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Nonlinear endochronic creep models taking into account the type of stress state

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Abstract. In the framework of the endochronic approach, two variants of the tensor-parametric nonlinear constitutive relations of the creep theory are proposed. These equations take into account the dependence of the materials behavior on the type of stress state, as well as large deformations and rotations. The constitutive equations of proposed models are applied to explain some effects in the classic creep. It is shown that the results of simulating of uniaxial isothermal creep are in agreement with experimental observations. Presented examples demonstrate the possibilities and a potential of the approach.

1. Introduction

Endochronic theory in integral form was proposed by Valanis [1] and was applied for describing of creep phenomena by some researchers [2-5]. In these studies, the deformations were considered as small. In the field of large deformations, the Valanis endochronic theory was used to solve the problems of viscoelasticity [6], elastoplasticity [7] and viscoplasticity [8].

The experimental data [9, 10] show that creep deformations of some materials may depend on the type of the stress state. Some theoretical results that describe such experiments in the framework of a geometrically linear incremental theory were presented in [11-13]. Endochronic theory of plasticity in Valanis's form accounting for the type of the stress state was proposed in [14].

In our paper, the endochronic theory of inelasticity in differential form [15] takes into account the type of the stress state and generalizes the constitutive equations for the finite deformations.

2. Endochronic creep theory accounting the stress state type

Let σ_{ij} is the tensor of active stress then $\sigma_{ij}^{'} = \sigma_{ij} - (\sigma_{ii}\delta_{ij})/3$ is its deviator, σ_2, σ_3 and ξ are the second, the third invariants and the angle of the type of the stress state, correspondently, so as

$$\sigma_2 = \sigma_{ij}^{'}\sigma_{ij}^{'}, \qquad \sigma_3 = \sigma_{ij}^{'}\sigma_{jk}^{'}\sigma_{ki}^{'}, \qquad \sin 3\xi = -\sqrt{6}\,\sigma_3 \cdot \sigma_2^{-3/2}, \qquad -\pi/6 \le \xi \le \pi/6.$$

If the material is loading by uniaxial tension, the torsion or uniaxial compression then $\xi = -\pi/6$, $\xi = 0$ and $\xi = \pi/6$, respectively.

By analogy with the incremental theory [13], the deviator of transformed stress is introduced for accounting of the type of the stress state

$$\sigma_{ij}^{*} = \sigma_{ij}^{'} \cdot \phi(\xi),$$ (1)

where a function $\varphi(\xi)$ shows the deviation of active stresses from shear stress when $\phi(\xi) = 1$.

Using these transformed stresses, the differential constitutive equations of endochronic theory [15] are formulated now in the physical nonlinear form

$$\frac{\sigma_{ij}^{*}}{2G} + \frac{\alpha\tau}{2G}\frac{d\sigma_{ij}^{*}}{dr} = \tau\frac{dr_{ij}}{dr} + \frac{r_{ij}}{g+\alpha}, \qquad K\,\varepsilon_{ii} = \sigma_{ii},$$

(2)

$$r_{ij} = \varepsilon_{ij}^{'} - (1-\alpha)\frac{\sigma_{ij}^{'}}{2G}, \qquad dr = \sqrt{dr_{ij} : dr_{ij}}, \qquad r^{\bullet} = \sqrt{\frac{dr_{ij}}{dt} : \frac{dr_{ij}}{dt}},$$

$$\tau = \tau(r, r^{\bullet}), \qquad g = g(r, r^{\bullet}), \qquad r = \int dr.$$

Here ε_{ii} and $\varepsilon_{ij}^{'}$ denote the volumetric and deviatoric parts of the strain tensor, r_{ij} is the deviator of the parametric tensor, τ is an analogy of the strain yield point, g is an analogy of the hardening factor of the material, α is the endochronic parameter ($0 \le \alpha \le 1$), G is the shear modulus, K is the bulk modulus, t is the physical time, the sign «:» is the double contraction of the tensor product.

If the endochronic parameter $\alpha = 1$ then the parametric tensor r_{ij} and its invariants r and $r^{\bullet}$ degenerate into the strain tensor ε_{ij} with invariants ε and $\varepsilon^{\bullet}$. Therefore, the constitutive relations (2) (while maintaining endochronic characteristics) take the form

$$\frac{\sigma_{ij}^{*}}{2G} + \frac{\tau}{2G}\frac{d\sigma_{ij}^{*}}{d\varepsilon} = \tau\frac{d\varepsilon_{ij}^{'}}{d\varepsilon} + \frac{\varepsilon_{ij}^{'}}{g+1},$$ (3)

$$d\varepsilon = \sqrt{d\varepsilon_{ij}^{'} : d\varepsilon_{ij}^{'}}, \qquad \varepsilon^{\bullet} = \sqrt{\frac{d\varepsilon_{ij}^{'}}{dt} : \frac{d\varepsilon_{ij}^{'}}{dt}},$$

$$\tau = \tau(\varepsilon, \varepsilon^{\bullet}), \qquad g = g(\varepsilon, \varepsilon^{\bullet}), \qquad K\,\varepsilon_{ii} = \sigma_{ii}.$$

Multiplying and dividing the equation (3) by dt, we yield

$$\frac{\sigma_{ij}^{*}}{2G} + \frac{1}{2G}\frac{d\sigma_{ij}^{*}}{dt}\frac{\tau}{\varepsilon^{\bullet}} = \frac{d\varepsilon_{ij}^{'}}{dt}\frac{\tau}{\varepsilon^{\bullet}} + k\cdot\varepsilon_{ij}^{'}.$$ (4)

Here $1/(g+1)$ is denoted by k.

Material functions $\tau = \tau(\varepsilon, \varepsilon^{\bullet})$ and $g = g(\varepsilon, \varepsilon^{\bullet})$ are defined from experiments according to the history of loading at different strain rates. Within the theory of elastoplastic processes and the incremental theory, a dependence of the strain yield point τ from intensity of the strain rate $\varepsilon^{\bullet}$ usually is approximated by the functions: $\tau = a + \tau_0 \cdot \varepsilon^{\bullet}$, $\tau = a + \tau_0 \cdot (\varepsilon^{\bullet})^{\delta}$, $\tau = a + \tau_0 \cdot \ln(1 + b\cdot\varepsilon^{\bullet})$, etc. Note that all of the aforesaid is relating to the strain rates are not above $\varepsilon^{\bullet} \sim 1$ sec^{-1}, i.e. quasi-static loading.

Suppose that material hardening doesn't depend on the time and loading history in the inelastic deformation and assume that $g = const$. For the parameter τ the approximation $\tau = \tau_0 \cdot (\varepsilon^{\bullet})^{\delta}$ is chosen. Then relation (4) will be changed into

$$\frac{\sigma_{ij}^{*}}{2G} + \frac{1}{2G}\frac{d\sigma_{ij}^{*}}{dt}\tau_0\cdot(\varepsilon^{\bullet})^{\delta-1} = \frac{d\varepsilon_{ij}^{'}}{dt}\tau_0\cdot(\varepsilon^{\bullet})^{\delta-1} + k\cdot\varepsilon_{ij}^{'}.$$ (5)

For using the equations of model (5) we need to know material constants G, K, τ_0, g, δ and function $\varphi(\xi)$. To identify them, it is necessary to conduct torsion tests on thin tubular samples at three different constant strain rates as well as tension, compression and torsion tests at the same strain rate. In particular, the function $\varphi(\xi)$ can be approximated, for example, as [15]

$$\varphi(\xi) = \begin{cases} \exp[k_1(\cos 6\xi - 1)], & -\pi/6 \le \xi \le 0 \\ \exp[k_2(\cos 6\xi - 1)], & 0 \le \xi \le \pi/6 \end{cases}. \tag{6}$$

3. Geometrical nonlinear variant of the endochronic creep theory

Now by analogy with above and as per recommendations [17], the geometrical nonlinear endochronic variant of constitutive equations for materials with sensitivity to the stress state view is presented.

$$\frac{\sigma_{ij}^*}{2G} r^\bullet + \frac{\alpha\tau}{2G} \overset{\circ}{\sigma}_{ij}^* = \tau \overset{\circ}{r}_{ij} + \frac{r_{ij}}{g+\alpha} r^\bullet, \qquad K\varepsilon_{ii} = \sigma_{ii}, \tag{7}$$

$$\tau = \tau(r, r^\bullet), \qquad g = g(r, r^\bullet), \qquad G = G(r, r^\bullet),$$

$$\overset{\circ}{r}_{ij} = D_{ij} - (1-\alpha)\frac{\overset{\circ}{\sigma}_{ij}^*}{2G}, \qquad \overset{\circ}{\varepsilon}_{ij} = D_{ij}, \qquad \Omega_{ij} = R_{ij}^\bullet R_{ij}^T,$$

$$\overset{\circ}{\sigma}_{ij}^* = \overset{\bullet}{\sigma}_{ij}^* + \sigma_{ij}^* \Omega_{ij} - \Omega_{ij}\sigma_{ij}^*, \qquad \overset{\circ}{\varepsilon}_{ij} = \overset{\bullet}{\varepsilon}_{ij} + \varepsilon_{ij}\Omega_{ij} - \Omega_{ij}\varepsilon_{ij}.$$

Here $\overset{\circ}{\sigma}_{ij}$, $\overset{\circ}{\varepsilon}_{ij}$ denote the objective derivatives of the deviators of the stresses and deformations tensors, respectively. Further, Ω_{ij} is the spin tensor, a proper orthogonal matrix R_{ij} is the rotation tensor, D_{ij} is the deformation-rate tensor (D_{ij} is the symmetric part of the velocity gradient L_{ij}),

$$R_{ij} = F_{ij} U_{ij}^{-1}, \qquad L_{ij} = F_{ij}^\bullet F_{ij}^{-1}, \qquad D_{ij} = (L_{ij} + L_{ij}^T)/2, \tag{8}$$

F_{ij} is the deformation gradient, the symmetric positive-definite tensor U_{ij} is the right-stretch tensor in the polar decomposition of the deformation gradient $F_{ij} = R_{ij} U_{ij}$, the upper sign «**T**» is the operation of matrix transposition.

Actually, equations (7)-(8) take into account both physical and geometrical nonlinearity in the constitutive relations of the inelasticity.

We will note some obvious facts:

- relations $\overset{\circ}{\varepsilon}_{ij} = D_{ij}$ are equations for determination of deformations, i.e. nonholonomic deformation measure is caused by neutral corotational derivative of Green-Naghdi type

$$\varepsilon = R \left(\int R^T D R \, dt \right) R^T; \tag{9}$$

- if $\phi(\xi) = 1$, $\alpha = 1$, $2G = 1$ and $\tau = k_0 \cdot \varepsilon^\bullet$, then equations (7) take a form of generalized Maxwell model [16]

$$k_0 \overset{\circ}{\sigma}_{ij} + \sigma_{ij} = k_0 \overset{\circ}{\varepsilon}_{ij} + k\varepsilon_{ij}, \qquad \overset{\circ}{\varepsilon}_{ij} = D_{ij}; \tag{10}$$

- from equations (10) it is clear that both axial deformations and shear ones are developed under the uniaxial loading, which is not appear in geometrical linear theories.

4. Simulation of creep strains

To demonstrate some possibilities of the presented approach, we consider the classical isothermal creep problem when the stresses are constant.

4.1. Modelling at small strains

Within model (5) suppose that $\sigma_{ij} = const$ then the type of the stress state doesn't change. Hence

$$\xi = const, \quad \sigma_{ij}^* = const, \quad d\,\sigma_{ij}^* \big/ dt = 0, \quad \varepsilon_{ij}^{'c} = \varepsilon_{ij}^{'} - \varepsilon_{ij}^{'e}, \quad \varepsilon_{ij}^{'e} = \sigma_{ij}^{'} \big/ E,$$

where $\varepsilon_{ij}^{'e}$, $\varepsilon_{ij}^{'c}$ are the deviators of elastic strain tensor and creep one, E is the Young modulus. Then relations (5) take a form

$$\frac{d\varepsilon_{ij}^{'}}{dt} = \frac{1}{\tau_0 \cdot (\varepsilon^\bullet)^{\delta-1}} \cdot \left(\frac{\sigma_{ij}^*}{2G} - k \cdot \varepsilon_{ij}^{'} \right). \qquad (11)$$

Figure 1 presents the computational results of intensity creep strain made accordingly to equations (11) at equal stress under tension (dashed line), torsion (solid line) and compression (dash dot line).
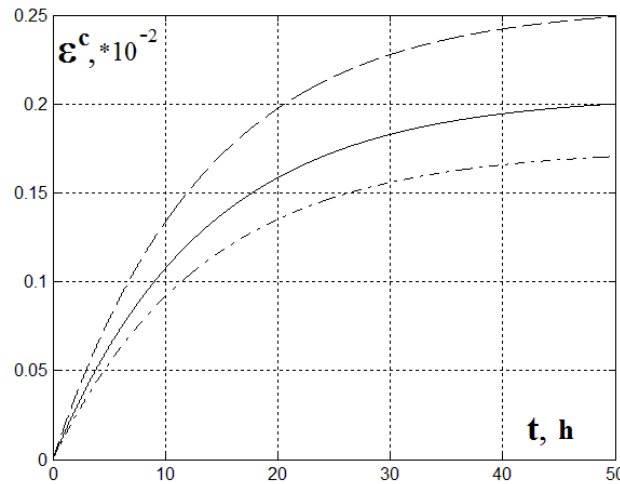

Figure 1. Creep strain intensity versus the time.

Calculations were performed at $2G = 195 GPa$, $\sigma = 400 MPa$, $k = 0,01$, $\tau_0 = 3,25 \cdot 10^{-3}$, $\delta = 2,65 \cdot 10^{-2}$. Function $\varphi(\xi)$ was taken in form (6) with $k_1 = -0,12$ and $k_2 = 0,08$.

Figure 1 demonstrates:

- the splitting of curves «creep strain ~ time» in depence on the type of the stress state;
- the existence of the first and the second stages of creep strain;
- the absence of the horizontal asymptotic values of creep.

4.2. Calculation of creep in finite deformations field

For the study of creep in the inelastic endochronic theory at large deformations (7), the deformation gradient F_{ij} is chosen in following form

$$F_{ij} = \begin{pmatrix} k_{11} & k_{12} & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \end{pmatrix}. \tag{12}$$

This type of the deformation gradient generates the orthogonal rotation tensor R_{ij} and the spin tensor Ω_{ij} with the next structures

$$R_{ij} = \begin{pmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \Omega_{ij} = \dot{\beta} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad tg\,\beta = \frac{k_{12}}{k_{11}+k_{22}}. \tag{13}$$

Using relations (8) and (12), the components of the tensor D_{ij} are connected with the components of the deformation gradient F_{ij} by the differential equations

$$\dot{k}_{11} = k_{11}\,D_{11}, \qquad \dot{k}_{22} = k_{22}\,D_{22}, \qquad \dot{k}_{33} = k_{33}\,D_{33}, \qquad \dot{k}_{12} = 2\,k_{22}\,D_{12} + k_{12}\,D_{11}. \tag{14}$$

Then substituting relations (13) into the objective derivatives of stress and deformation deviators and into formulas (7), we yield closed system of ordinary differential equations.

The isothermal creep strains in time were calculated in the frameworks of model (7), (8), (12) – (14) at the constant shear stress $\sigma_{12} = 400\,MPa$ and $\phi(\xi) = 1$. A development of the creep strain intensity ε^c versus time is demonstrated in figure 2. The deformations from geometrically nonlinear equations (dashed line) are greater than ones obtained by the linear relations (solid line) as it is observed in experiments [9], [16], [18].
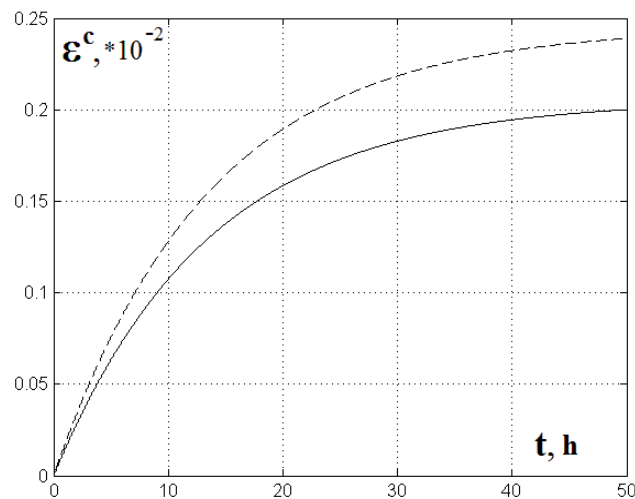

Figure 2. Creep curves under constant shear stress.

5. Conclusions

Proposed variants of the endochronic constitutive equations take into account the type of the stress state, large deformations and rotations. Solutions of the classical creep problems demonstrate the possibilities and a potential of these relations. Creep simulation results don't contradict experimental observations.

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