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Modelling of the technological process of multiple filtering suspensions with multi-layered filter

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Abstract. Although a technological process of filtration with multi-layered filters is well-known in industry (purification of such liquid solutions as spinning solutions, liquid fuels, drinking liquids, pharmaceuticals, etc.), very few studies have systematically quantified its mechanism, and even less have characterized the affect of its parameters on operating costs and the quality of the final product. In this study, there was developed a mathematical model of multiple filtration of suspensions with multi-layered filter, a computational algorithm for solving the problem as well as software for carrying out computational experiments. The developed mathematical software allows to carry out many-sided study and forecast the technological process of filtration in order to make appropriate management decisions.

1. Introduction

One of the main problem of complex non-stationary technological processes of filtering of liquid mixtures is to provide a greater perfomance of filtering machines taking into account the physicomechanical and chemical properties of mixtures, their composition, operating modes of the aggregates etc. In this regard, there is a necessity to identify possible ways to improve the considered technological processes on the basis of mathematical models and computational experiments (CE).

Wrong choice of the operating mode of the filtering units and technological regimes leads to significant losses of valuable raw materials and quality of the final product. The efficiency of the aggregates is determined by the state of the filter barriers, by means of which the solid phase particles or unwanted ions are separated from the liquid. The quality of the final product during the filtration process depends on the stability of the filtering arches with respect to the pressure inside the filter column.

Thus, the problem of research, forecasting and management of technological process (TP) is quite relevant. Therefore, it is necessary to analyze the processes with the help of effective methods and tools. One of the effective tools which provides ability to explore and manage technological processes is the mathematical models implemented in the form of software and algorithmic tools.

For nowdays, many researches have been carried out on TP and significant theoretical and practical results have been obtained. These problems were considered and solved by such scientists as Jing Wang, J.Zhang, Yu.Zhihui, B.Remy, J.W.Dufty, Duygu Kocaefe, Carlos Andre, Gitisa Vitaly, Andrii Safonyk and others.

In the works [1, 2] the issues of taking into account the reverse effect of the technological characteristics of the process (concentration of contamination of the liquid and sediment) and characteristics of the medium (coefficients of porosity, filtration, diffusion, mass transfer, etc.) are presented and solved by the example of cleaning the liquid in magnetic and sorption filters. An algorithm for the numerically asymptotic approximate solution of the corresponding model problems, which are described by a system of nonlinear singular differential equations of the "convection-diffusion-mass exchange" type, is presented. The results of the study of the complex mechanisms of sedimentation of particles in a filter candle are given in [2]. In order to study the filtering process and its modeling, the authors conducted filtration experiments with a suitable particle of an oil suspension in an experimental filter. While some filtering depth occurs at the beginning of the filter's service life, blocking and cake filtering layers are the main mechanisms responsible for filter clogging. In the research [3] developed a phenomenological model of deepwater infiltration. The proposed mathematical model is combined with the equation of adjectival dispersion and the nonlinear equation of the kinetics of the considered TP. The mathematical model (MM) is solved numerically using an explicit finite-difference scheme. The results obtained are compared with field experiments on the EPA facilities, produced by the Israeli company Mekorot.

The authors of the article [4] cosidered the task of filtering a two-particle suspension through a porous medium. A model is proposed based on the laws of conservation of mass for particles and for a liquid, as well as local laws of capture of particles described by kinetic equations. In contrast to the well-known model for particles of the same type, this model allows to take into account differences in the physical properties of particles (for example, their size).

The equations describing the flow of a weakly compressible fluid in a weakly deformable porous skeleton with a nonlinear filtration law with a limiting (initial) pressure gradient were analyzed in [5] and indicates a number of solutions expressed in elementary functions.

The theoretical principles of purification of low concentrated suspensions by the method of filtration, which are widely used in the technology of drinking water purification, are considered in [6]. A practical application of the theoretical foundations of filtering is the method of technological process modeling for solving the problem of improving the operation of filtering facilities at the water treatment plant in Astana. The results of studies to determine some of the technological parameters on a modular filter unit are presented.

2. Problem statement

Purification of public goods (for example drinking water) is performed by filtering it with multi-layered filters with different porosities and permeabilities. To derive the mathematical support of the process under consideration, we will use the basic laws of conservation of mass and momentum and kinetics of this phenomenon.
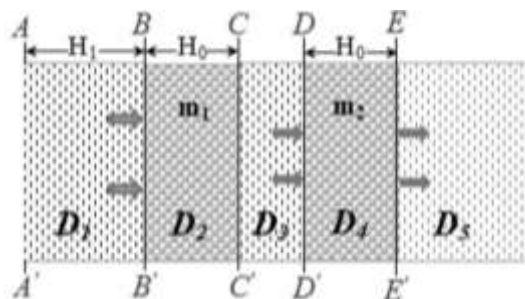

Figure 1. The design scheme of the two-layer filter.

As shown in figure 1, the filter partitions of the unit are empty parallel to each other. Areas $D^{(2)}$, $D^{(4)}$ occupied by a porous medium, and areas $D^{(1)}$ and $D^{(3)}$ - by filtered solution, moreover, the distance "BC" can be as small as desired, and in the case when "BC" is large enough, it should be considered as two single filtering and the mathematical model of the process is the same as in [7].

Let the filtrate, leaving the area, $D^{(2)}$ immediately falls into the area $D^{(4)}$, region $D^{(3)}$ so small that the change in pressure gradient is insignificant. If the filter barriers $D^{(2)}$ and $D^{(4)}$ at the same time they are an ion-exchange filter, then the ion-exchange filtering equations are written as [7]:

$$\left. \begin{array}{l} \dfrac{\partial W_j}{\partial t} + W_j \dfrac{\partial W_j}{\partial x} - \dfrac{W_j}{1-\theta_j^{(3)}} \dfrac{d\theta_j^{(3)}}{dt} = -\dfrac{\partial P_j}{\partial x} + \dfrac{\mu}{\rho} \dfrac{\partial^2 W_j}{\partial x^2} - \dfrac{W_j}{\rho HK_{0j}(1-\delta_j)^2}, \\[4mm] \dfrac{\partial \delta_j}{\partial t} = \lambda_j(\theta_j - \gamma_j \delta_j), \quad \dfrac{\partial \theta_j m_j}{\partial t} + \dfrac{\partial W_j \theta_j}{\partial x} = 0, \\[4mm] \dfrac{\partial \theta_j^{(3)}}{\partial t} = \dfrac{W_{0j}(1-\theta_j^{(3)})}{H_0(1-\theta_j^{(1)})}(\dfrac{1}{m_{1j}}-1) + \lambda_j(1-m_{oj})e^{\lambda_j\gamma_j t} - \int_0^{H_0}(\theta_j - \gamma_j \delta_j)dx, \end{array} \right\} \tag{1}$$

$$\left. \begin{array}{l} \dfrac{\partial n_{ij}}{\partial t} + \dfrac{\partial W_j n_{ij}}{\partial x} + \dfrac{\partial N_{ij}}{\partial t} = D_j \dfrac{\partial^2 n_{ij}}{\partial x^2}, \\[4mm] \dfrac{\partial N_{ij}}{\partial t} = \beta_j(n_{ij} - n'_{ij}), \\[4mm] N_{ij} = \dfrac{n'_{ij}}{a_j + b_j n'_{ij}}, \quad m_j = m_{ij} + \delta_j(m_{0j} - m_{ij}), \quad i=1,2; \ j=1,2. \end{array} \right\} \tag{2}$$

there W_j – filtration rate; P_j – pressure; ρ and μ – filtrate density and viscosity; K_{0j} – permeability coefficients; δ_j – the concentration of particles deposited in the pores of the filter; θ_j – concentration of substances in the moving mixture; $\theta_j^{(3)}$ – the concentration of particles flowing through the filter walls in areas $D^{(3)}$ and $D^{(5)}$ respectively; m_j – filter porosity; n_{ji} – non-equilibrium concentrations of exchanging ions in solution; N_{ji} – nonequilibrium concentrations of exchanging ions in the sorbent; nonequilibrium concentrations of exchanging ions in the sorbent; n'_{ji} – concentration of ions in solution, corresponding to equilibrium with concentration; N_{ji}; D_j – diffusion coefficients; β_j – effective rate constants of the exchanging ions; a_j and b_j – permanent isotherms.

The boundary conditions of the problem (1) - (2) are:

$$\left. \begin{array}{l} W_1 = W_0, \ W_2 = 0, \ \delta_1 = \delta_2 = 0, \\[2mm] \theta_1^{(3)} = \theta_2^{(3)} = 0, \ \theta_1 = \theta_0^{(1)}, \ \theta_2 = 0, \\[2mm] n_1 = n_2 = 0, \ N_1 = N_{10}, \ N_2 = N_{20}(t=0); \end{array} \right\} \tag{3}$$

on the border AA' (4), on the border BB' (5), on the border CC' (6), on the border DD' (7):

$$W_1 = W_0, \ \theta_1 = \theta_0^{(1)}, \ n_1 = 0, \ P_1 = 0; \tag{4}$$

$$\frac{\partial W_1}{\partial x} = a_1; \tag{5}$$

$$\left. K_2 \frac{\partial W_2}{\partial x}\right|_{x=H_0+0} = \left. K_1 \frac{\partial W_1}{\partial x}\right|_{x=H_0-0},$$

$$\left. m_2 \frac{\partial \theta_2}{\partial x}\right|_{x=H_0+0} = \left. m_1 \frac{\partial \theta_1}{\partial x}\right|_{x=H_0-0}, \Bigg\} \qquad (6)$$

$$\left. K_2 \frac{\partial P_2}{\partial x}\right|_{x=H_0+0} = \left. K_2 \frac{\partial P_1}{\partial x}\right|_{x=H_0-0};$$

$$\frac{\partial W_2}{\partial x} = a_2. \qquad (7)$$

3. Solution method

Problems (1) - (2) with conditions (3) - (7) are described by a system of nonlinear partial differential equations. Since this problem is solved analytically by difficulty, for numerical integration in system (1) - (7), we introduce dimensionless variables by the formulas:

$$\bar{W}_j = \frac{W_j}{W_0}, \ \bar{\delta}_j = \frac{\delta_j}{\theta_0^{(1)}}, \ \bar{\theta}_j = \frac{\theta_j}{\theta_0^{(1)}}, \ \bar{\theta}_j^{(3)} = \frac{\theta_j^{(3)}}{\theta_0^{(1)}}, \ \bar{P}_j^{(3)} = \frac{P_j}{P_0}, \ \bar{n}_j = \frac{n_j}{n_{10}},$$

$$\bar{N} = \frac{N_j}{N_{10}}, \ \tau = \frac{t}{\alpha_\tau}, \ \bar{x} = \frac{x}{H_0}, \ \alpha_\tau = \frac{\rho H K_1}{\mu H_0} \times 1\,hour$$

and omitting dashes above the dimensionless variables, we get:

$$\frac{\partial W_j}{\partial \tau} + \mathrm{Re}\, W_j \frac{\partial W_j}{\partial x} - \frac{W_j}{1-\theta_j^{(3)}} \frac{d\theta_j^{(3)}}{d\tau} = -Eu\,\mathrm{Re}\frac{\partial P_j}{\partial x} + \frac{HK_1}{H_0^3}\frac{\partial^2 W_j}{\partial x^2} - \frac{W_j}{(1-\theta_j^{(3)})(1-\delta_j)^2}, \Bigg\}$$

$$\frac{\partial \delta_j}{\partial \tau} = \lambda_j \alpha_\tau (\theta_j - \gamma_j \delta_j), \frac{\partial \theta_j m_j}{\partial \tau} + \mathrm{Re}\frac{\partial W_j \theta_j}{\partial x} = 0, \frac{\partial P_j}{\partial x} = \frac{1}{Eu_1}\frac{W_j}{(1-\sqrt{\delta_j})^3}, \frac{d\theta_j^{(3)}}{d\tau} = q_j(\tau), \Bigg\} \qquad (8)$$

$$\frac{\partial m_j n_{ij}}{\partial \tau} + \mathrm{Re}\frac{\partial W_j n_{ij}}{\partial x} + \frac{\partial N_{ij}}{\partial \tau} = P_{r\partial}\frac{\partial^2 n_{ij}}{\partial x^2}, \Bigg\}$$

$$\frac{\partial N_{ij}}{\partial \tau} = \beta_j \alpha_\tau \left(n_j - \frac{a_j N_{ij}}{1-b_j N_{ij}} \right), \Bigg\} \qquad (9)$$

with the following boundary conditions:
at $\tau = 0$

$$W_1 = 1, \quad W_2 = 0, \quad \delta_1 = \delta_2 = 0, \ \theta_1^{(3)} = \theta_2^{(3)} = 0,$$

$$\theta_1 = 1, \quad \theta_2 = 0, n_1 = n_2 = 0, \ N_1 = 1, \quad N_2 = \frac{N_{20}}{N_{10}}, \qquad (10)$$

on the border AA':

$$W_1 = 0, \ \theta_1 = 1, \ n_1 = 0, \ P_1 = 1, \qquad (11)$$

on the border BB':

$$\frac{\partial W_1}{\partial x} = \bar{a}_1, \tag{12}$$

on the border CC' :

$$\left.\begin{array}{r}
K_2 \left.\dfrac{\partial W_2}{\partial x}\right|_{x=1+0} = K_1 \left.\dfrac{\partial W_1}{\partial x}\right|_{x=1-0}, \\[2ex]
m_2 \left.\dfrac{\partial \theta_2}{\partial x}\right|_{x=1+0} = m_1 \left.\dfrac{\partial \theta_1}{\partial x}\right|_{x=1-0}, \\[2ex]
K_2 \left.\dfrac{\partial P_2}{\partial x}\right|_{x=1+0} = K_2 \left.\dfrac{\partial P_2}{\partial x}\right|_{x=1-0}.
\end{array}\right\} \tag{13}$$

there $Re = \dfrac{\rho K_0 W_0}{\mu H_0}$ - Reynolds number; $Eu = \dfrac{P_0 H K_0}{\mu H^2_0 W_0}, Eu_1 = \dfrac{P_0 K_0}{\mu H_0 W_0}$ - Euler number; $P_{r\partial} = \dfrac{Re}{Peg}$ -

Prandtl number; $Peg = \dfrac{D_j}{W_0 H_0}$ - Peclet diffusion number; $K_j = K_{0j}(1 - \sqrt{\delta_j})^3$.

The filtering equation (8) for each filter layer will be applied separately by a vector difference scheme [8]. For the numerical integration of the tasks using the Samara-Fryazinov vector scheme and introducing the notation

$$W_1 = \begin{pmatrix} U_{1i} \\ V_{1i} \end{pmatrix}, \quad \delta_1 = \begin{pmatrix} \delta_{1i} \\ \bar{\delta}_{1i} \end{pmatrix}, \quad \theta_1 = \begin{pmatrix} \theta_{1i} \\ \bar{\theta}_{1i} \end{pmatrix}, \quad \theta_1^{(3)} = \begin{pmatrix} \theta_{1i}^{(3)} \\ \bar{\theta}_{1i}^{(3)} \end{pmatrix}, \quad m_1 = \begin{pmatrix} m_{1i} \\ \bar{m}_{1i} \end{pmatrix}, \quad W_2 = W_{2i} = \begin{pmatrix} U_{2i} \\ V_{2i} \end{pmatrix}, \quad \delta_2 = \begin{pmatrix} \delta_{2i} \\ \bar{\delta}_{2i} \end{pmatrix},$$

$$\theta_1 = \begin{pmatrix} \theta_{2i} \\ \bar{\theta}_{2i} \end{pmatrix}, \quad \theta_2^{(3)} = \begin{pmatrix} \theta_{2i}^{(3)} \\ \bar{\theta}_{2i}^{(3)} \end{pmatrix}, \quad m_2 = \begin{pmatrix} m_{2i} \\ \bar{m}_{2i} \end{pmatrix},$$

together systems of equations (14), (15), we obtain, respectively, a system of equations describing the first and second stages of the solution filtration process through porous sphere.

4. Results and discussion
On the basis of the developed mathematical software, problems (1) - (7) were carried out a computational experiment. The calculation results are shown in figures 2-3.
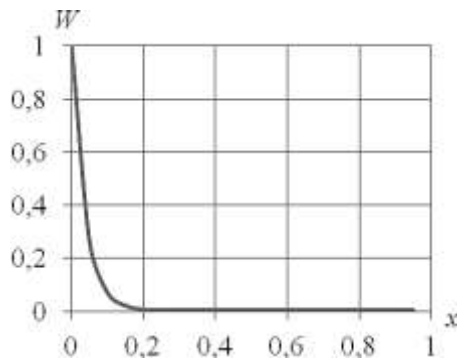

Figure 2. The changing of filtration rate by filter depth at the first filtration stage (t=3.85 h).
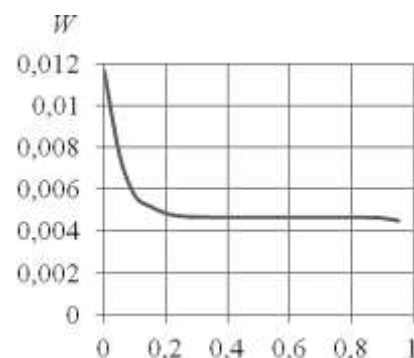
Figure 3. The changing of filtration rate by filter depth at the second filtration stage (t=3.85 h).

Numerical calculations showed that at the initial time t=0.01 h in the first stage, the filtration rate decreases sharply due to the formation of a sediment layer on the surface of the filter and filling the pores of the filter with gel particles. This is especially noticeable in the upper layer of the filter, and in the second stage, the filtration rate almost does not change on the upper layer of the filter. With t=3.85 h in the first stage, the same thing happens in the same way (figure 2); in the second stage, the filtering rate almost does not decrease, especially after the first half of the filter thickness (figure 3).

The performed numerical calculations on a computer showed that at the initial time of filtering (with t=0.01 h) Changes in concentration at both stages of the process have almost the same character. According to computer numerical calculations, the concentration of both stages of the filtering process is reduced by the depth of the filter septum. At filtering time t=3.85h changes in the concentration on the upper layers of the filter at both stages are reduced by the filter bus only.

The numerical calculations showed that the sedimentation rate of gel particles on the filter pore space at the initial stage of the process increases dramatically, and then it will decrease with time. in the second stage.

As can be seen from the analysis of numerical calculations of the deposition rate of the concentration of particles in the pore space of the filter with time will gradually decrease.

On the basis of the HE carried out, it was established that as a result of clogging of suspended particles in the pores of the ion filter, the rate of ion exchange and the switching time of the filter unit decrease with time. The theoretical (calculated) switching time in the absence of suspended matter concentration in water is 16.6 hours, while adding concentration $\theta = 0.0001$, switching time is 13.7 an hour at concentration $\theta = 0.0003$ - 15.1 hour. With an increase in the concentration of gel-particles in the solution, the filter switching time decreases accordingly according to a linear law.

Numerical calculations have shown that in solving the problem of mass transfer with the clogging of the pores of a gel-particle filter, despite of fact that the filter septum is not fully saturated with ions, there is a delay in switching the filter unit. This effect is explained by the fact that gel particles, clogging the ion - exchange filter, as if "isolate" the ionite grains and thus represent contact with the liquid phase.

To study the adequacy of the filtering process of ionized solutions developed by MM, the results of the obtained calculations were compared with the experimental data of Elissian and those obtained at the Fergana Chemical Plant for the preparation of artificial fibers.

The analysis of these assessments showed that the developed MM adequately describes the process as a whole.

5. Conclusion

The analysis of the numerical calculations on a computer showed that, when filtering a suspension with a high degree of contamination in a wide range of changes on the size of gel particles, it is necessary to pass the solution through multi-layer filter partitions, which lead to an increase in the unit's performance and a high degree of purification of the output product. The carried out numerical calculations have established that one of the main indicators of the technological process of filtering, which plays a significant role, is porosity and the thickness of the filter partition of the unit. They must be selected depending on the diameters of the main mass of particles contained in the composition of the partial suspension.

Computational experiments have established that in order to increase the degree of purification of the suspension from gel particles and increase the productivity of the unit, the thickness of the second filter partition and its porosity must be selected depending on the output concentration of the suspension passing through the first purification degree.

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