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# The comparison of several approaches to the interpolation of a trajectory of a navigation satellite

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**Abstract.** Since a satellite orbit is quite smooth, polynomial techniques can be widely used for the interpolation of satellite positions in real-time applications. The paper is devoted to the comparison of different approaches to the polynomial interpolation of the trajectory of a satellite using available data. All approaches have been examined for test and actual data.

## Introduction

At the present time, the global navigation satellite systems (GNSS), such as GPS or GLONASS, are mainly used to determine location, exact time and other traffic parameters for land, water and air objects. Even the simplest GNSS consumer receivers have to calculate the current position of each visible satellite based on data obtained from these satellites. Moreover, there are many tasks associated with accurate positioning of objects, cartography, geodesy, tectonics monitoring, weather forecasting, which require precise satellite ephemeris. Thus, most navigation problems are based on the definition, prediction, and correction of satellite trajectories [1–2].

For applications, navigation systems typically provide several data sets for determining the position of a satellite and its orbital velocity. These are almanac data, broadcast data, ultra-rapid data (observed and predicted half), rapid and final satellite ephemerides and satellite clocks. These data differ in accuracy and time when they become available [3–4]. For example, the almanac data are updated at least once every few days, but their accuracy is about several kilometres. The broadcast ephemerides are based on observations at the monitoring stations at the corresponding time interval [5], they are known with an accuracy of 1 m in real-time transfer. The most accurate orbital information is provided by the data set in the form of final ephemerides (~0.025 m) and accompanying data. These are the post-mission satellite orbits, obtained by collecting data from stations around the world.

Files with these data are freely available for all users, for instance, refer to International GNSS Service (IGS) for GPS and GLONASS global navigation satellite systems and Galileo (Europe), BeiDou (China), Quasi Zenith Satellite System (Japan), and IRNSS (India) regional ones [4]. These files are presented in a special sp3 version c format and contain satellite positions and a clock correction at equidistant epochs, as well as optional additional records about satellite velocities, standard deviations of positions and velocities, the correlation coefficients between the satellite position and the satellite clock correction values, the correlation coefficients between the satellite velocities and clock-rate correction values. The typical spacing of the data is 15 minutes, which is sometimes not sufficient for applications.



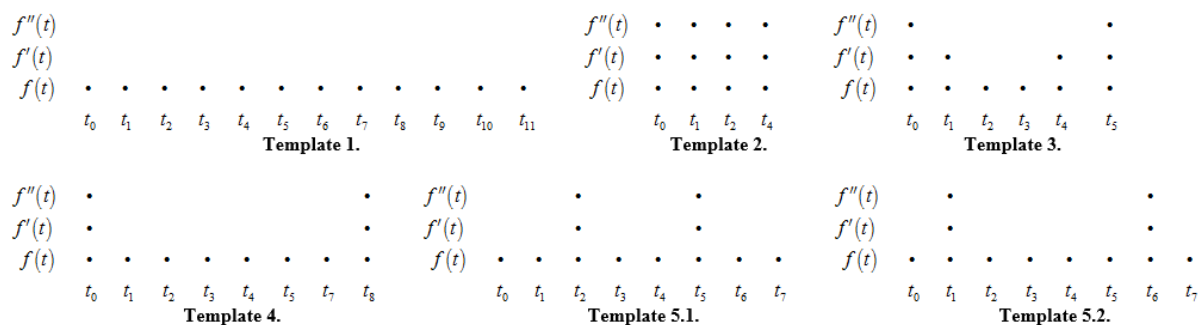
The position of a satellite between two given epochs may be obtained by interpolation. The most rigorous approach to interpolation was discussed in [6] and is based on solving the differential equation of a satellite motion in the perturbed gravitational field. But this approach is more complicated than the polynomial one. Most studies use the Lagrange polynomials of the 9th or 12th degree [7]. In this case, using the final ephemerides, one can achieve an interpolation error about 1 cm on time interval of 30 minutes [7–8]. Feng in [9–10] compared three interpolation methods: Chebyshev, Neville and trigonometric ones. The interpolation results for the first two methods are the same as in the case of the Lagrange polynomial. The 9th degree trigonometric method gives an accuracy about 5 cm, and the 19th degree trigonometric method (for full satellite orbit) allows one to obtain an accuracy about 1 cm. The disadvantage of this method is a large interpolation error near the ends of the data range. In [11], an algorithm for real-time processing software that calculates the position and velocity of GPS satellites from both broadcast and final ephemerides was tested for Lagrange polynomials of different degrees.

The Lagrange interpolation is used because a satellite ephemeris is the most available information. However, in some applications, along with ephemeris, satellite velocity and even acceleration can be known. In these cases, interpolation is possible with other polynomials, including the Hermite approach.

In the present research, a series of interpolation patterns, based on the Lagrange and Hermite polynomials, is tested.

### Interpolation patterns

Consider the following polynomial interpolation problem. Divide a segment  $[T_0, T]$  by equidistant time instants  $T_0=t_0 < t_1 < \dots < t_N=T$ ,  $t_i = t_0 + it$  where  $t = 1/N$  is a time step,  $N \geq 2$ ,  $N+1$  is a number of interpolation nodes in a template. Let the values of a grid-function  $f(t_i)$  and, possibly, its first and second order derivatives  $f'(t_i)$  and  $f''(t_i)$  be known. Moreover, let  $3K$  values of the grid-function and its derivatives be only given where  $K \leq N+1 \leq 3K$ . Compare the accuracy of interpolation polynomials of degree  $3K-1$ , based on the following set of templates (figure 1).



**Figure 1.** The interpolation templates for polynomials of the 11th degree.

Each of these templates has some advantages and disadvantages. Template 1 is a Lagrange polynomial template. This template is the widest among all considered ones for the same time step, i.e., to approximate the full orbit, the smallest number of the interpolation polynomials is required. To construct the Lagrange polynomial, only the function values are needed, which is preferable in many applications approximating the satellite positions. The main disadvantage of Lagrange polynomials is a loss of accuracy at the ends of the data range.

Template 2 is a full Hermite polynomial. To construct it, we need the values of derivatives of the function. When we interpolate satellite positions, the derivatives represent the satellite velocity and acceleration, which are not always possible to be calculated with sufficient accuracy. However, in some cases they are available. Template 2 is the shortest among all considered ones for the same time step, i.e., to approximate the full orbit, the largest number of the interpolation polynomials is required.

Templates 3 and 4 are modifications of template 2, where the required number of the derivative values is reduced with increasing the width of the template. At the same time, these templates improve the approximation at the ends of the data range as compared to the Lagrange polynomial and provide a smooth splicing.

For templates 5.1 and 5.2, the interpolation is reasonable only in the interval  $[t_k, t_{N-k+1}]$ , where  $k$  is a parameter of template. In this case, the gluing of the neighboring templates should be carried out at the ends of this interval (at the nodes  $t_k$  and  $t_{N-k+1}$ ). Moreover, to improve the smoothness of gluing at these points, not only the values of the function, but also the values of its derivatives are used. The number of insignificant nodes (parameter  $k$ ) can vary.

In figure 1, the templates for the 11th degree polynomials are shown, but the result can be generalized to polynomials of any degree  $3K-1$ . For an interpolation polynomial of degree  $n$  constructed using the  $j$ th template,  $j = \{1, 2, 3, 4, 5.1, 5.2\}$ , we introduce the notation  $j^n(t)$ .

## Results and discussion

To complete covering an orbit, it is necessary to use several glued templates, and an interpolation polynomial is constructed on each template independently. To provide smoothness of the approximation of the satellite trajectory, the neighboring templates have to intersect at least at one node. The values of the interpolation polynomials and, possibly, their derivatives (if they are used in the template) coincide at the gluing nodes. For templates 5.1 and 5.2, the gluing is performed at the ends of a significant interval  $[t_k, t_{N-k+1}]$ , i.e., the interpolation polynomials have  $2(k+1)$  common nodes.

*Test 1.* We compare the accuracy of the interpolation polynomial constructed with the use of the templates for the test function  $f(t) = \sin t$  on the interval  $t \in [p/3, 7p/3]$ , where the function is nonsymmetric.

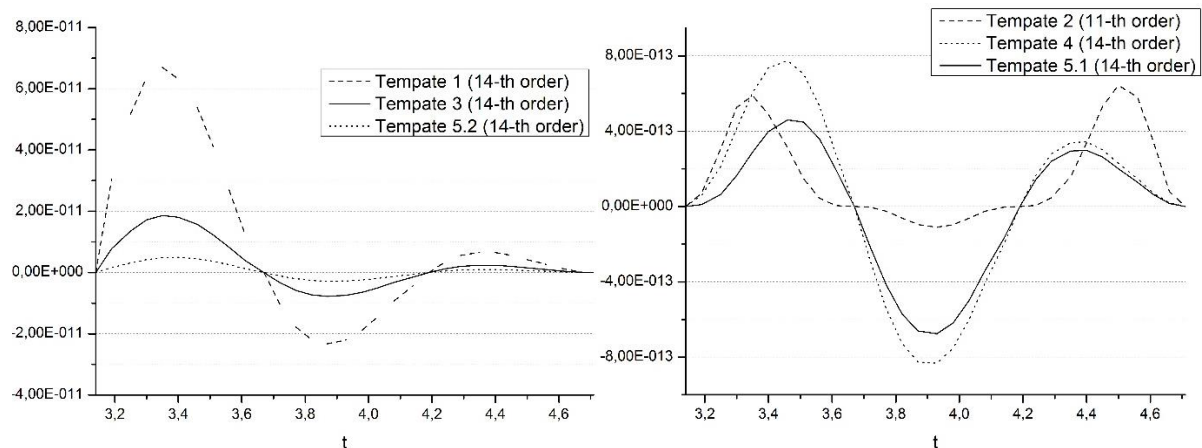
Since the goal is the interpolation of satellite positions, additional restrictions have to be put in the interpolation problem. Firstly, the most accurate satellite positions are spaced at 15 minutes intervals. If we coarsely estimate the circuit time as 12 hours, then the step in the template should not be less than  $p/24$ . In numerical experiments, the step  $p/6$  is chosen, which corresponds to the satellite orbital period of little less than an hour. Secondly, the expected interpolation error for the test function should not exceed  $10^{-12}$ , which corresponds to the accuracy of a fraction of mm of the orbit.

The interpolation errors  $\sin t - j^n(t)$  are shown in figure 2 for polynomials constructed with the use of all considered templates. For a fixed time step, the templates are of different widths, and the Hermite polynomial is of the smallest template width. In numerical experiments, the interpolation intervals for each case were chosen in such a way that all of them contain the Hermite interpolation interval in the middle of their own one. The errors of all polynomials are shown in figure 2 only on the segment that coincides with the Hermite polynomial template. Observe that the Hermite polynomial of the 11th degree gives the smallest error among the examined versions. Moreover, the interpolation polynomials of the 14th degree constructed with the use of templates 4 and 5.1 give the error of the same order as the Hermite polynomial.

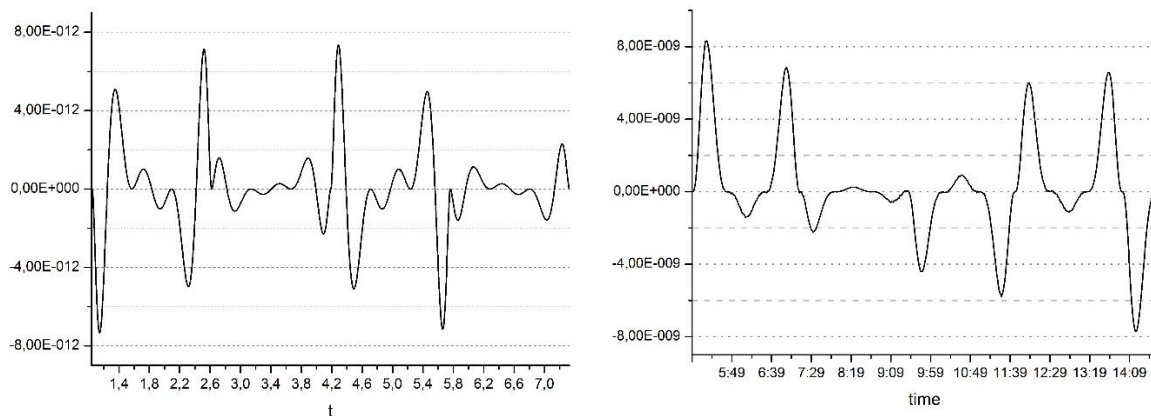
The derivative approximation error  $\cos t - (j_{11}^1)'(t)$  is shown in figure 3. The interpolation on the interval  $[p/3, 7p/3]$  was constructed with four Hermite 11th degree polynomials for time step  $p/6$ . These polynomials are glued at the ends of the templates. We can see that the derivative approximation error does not exceed  $10^{-11}$ .

*Test 2.* The recovery of the satellite position from the data calculated using a mathematical model. We use the initial position and velocity of the satellite from a sp3 file [4]. Further, according to the model of satellite motion, taking into account the non-sphericity of the geopotential and the influence of the Sun and the Moon, the ephemeris, velocity, and acceleration of the satellite were calculated.

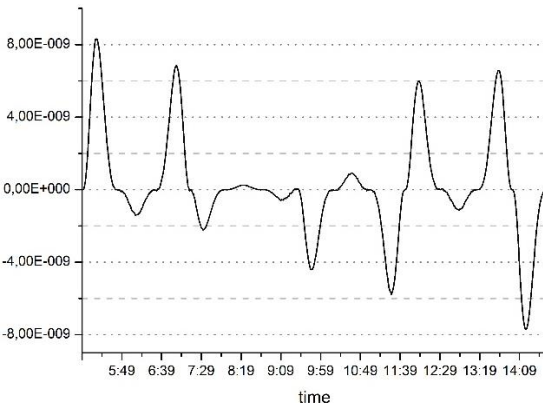
According to this data, the Hermite polynomial of the 11th degree was constructed for time step of 45 minutes. The interpolation error of the x-coordinate is shown in figure 4. Here we use four glued Hermite polynomials (13 nodes) to approximation 9 hours of the satellite motion. The graph shows that the error does not exceed a fraction of mm. The differentiation of the interpolation polynomial allows one to obtain the approximation of the satellite velocity at the same calculated interval within 1 mm/s.



**Figure 2.** The interpolation error  $\sin t - j_j^n(t)$  for polynomials constructed with the use of all considered templates (figure 1),  $t = p/6$ , the segment is  $[p, 3p/2]$ .



**Figure 3.** The derivative approximation error  $\cos t - (j_2^{11})'(t)$  of 4 gluing Hermite polynomials of the 11th degree,  $t = p/6$ , the segment is  $[p/3, 7p/3]$ .



**Figure 4.** The interpolation error for the x-coordinate for the glued Hermite polynomials of the 11th degree for time step of 45 minutes. The data are calculated using a mathematical model.

## Conclusions

Although the interpolation using the Hermite polynomial gives the best approximation of ephemeris, it has some obvious drawbacks. First, it is necessary to know the values of first and second order derivatives at each interpolation node, which are difficult to obtain using the standard data on the position of the satellite. Second, the Hermitian template is the shortest among the considered ones for the same template-size.

In some applications, along with ephemeris, satellite velocity and even acceleration can be known or may be recovery from available data. In the absence of data, it is recommended to use templates 3 or 5, which, within reasonable accuracy, expand the template and do not require the values of derivatives at each node.

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