among them in such a way that the relative integration effects are approximately equal. This imputation is presented in table 2.

Table 2. An element of the Core.

	Imputation(costs)		Isolated	Integration effect	
	$ million	%	$ million	%	$ million
Russia	6853	2.06	7591	-9.72	-738
Mongolia	837	0.25	909	-7.95	-72
China	135801	40.78	147757	-8.09	-11956
North Korea	4662	1.40	5047	-7.62	-385

South Korea	52574	15.79	57153	-8.01	-4580
Japan	132274	39.72	138441	-4.45	-6167
Total	333001	100.00	356899	-6.70	-23898

It is noticeable that for our problem the Shapley value (17) turns out to be an element of the Core. Consequently, it inherits the features of the Core imputations in addition to its own properties which were briefly discussed above. Information on the Shapley value is gathered in table 3.

Table 3. The Shapley value.

	Imputation(costs)		Isolated	Integration effect	
	$ million	%	$ million	%	$ million
Russia	3867	1.16	7591	-49.07	-3725
Mongolia	350	0.11	909	-61.46	-559
China	141328	42.44	147757	-4.35	-6429
North Korea	527	0.16	5047	-89.56	-4520
South Korea	52578	15.79	57153	-8.01	-4575
Japan	134351	40.35	138441	-2.95	-4090
Total	333001	100.00	356899	-6.70	-23898

Tables 1, 2, and 3 show that imputations provide decreasing of cost for each country comparing with isolated playing. In other words, the individual rationality condition (15) holds.

5. Effects of the grand coalition

In this section, we will briefly discuss effects of the grand coalition as compared to the isolated operation. These results do not depend on a particular imputation and characterize the changes that occur in the integrated power system when the national power systems cooperate in the grand coalition. The effects are presented for the individual countries as well as for the whole coalition.

The capacity effect describes the amount of new generation capacities introduced in the system (see table 4). The second column of the table contains initial capacities, the third and the forth ones show the introduced capacities for the isolated case, and the last two columns relate to the grand coalition.

Table 4. New generation capacities.

	Initial	Isolated work		Grand coalition	
	GW	Δz, GW	Δz, %	Δz, GW	Δz, %
Russia	71.0	3.9	5.56	4.9	6.96
Mongolia	1.9	2.2	113.49	0	0
China	449.0	301.9	67.25	264.3	58.86
North Korea	10.9	8.1	74.31	0.3	2.71
South Korea	127.7	22.1	17.29	21.3	16.67
Japan	210.1	30.4	14.48	10.9	5.21
Total	870.6	368.7	42.35	301.7	34.66

Note that the grand coalition introduces less amount of new capacities than the isolated countries do in total. Mongolia, when in the grand coalition, does not install any new capacities at all receiving power from other countries in the case of a power shortage.

When countries are isolated, the capacities of the international power lines are equal to zero. In the case of the grand coalition, 85.9 GW of new power lines appear in the system. It totals to $39,278 million. Table 5 presents the line capacities between the possible pairs of the countries. A dash means that no line between those countries is allowed ($a_{jj'} = 0$).

Table 5. Power lines in the grand coalition (GW).

	Russia	Mongolia	China	N. Korea	S. Korea	Japan
Russia		9.3	16.3	1.5	–	5.3
Mongolia	9.3		8.5	–	–	–
China	16.3	8.5		15	–	–
North Korea	1.5	–	15		15	–
South Korea	–	–	–	15		15
Japan	5.3	–	–	–	15	

In table 6, we show the annual amount of the generated power. Table 7 contains the values of the annual operating costs (the fuel effect). The last column in tables 6, 7 shows the differences between the amount of power generated within the grand coalition and in the course of isolated operation. The bottom line of table 6 indicates that more energy were generated in the coalition than during the isolated work. This is caused by the transmission losses defined by coefficients $\delta_{jj'}$. China and Russia are the only countries that have increased the annual power output in the coalition. Also, it is worth to note that Japan is the largest power importer due to its high demand in electricity. The lack of initial generation capacities results in a power shortage. This fact along with the high capital investments in Japan explains the large total cost for this country. China plays the role of the largest power exporter in the coalition.

Table 6. Annual power generation.

	Isolated work, GWh	Grand coalition, GWh	Δ, %
Russia	421816	434810	3.08
Mongolia	19632	13304	-32.23
China	3845569	4016314	4.44
North Korea	78062	50030	-35.91
South Korea	792851	786193	-0.84
Japan	1139897	1032821	-9.39
Total	6297826	6333473	0.57

Table 7. Annual operating costs (fuel effect).

	Isolated work, $ million	Grand coalition, $ million	Δ, %
Russia	4911	5409	10.14
Mongolia	436	305	-30.10
China	82988	89085	7.35
North Korea	2782	1279	-54.01
South Korea	36734	36272	-1.26
Japan	88171	74359	-16.18
Total	216569	206710	-4.55

6. Conclusion

We considered the power grid interconnections in the Northeast Asian region. The countries under consideration were Russia, Mongolia, China, North Korea, South Korea, and Japan. Since the problem of dividing the cooperation benefit between countries gains not much attention in the literature, we proposed a game theory approach to allocation of the total surplus of the coalition over the participants. The study is entirely based on the ORIRES model. This model accounts for the development of generation capacities and transmission lines in the year 2030. The corresponding cooperative game in characteristic function form was determined. In order to specify the characteristic function we solved 32 linear programming problems with 42000 variables and 56000 constraints. We specified two allocation methods satisfying some reasonable conditions: the Core and the Shapley value. The Core turned out to be a nonempty set with infinite number of elements. This fact inspires our future work. It is worth to suggest and examine some additional conditions for a Core allocation. For example, an allocation may be chosen as a solution of an optimization problem with an expertly defined objective function over the Core. However, such a problem would be valid only in the case of a nonempty Core. If the Core is empty for some input data, one may use the Shapley value as an allocation, since this imputation always exists.

Acknowledgements
This work is a part of the project III.17.3.1 of the basic research program of Siberian Branch of the Russian Academy of Sciences, no. AAAA-A17-117030310444-2. The author would like to thank the anonymous reviewers for their valuable comments.

References
[1] Belyaev L S, Podkovalnikov S V, Saveljev V A and Chudinova L Y 2008 *Efficiency of International Electric Connections* (Novosibirsk: Nauka)
[2] Podkovalnikov S V and Khamisov O V 2011 *Bulletin of Russian Academy of Sciences. Energy* **2** 57-76
[3] Otsuki T, Mohd Isa A B and Samuelson R D 2016 *Energ. Pol.* **89** 311-29
[4] Bogdanov D and Breyer C 2016 *Energ. Convers. Manag.* **112** 176-90
[5] Belyaev L S, Chudinova L Y, Khamisov O V, Kovalev G F, Lebedeva L M, Podkovalnikov S V and Savelyev V A 2002 *Int. J. Global Energy Issues* **17** 228-49
[6] Chung K H and Kim B H H 2007 *Journal of Electrical Engineering and Technology* **2** 452-60
[7] Podkoval'nikov S V, Savel'ev V A, Khamisov O V and Chudinova L Y 2018 *Autom. Rem. Cont.* **79** 1756-66
[8] Gakh S 2016 *Suppl. Proc. of the 9th Int. Conf. on Discrete Optimization and Operations Research and Scientific School (DOOR 2016)* 606-17
[9] Gilles R P 2010 *The Cooperative Game Theory of Networks and Hierarchies* (Berlin Heidelberg: Springer-Verlag)
[10] IBM ILOG CPLEX Optimization Studio Available from https://www.ibm.com/analytics/cplex-optimizer