PAPER • OPEN ACCESS

Material evaporation with ultrashort laser exposure

To cite this article: I I Latypov *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **537** 022068

IOP Conf. Series: Materials Science and Engineering **537** (2019) 022068 doi:10.1088/1757-899X/537/2/022068

Material evaporation with ultrashort laser exposure

I I Latypov, L A Bigaeva, V V Chudinov, A Y Gilev and F R Gaisin

Bashkir State University, 32, Validy str., Ufa, Russia

E-mail: latypovii@rambler.ru

Abstract. The task is to analyse the temperature distribution in the material under ultrashort laser irradiation. The mathematical model is built on the basis of a two-temperature model describing transition phenomena in a nonequilibrium electron gas and lattice with an ultrashort laser effect on the material. The vaporized body is considered as a thin plate and the problem is formulated as a system of one-dimensional boundary-value problems of the heat equation written for the electron and lattice components. The initial problem is reduced to solving a system of singularly perturbed boundary problems of the heat equation with nonlinear boundary conditions on moving boundaries, an approximate solution of which is obtained in the form of an asymptotic expansion of the solution in the Poincaré sense in powers of small parameters.

1. Introduction

The rapid development of experimental technology has led to the fact that more and more attention is paid to ablation under the action of ultrashort laser pulses of the picosecond and femtosecond ranges, for which the quasistationary ablation regime is not achieved.

When using shorter (subpicosecond and femtosecond) laser pulses, features arise in the ablation kinetics that cannot be described within the framework of the conventional thermal model. However, ablation using ultrashort laser pulses is used in high-tech materials processing. In the case of a metal, one of the features is the effects associated with the electron-phonon interaction and the phenomena caused by a hot electron gas in a substance [1-3]. The task of studying the influence of boundary conditions and pulsed irradiation concerning the temperature distribution in the material is also very important.

The task is to analyse the temperature distribution in the material during subpicosecond laser ablation. The vaporized body is considered as a thin plate and the problem is formulated as a system of one-dimensional boundary-value problems of the heat equation [2], written for the electron and lattice components.

2. Problem statement

The two-temperature model [2] describes the transport of energy inside a metal using a system of heat conduction equations for the temperature of T_e electrons and T_i lattice:

$$c_e \frac{\partial T_e}{\partial t} = c_e v \frac{\partial T_e}{\partial z} + \frac{\partial}{\partial z}\left(\chi_e \frac{\partial T_e}{\partial z} \right) + D_k \cdot Q - \mu \left(T_e - T_i \right),\ (z,t) \in \Omega, \tag{1}$$

$$c_i \frac{\partial T_i}{\partial t} = c_i v \frac{\partial T_i}{\partial z} + \frac{\partial}{\partial z}\left(\chi_i \frac{\partial T_i}{\partial z} \right) + \mu \left(T_e - T_i \right),\ (z,t) \in \Omega, \tag{2}$$

IOP Conf. Series: Materials Science and Engineering **537** (2019) 022068 doi:10.1088/1757-899X/537/2/022068

$$T_e\big|_{t=t_k} = T_{e,k}(z), \quad T_i\big|_{t=t_k} = T_{i,k}(z), \tag{3}$$

$$-\chi_e \frac{\partial T_e(z,t)}{\partial z}\bigg|_{z=0} = J_e, \qquad J_e = -k_0 b_0 \left(T_{i,s} + T_0\right)^2 \exp\left\{-\frac{T_u}{T_{i,s} + T_0}\right\}, \tag{4}$$

$$-\chi_i \frac{\partial T_i(z,t)}{\partial z}\bigg|_{z=0} = J_i = -\rho \nu L, \tag{5}$$

$$-\chi_i \frac{\partial T_i(z,t)}{\partial z}\bigg|_{z=H} = \psi_i(t), \; -\chi_e \frac{\partial T_e(z,t)}{\partial z}\bigg|_{z=H} = \psi_e(t), \tag{6}$$

$$T_e\big|_{z=0} = T_{e,s}, \; T_i\big|_{z=0} = T_{i,s}, \; D_k = \begin{cases} 1, & если \;\; k = 2p, \\ 0, & если \;\; k = 2p+1, \end{cases} \; p = \left\{0,1,\ldots,\left[\frac{n}{2}\right]\right\}, \tag{7}$$

$$\Omega = \left\{(z,t): 0 < z < H, 0 < t < \overline{t_0}\right\}, \; \nu = \nu_0 \exp\left\{-\frac{T_a}{T_{i,s} + T_0}\right\}, \; 0 = t_0, t_1, t_2, \ldots, t_n = \overline{t_0},$$

where $\mu_e = c_e / t_{rel}$ is coefficient of the rate of energy exchange between the electronic and lattice subsystems (t_{rel} -characteristic exchange time for the electronic subsystem); b_0 is Richardson constant; T_a is output work; $k_0 = k_b(T_e + T_0)/e$ is J_e power flux density conversion factor into to energy units; H, ρ, L - thickness, density and specific heat of material melting; ν_0, T_u - constants characterizing the model of evaporation, which are taken from reference books; $r_k = \left(t_{2k+1} - t_{2k}\right)$, $k = \overline{0, m-1}$ - time of k-th laser pulse, $\left(t_{2k} - t_{2k-1}\right)$, $k = \overline{1, m}$ - time between pulses ($n = 2m$); $T_e\big|_{t=t_0} = T_{e,0}(z) = T_0$, $T_i\big|_{t=t_0} = T_{i,0}(z) = T_0$; ψ_e, ψ_i functions determine the modes of heat transfer on the back of the plate. Q heat source is defined in the following equation: $Q = -\frac{\partial I}{\partial z} = \alpha \cdot I(0,t)$, $I(0,t) = I_{z=0}(t), t \in \left[0, \; r_k\right]$, $k = \overline{0, m-1}$, where α, I_0 - absorption coefficient and radiation intensity on the surface of the material ($z = 0$), and, $I_{z=0}(t) = A \cdot I(t)$ depends on the shape of the laser pulse, for example: $I(t) = I_0 \cdot \frac{t}{\overline{t_1}} \exp\left\{-\frac{t}{\overline{t_1}}\right\}$.

This model is applicable in the case when it is possible to use the classical Fourier laws, that is, it is applicable for times much longer than τ_e characteristic time for establishing an equilibrium distribution in the electron gas. τ_e time - depends on the electron temperature (energy density in a laser pulse), in most problems it is several hundred femtoseconds. The propagation of electron temperature arises on a spatial scale larger than the electron mean free path l_e [1, 2].

3. Problem solution

We will reduce the considered two-temperature model to a dimensionless form. For this, we will put $z = x \cdot z_0$, $t = s \cdot t_0$, where z_0 is the characteristic (length) thickness of the heating (□ 10^{-6} $м$), $t_0 > t_{rel}$, or; additionally we will assume that $c_e, c_i, \chi_e, \chi_i, \mu, \alpha, A$ do not depend on temperature.

We will obtain the following dimensionless singularly perturbed boundary value problem:

$$\frac{\partial \overline{T}_e}{\partial s} = \overline{\nu}_0 \cdot \exp\left\{-\frac{T_a}{\overline{T}_{i,s} + T_0}\right\} \frac{\partial \overline{T}_e}{\partial x} + p_e \frac{\partial^2 \overline{T}_e}{\partial x^2} + \overline{A} \cdot \tau \cdot \exp\left\{-\overline{t}_0 \cdot s\right\} - \mu_0 \left(\overline{T}_e - \overline{T}_i\right), \; (x,s) \in \Omega', \tag{8}$$

$$\frac{\partial \bar{T}_i}{\partial s} = \bar{v}_0 \cdot \exp\left\{-\frac{T_a}{\bar{T}_{i,s}+T_0}\right\}\frac{\partial \bar{T}_i}{\partial x} + p_i \frac{\partial^2 \bar{T}_i}{\partial x^2} + \mu_0\left(\bar{T}_e - \bar{T}_i\right),\ (x,s)\in\Omega', \tag{9}$$

$$T_e\big|_{s=\tilde{t}_k} = \bar{T}_{e,k}\left(x\right),\quad T_i\big|_{s=\tilde{t}_k} = \bar{T}_{i,k}\left(x\right), s \square\ \ 0, \tag{10}$$

$$-\frac{\partial \bar{T}_e}{\partial x} = -k_e\left(\bar{T}_{e,s}+T_0\right)^3 \exp\left\{-\frac{T_u}{\bar{T}_{e,s}+T_0}\right\},\ -\frac{\partial \bar{T}_i}{\partial x} = -k_i \exp\left\{-\frac{T_u}{\bar{T}_{i,s}+T_0}\right\},\ x=0, \tag{11}$$

$$-\frac{\partial \bar{T}_i(x,s)}{\partial x}\bigg|_{x=h} = \varphi_i(s),\ -\frac{\partial \bar{T}_e(x,s)}{\partial x}\bigg|_{x=h} = \varphi_e(s), \tag{12}$$

$$\bar{T}_e(x,\tau) = T_e(x\cdot z_0, \tau\cdot t_0), \bar{T}_i(x,\tau) = T_i(x\cdot z_0,\tau\cdot t_0), \bar{t}_0 = {t_0}/{t_1},\ p_e = \frac{\chi_e}{c_e}\frac{t_0}{z_0^2},\ p_i = \frac{\chi_i}{c_i}\frac{t_0}{z_0^2},\ 0<p_e<<1,$$

$$0<p_i<<1,\ \varphi_e(\tau) = \frac{1}{\chi_e}\psi_e(\tau\cdot t_0),\ \varphi_i(\tau) = \frac{1}{\chi_i}\psi_i(\tau\cdot t_0),\ h = \frac{H}{z_0},\ k_e = \frac{k_b}{\chi_e}\frac{b_0}{e}z_0,\ k_i = \frac{\rho\cdot z_0}{\chi_i}v_0 L,$$

$$\bar{A} = A\cdot\alpha\cdot I_0\frac{t_0^2}{c_e \bar{t}_1},\ \bar{v}_0 = v_0\frac{t_0}{z_0}, \mu_0 = t_0\mu,\ \bar{T}_e\big|_{x=0} = \bar{T}_{e,s},\ \bar{T}_i\big|_{x=0} = \bar{T}_{i,s},\ \tilde{t}_k = {t_k}/{t_0},\ \bar{T}_{e,k}\left(x\right) = T_{e,k}\left(\frac{z}{x_0}\right),$$

$$\bar{T}_{i,k}\left(x\right) = T_{i,k}\left(\frac{z}{x_0}\right),\ \Omega' = \left\{(x,s): 0<x<h,\ 0<s<\tilde{t}_n\right\}.$$

The solution of this boundary value problem is sought according to the scheme of the splitting method, for which we assume that in the first approximation $\bar{T}_i = \bar{T}_i(x,0) = \bar{T}_{i,s} = T_0 = const$ and we find it as a solution of the boundary value problem for equation (8) with the corresponding initial and boundary conditions; then, using the obtained solution $\bar{T}_e(x,s)$, we define as the solution of the boundary value problem (9), (10), (11), (12). Continuing this process n times we obtain the n-th approximation of the desired solutions $\bar{T}_e^{(n)}(x,s)$, $\bar{T}_i^{(n)}(x,s)$.

In the first approximation for the initial pulse to determine $\bar{T}_e^{(1)}(x,s)$, $\bar{T}_i^{(1)}(x,s)$ (if $\bar{T}_i = \bar{T}_i(x,0) = \bar{T}_{i,s} = T_0$) we have the following boundary value problem

$$\frac{\partial \bar{T}_e^{(1)}}{\partial s} = \bar{v}_0 \exp\left\{-\frac{T_a}{T_0+T_\infty}\right\}\frac{\partial \bar{T}_e^{(1)}}{\partial x} + p_e\frac{\partial^2 \bar{T}_e^{(1)}}{\partial x^2} + \bar{A}s\exp\left\{-\bar{t}_0 s\right\} - \mu_0\left(\bar{T}_e^{(1)} - T_0\right),\ (x,s)\in\Omega' \tag{13}$$

$$\bar{T}_e^{(1)}(x,s) = T_0,\ s=0, \tag{14}$$

$$-\frac{\partial \bar{T}_e^{(1)}}{\partial x} = -k_e\left(\bar{T}_e^{(1)}+T_\infty\right)^3\exp\left\{-\frac{T_u}{\bar{T}_e^{(1)}+T_\infty}\right\},\ x=0,\ -\frac{\partial \bar{T}_e^{(1)}}{\partial x} = \varphi_e(s),\ x=h, \tag{15}$$

$$\frac{\partial \bar{T}_i^{(1)}}{\partial s} = \bar{v}_0\exp\left\{-\frac{T_a}{T_0+T_\infty}\right\}\frac{\partial \bar{T}_i^{(1)}}{\partial x} + p_i\frac{\partial^2 \bar{T}_i^{(1)}}{\partial x^2} + \tilde{\mu}_0\left(\bar{T}_e^{(1)} - \bar{T}_i^{(1)}\right), (x,s)\in\Omega', \tag{16}$$

$$\bar{T}_i^{(1)}\left(x,s\right) = T_0,\quad s=0, \tag{17}$$

$$-\frac{\partial \bar{T}_i^{(1)}}{\partial x} = -k_i\exp\left\{-\frac{T_a}{\bar{T}_i^{(1)}+T_\infty}\right\},\ x=0,\ -\frac{\partial \bar{T}_i^{(1)}}{\partial x} = \varphi_i(s),\ x=h. \tag{18}$$

If we make a change of variables $z = x + s\cdot\bar{v}_0 \exp\left\{-\frac{T_a}{T_0+T_\infty}\right\}$, $t = s$, then we get the following boundary value problem (hereinafter, to simplify, the superscript in the expressions will be omitted i.e. $\bar{T}_e^{(1)}(x+s\cdot\tilde{v}_0, s) \to \bar{T}_e(z,t)$, $\bar{T}_i^{(1)}(x+s\cdot\tilde{v}_0, s) \to \bar{T}_i(z,t)$

$$\frac{\partial \bar{T}_e(z,t)}{\partial t} = p_e \frac{\partial^2 \bar{T}_e(z,t)}{\partial z^2} + \bar{A}\cdot t\cdot\exp\{-t\cdot\tilde{t}_0\} + \mu_0\left(\bar{T}_e(z,t) - T_0\right),\ (z,t)\in\Omega'', \tag{19}$$

$$\bar{T}_e(z,t) = T_0,\ t = 0, \tag{20}$$

$$-\frac{\partial \bar{T}_e(z,t)}{\partial z} = -k_e\left(\bar{T}_e(z,t) + T_\infty\right)^3 \exp\left\{-\frac{T_u}{\bar{T}_e(z,t)+T_\infty}\right\},\ z = \tilde{v}_0\cdot t, \tag{21}$$

$$-\frac{\partial \bar{T}_e(z,t)}{\partial z} = \psi_e(t),\ z = h + \tilde{v}_0\cdot t, \tag{22}$$

$$\frac{\partial \bar{T}_i(z,t)}{\partial t} = p_i \frac{\partial^2 \bar{T}_i(z,t)}{\partial z^2} + \tilde{\mu}_0\left(\bar{T}_e(z,t) - \bar{T}_i(z,t)\right),\quad (z,t)\in\Omega'', \tag{23}$$

$$\bar{T}_i(z,t) = T_0,\ t = 0, \tag{24}$$

$$-\frac{\partial \bar{T}_i(z,t)}{\partial z} = -k_i \exp\left\{-\frac{T_a}{\bar{T}_i(z,t)+T_\infty}\right\},\ z = \tilde{v}_0\cdot t, \tag{25}$$

$$-\frac{\partial \bar{T}_i(z,t)}{\partial z} = \psi_i(t),\ z = h + \tilde{v}_0\cdot t, \tag{26}$$

$$(z,t)\in\Omega'' = \left\{(z,t): \tilde{v}_0\cdot t < z < h + \tilde{v}_0\cdot t,\ 0 < t < \tilde{t}_0\right\},\ \tilde{v}_0 = \bar{v}_0 \exp\left\{-\frac{T_a}{T_0+T_\infty}\right\}.$$

We will introduce the notation

$$P_e\left[\bar{T}_e(z,t)\right] = \bar{A}\cdot t\cdot\exp\{-t\cdot\tilde{t}_0\} + \mu_0\left(\bar{T}_e(z,t) - T_0\right) = \bar{Q}(z,t) + \mu_0\bar{T}_e(z,t),$$

$$P_i\left[\bar{T}_i(z,t)\right] = \tilde{\mu}_0\left(\bar{T}_e(z,t) - \bar{T}_i(z,t)\right).$$

To solve boundary problems, we use integral representations of solutions $\bar{T}_e(z,t)$, $\bar{T}_i(x,t)$, written using the Green function [4-6]

$$\bar{T}_e(z,t) = \int_0^h T_0\cdot\Gamma\left(z,t;y,0\right)dy + p_e\int_0^t \vartheta_1(\tau)\cdot\Gamma\left(z,t;\tilde{v}_0 t,\tau\right)d\tau - p_e\int_0^t \vartheta_2(\tau)\cdot\Gamma\left(z,t;\tilde{v}_0 t + h,\tau\right)d\tau$$
$$+\int_0^t \int_{\tilde{v}_0\cdot\tau}^{\tilde{v}_0\tau+h} P_e\left[\bar{T}_e(y,\tau)\right]\cdot\Gamma\left(z,t;y,\tau\right)dyd\tau, \tag{27}$$

$$\bar{T}_i(z,t) = \int_0^h T_0\cdot\Gamma\left(z,t;y,0\right)dy + p_i\int_0^t \omega_1(\tau)\cdot\Gamma\left(z,t;\tilde{v}_0 t,\tau\right)d\tau - p_i\int_0^t \omega_2(\tau)\cdot\Gamma\left(z,t;\tilde{v}_0 t + h,\tau\right)d\tau$$
$$+\int_0^t \int_{\tilde{v}_0\cdot\tau}^{\tilde{v}_0\tau+h} P_i\left[\bar{T}_i(y,\tau)\right]\cdot\Gamma\left(z,t;y,\tau\right)dyd\tau. \tag{28}$$

The Green function of this boundary value problem is written out explicitly [4-6]. To find the unknown functions $\vartheta_1(t)$, $\vartheta_2(t)$, $\omega_1(t)$, $\omega_2(t)$ we use the boundary conditions (21), (22), (25), (26). As a result, we obtain these functions in the form of asymptotic expansions

$$\vartheta_j(t) = a_e\sum_{k=0}^{m_e} C_{e,k}^j p_e^k + O\left(p_e^{m_e+1}\right),\ p_e\to 0,\ \omega_j(t) = a_i\sum_{k=0}^{m_i} C_{i,k}^j p_i^k + O\left(p_i^{m_i+1}\right),\ p_i\to 0,\ j = 1,2;$$

where the coefficients $C_{e,k}^{j}$, $C_{i,k}^{j}$ are calculated explicitly: $a_e = k_e \cdot \exp\left(-\frac{T_u}{A}\right)$, $A = T_0 + T_\infty$, $a_i = k_i \cdot \exp\left(-\frac{T_a}{A}\right)$, $B_e = \frac{a_e}{\bar{v}_0}$, $B_i = \frac{a_i}{\bar{v}_0}$, $C_{e,0} = A^3$, $C_{e,1} = -A^6\left(T_u A + 3A^2\right)B_e$, $C_{i,0} = 1$, $C_{e,2} = \left(A^9\left(T_u A + 3A^2\right) + \frac{A^{11}}{2}\left(T_u^2 + 4T_u A + 6A^2\right)\right)B_e^{\,2}$, $C_{i,1} = -\frac{T_a}{A^2}B_e$, $C_{i,2} = \left(-\frac{T_a}{2A^3} + \frac{T_a^2}{2A^4}\right)B_e^{\,2}$.

Using the obtained expressions for $\vartheta_j(t)$, $\omega_j(t)$, $j = 1, 2$, we find the asymptotic expansions of solutions $\bar{T}_e(z,t)$, $\bar{T}_i(z,t)$ according to the following iterative scheme:

$$\bar{T}_e^{(n+1)}(z,t) = \tilde{T}_e(z,t) + \int_0^t \int_{\tilde{v}_0 s}^{\tilde{v}_0 s + h} \bar{Q}(y,s)\cdot \mathit{\Gamma}(z,t,y,s)dyds - \mu_0 \int_0^t \int_{\tilde{v}_0 s}^{\tilde{v}_0 s + h} \bar{T}_e^{(n)}(y,s)\cdot \mathit{\Gamma}(z,t;y,s)dyds,$$

$$\bar{T}_i^{(n+1)}(z,t) = \tilde{T}_i(z,t) - \tilde{\mu}_0\left(\int_0^t \int_{\tilde{v}_0 s}^{\tilde{v}_0 s + h} \bar{T}_i^{(n)}(y,s)\mathit{\Gamma}(z,t;y,s)dyds - \int_0^t \int_{\tilde{v}_0 s}^{\tilde{v}_0 s + h} \bar{T}_e^{(n+1)}(y,s)\mathit{\Gamma}(z,t;y,s)dyds\right),$$

$$n = 1,2,3,\ldots,\ \bar{T}_e^{(0)}(z,t) = \tilde{T}_e(z,t) = T_0 + p_e\left[a_e \cdot \sum_{k=0}^{m_e} \bar{C}_{e,k} p_e^k + O\left(p_e^{m_e+1}\right)\right],$$

$$\bar{T}_i^{(0)}(z,t) = \tilde{T}_i(z,t) = T_0 + p_i\left[a_i \cdot \sum_{k=0}^{m_i} \bar{C}_{i,k} p_i^k + O\left(p_i^{m_i+1}\right)\right].$$

Continuing this process further, we obtain the solution in the form of asymptotic expansions in powers of small parameters [4, 6]. Then the following statement is true

Theorem. The asymptotic expansion of the solution to the boundary value problem (19) - (26) has the following form:

1) In the "*boundary layer*" of the border $z = \tilde{v}_0 t$, that is, if the following condition is fulfilled $z - \tilde{v}_0 t = O\left(p_e^{\,p}\right)$, $z - \tilde{v}_0 t = O\left(p_i^{\,p}\right)$, $p > 1$,

$$\bar{T}_{q,pgr}(z,t) = \sum_{k=0}^{N}\sum_{j=0}^{\bar{N}} d_{q,k,j}^{(0)}(z,t)\cdot\left(\frac{z - \tilde{v}_0 t}{p_q}\right)^j \cdot p_q^{\,k} + O\left(\left(\frac{z - \tilde{v}_0 t}{p_q}\right)^{\bar{N}+1}\cdot p_q^{\,N+1}\right),\ p_q \to 0,\ q = \{e,i\}.$$

2) In the *"intermediate layer"* of the border $z = \tilde{v}_0 t$, that is, if the following condition is fulfilled $z - \tilde{v}_0 t = O\left(p_e\right)$, $z - \tilde{v}_0 t = O\left(p_i\right)$, $p_e \to 0$, $p_i \to 0$,

$$\bar{T}_{q,pr}(x,t) = \sum_{k=0}^{M} d_{q,k}^{(1)}(z,t)\cdot p_q^{\,k} + O\left(p_q^{\,M+1}\right),\ p_q \to 0,\ q = \{e,i\}.$$

3) In the *"remote area"* $z = \tilde{v}_0 t$, $z = \tilde{v}_0 t + h$ *points*", where the condition is fulfilled $p < 1$, $z - \tilde{v}_0 t = O\left(p_e^{\,p}\right)$, $z - \tilde{v}_0 t - h = O\left(p_e^{\,p}\right)$, $z - \tilde{v}_0 t = O\left(p_i^{\,p}\right)$, $z - \tilde{v}_0 t - h = O\left(p_i^{\,p}\right)$, $p_e \to 0$, $p_i \to 0$, $\bar{T}_{e,ud}(z,t) = T_0 + \bar{A}\left(t_0^{-2} - \frac{t\cdot t_0 + 1}{t_0^2}\exp\left(-t\cdot t_0\right)\right)$, $\bar{T}_{i,ud}(z,t) = T_0$.

4) In the *"intermediate layer"* of the border $z = \tilde{v}_0 t + h$, that is, if the following condition is fulfilled $z - \tilde{v}_0 t - h = O\left(p_q\right)$, $\bar{T}_{q,pr}(x,t) = \sum_{k=0}^{M} d_{q,k}^{(2)}(z,t)\cdot p_q^{\,k} + O\left(p_q^{\,M+1}\right)$, $p_q \to 0$, $q = \{e,i\}$.

IOP Conf. Series: Materials Science and Engineering **537** (2019) 022068 doi:10.1088/1757-899X/537/2/022068

5) In the *"boundary layer"* of the border $z=\tilde{v}_0 t+h$, that is, if the following condition is fulfilled $z-\tilde{v}_0 t-h=O\left(p_e{}^p\right)$, $z-\tilde{v}_0 t-h=O\left(p_i{}^p\right)$, $p>1$,

$$\overline{T}_{q,pgr}(z,t)=\sum_{k=0}^{N}\sum_{j=0}^{\overline{N}} d_{q,k,j}^{(3)}(z,t)\left(\frac{z-\tilde{v}_0 t-h}{p_q}\right)^j p_q{}^k+O\left(\left(\frac{z-\tilde{v}_0 t-h}{p_q}\right)^{\overline{N}+1} p_q{}^{N+1}\right),\ p_q\to 0,\ q=\{e,i\}.$$

The coefficients of the asymptotic expansions $d_{e,k,j}^{(0)}$, $d_{i,k,j}^{(0)}$, $d_{e,k}^{(1)}$, $d_{i,k}^{(1)}$, $d_{e,k}^{(2)}$, $d_{i,k}^{(2)}$, $d_{e,k,j}^{(3)}$, $d_{i,k,j}^{(3)}$ are computed explicitly [4-6] and do not depend on the small decomposition parameters, i.e. we have an asymptotic expansion in the Poincaré sense. The resulting expansions satisfy the initial and boundary conditions of the problem.

It should be noted that in the above theorem only consistent asymptotic scales are used. In the extended formulation, it is necessary to present the results based on a combination of different asymptotic scales.

The brief algorithm for the asymptotic analysis of a boundary value problem for a single laser irradiation pulse is similarly carried out for the entire cycle of pulsed exposure to ultrashort radiation on the material under study.

4. Conclusion

The obtained asymptotic expansions of the problem solution allow to carry out a parametric analysis of the process under study, to reveal the influence of the boundary conditions and the pulsed mode of irradiation on the temperature distribution in the material.

References

[1] Prokhorov A M, Konov V I, Ursu I and Mikheilesku I N 1988 Interaction of laser radiation with metals (Moscow: Nauka)

[2] Anisimov S I and Lukiyanchuk B S 2002 Selected questions of the theory of laser ablation *Advances in physical sciences* **172(3)**

[3] Volkov N B, Kundikova N D, Leivi A Ya, Mayer A E and Yalovets A P 2017 On the effect of high-power ultrashort electron beams on metal targets *Letters in ZhTF* **33(2)** 43

[4] Kravchenko V F, Nesenenko G A, Latypov I I 1999 An application of integral equations to a singularly perturbed nonstationary boundary value problem for the heat equation in a domain with moving boundaries *Differential Equations* **35(9)** 1184-92

[5] Latypov I I, Shikirov R A and Ulitin N V 2014 Approximate solution of the problem of finding the temperature in the active elements of solid-state lasers *Bulletin of Kazan Technological University* **17(5)** 80-6

[6] Latypov I I 2017 Approximate solution to a singular perturbed boundary value problem of thermal shielding *J. Phys.: Conf. Ser.* **918** 012005