PAPER • OPEN ACCESS

Flow with heat transfer in a rotating cavity

To cite this article: A A Zuev *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **537** 022026

IOP Conf. Series: Materials Science and Engineering **537** (2019) 022026 doi:10.1088/1757-899X/537/2/022026

Flow with heat transfer in a rotating cavity

A A Zuev[1], A A Arngold[2], M I Tolstopyatov[1], E V Khodenkova[1] and V P Nazarov[1]

[1]Department of Aircraft Engines, Reshetnev Siberian State University of Science and Technologies, 31 Krasnoyarsky Rabochy Av., 660037, Krasnoyarsk, Russian Federation
[2]Department of Special Connectors and Instruments, Krasnoyarsk Machine-Building Plant, 29 Krasnoyarsky Rabochy Av., 660123, Krasnoyarsk, Russian Federation

E-mail: dla2011@inbox.ru

Abstract. An integral relation of the temperature spatial boundary layer energy equation, which allows integration over the surface of any shape to determine the thickness of the energy loss, was obtained. The equations for determining the energy loss of the temperature spatial boundary layer thickness are necessary to determine the local heat transfer coefficients for the characteristic cases of flow, taking heat exchange into account. The corresponding flows in the power unit cavities are considered. Turbulent flows of a gaseous flow were considered. Calculations for local heat transfer coefficients are defined. The local heat transfer coefficient for a rectilinear flow, a rotational flow according to the law of a solid, and a rotational flow according to the law of a free vortex were determined. Calculations for local heat transfer coefficients are defined as Stanton Criteria.

1. Introduction

Consideration of the heat transfer features in flow-through parts of turbopump units (TPU) of liquid-propellant rocket engines (LRE) is an important task. At present, the flow characteristics consideration with heat transfer in the implementation of potential and vortex rotational flow in turbine setting is mainly carried out by the following methods: using empirical equations, numerical and analytical methods for solving partial differential equations [1].

The first method does not always provide the required accuracy of the calculation of the hydrodynamic and thermal characteristics of rotational flows, taking into account heat transfer, and requires additional experimental refinements. As a rule, this entails a rather large expenditure of time and resources for research.

Numerical methods are rather difficult to use when carrying out engineering calculations and require their implementation in specialized software. Numerical methods use direct numerical simulation (DNS method) and Reynolds-averaged Navier-Stokes equation (RANS method). The choice of method depends on the complexity of the problem and the results accuracy. RANS method is quite often used with the use of k-ε and k-ω turbulence models [2-6].

The analytical method allows obtaining analytical dependencies applicable for engineering calculations in a wide range of possible variations of design and operational parameters. Analytical methods, as a rule, were developed for rectilinear uniform flow, and they have several limitations. One of the early studies is the work of E.L. Knuth [7]; his analysis is based on an extended Reynolds analogy with the transfer of heat, mass, and momentum in a developed turbulent flow in a pipe. The use of the velocity and temperature distribution profile in the boundary layer is proposed by W.D. Rannie [8] and modified by D.L. Turcotte [9]. Turcotte's analysis of the underlayer took into account the effect of heat transfer on turbulence. Analytical methods for determining the heat transfer coefficients proposed in [10, 11] take into account convective heat transfer in the liquid propellant rocket engines (LRE) chambers and are designed for rectilinear turbulent flow. A one-dimensional analytical model for subcritical conditions was also proposed by S.R. Shine [12].

2. The object of the study.

When designing the flow parts of the turbopump of liquid-propellant rocket engines (LRE), it is necessary to take into account the temperature change of the working fluid flow along the length of the working channel. And the viscosity parameter acts as a function of temperature and determines the flow regime and, as a consequence, loss, in particular disk friction and hydrodynamic losses in the flow part.

On the one hand, the mode of operation of the supply units is possible, in which a slight heating of the cryogenic working fluid can cause boiling. In this case, there is a drop in performance objectives and possible loss of the unit tightness as a whole. On the other hand, insufficient heating in the flow part of some working fluids leads to their high viscosity and decrease the overall efficiency of the turbine unit.

The main object of the study, where the potential and vortex rotational flow are realized, is the structural elements of gas turbines and centrifugal pumps: inlet and outlet devices, cavities between the stator and the impeller, auxiliary hydraulic path [13].

3. Setting a research problem.

In the generalized problem formulation of fluid flow during heat exchange with the surface of units, such as compressors, expanders, pumps of cryogenic components, etc., it is necessary to take into account the flow temperature change along the length of the working channel, since viscosity, as a temperature function, mainly determines the flow regime and, as a result, hydraulic losses [14].

For the case of the incompressible fluid flow, a joint solution of the equations of motion and energy in the boundary conditions of the spatial boundary layer is necessary [15]. For a compressible fluid, the system must be supplemented with an equation of state.

4. Flow with heat transfer in a rotating cavity.

Taking into account the analysis of the values scale and given the internal heat absence $\varepsilon = 0$, integrating the energy equation along the coordinate y within the thickness of the boundary layer, the equation for the integral relation of the spatial boundary layer energy equation (SBL) is obtained:

$$\frac{1}{H_\varphi}\frac{\partial\left(\delta^{**}_{t\varphi}\right)}{\partial\varphi}+\frac{1}{H_\psi}\frac{\partial\left(\delta^{**}_{t\psi}\right)}{\partial\psi}+\frac{1}{H_\varphi H_\psi}\frac{\partial H_\psi}{\partial\varphi}\delta^{**}_{t\varphi}+\frac{1}{H_\varphi H_\psi}\frac{\partial H_\varphi}{\partial\psi}\delta^{**}_{t\psi}=\frac{\alpha}{\rho C_p U}-\frac{\tau_{\varphi_0}\left(1+\varepsilon^2\right)}{\rho C_p\left(T_\delta-T_0\right)}, \quad (1)$$

where $\delta^{**}_{t\varphi}$ - the temperature spatial boundary layer in the longitudinal direction energy loss thickness; $\delta^{**}_{t\psi}$ - the temperature of SBL in the transverse direction energy loss thickness.

This paper investigates the power turbulent distribution profile of the SBL velocity diagram.

$$\overline{u} = \overline{y}^{\frac{1}{m}} . \quad (2)$$

When integrating the SBL power profile velocity distribution, the extrusion thicknesses and impulse losses were obtained.

To solve the problem of heat exchange with the surface, the heat transfer law is written down:

$$St = \frac{q_0}{\rho C_p U (T_\delta - T_0)} = \frac{\lambda \left(\dfrac{\partial T}{\partial y} \right)_{y=0}}{\rho C_p U \left(T_\delta - T_0 \right)} = \frac{\lambda}{\rho C_p U} \left[\frac{\partial}{\partial y} \left(\frac{T - T_0}{T_\delta - T_0} \right) \right]_{y=0},$$

where $\alpha = \dfrac{q_0}{\left(T_\delta - T_0 \right)}$.

With Prandtl criteria $\Pr = 1$ thickness of the dynamic and thermal boundary layers are the same. To obtain an additional equation relating the thickness of the energy loss of SBL temperature $\delta_{t\varphi}^{**}$ and the law of heat transfer the velocity distribution by a power (2) function is approximated. For the velocity distribution (2) power profile it is impossible to determine the derivative of the temperature boundary layer on the wall taking into account the laminar sublayer:

$$\frac{\partial}{\partial y} \left(\frac{T - T_0}{T_\delta - T_0} \right)_{y=0} = \frac{\partial}{\partial y} \left(\frac{u}{U} \right)_{y=0} = \frac{\partial}{\partial y} \left(\frac{y}{\delta} \right)_{y=0}^{\frac{1}{m}} = \left(\frac{y^{\frac{1-m}{m}}}{\delta^{\frac{1}{m}} m} \right)_{y=0} = 0.$$

Obviously, when m<1 the derivative (11) does not exist formally.

In order to obtain the heat transfer law, a solution of a turbulent power profile and a laminar viscous sublayer is used, which corresponds to a linear dependence:

$$u = \frac{\left(U^* \right)^2 y}{\nu},$$

where $U^* = \sqrt{\dfrac{\tau_0}{\rho}}$ dynamic velocity.

At the boundary of the viscous sublayer, the power law is conjugated with a linear $\dfrac{\left(U^* \right)^2 y}{\nu} = U \left(\dfrac{y}{\delta} \right)^{\frac{1}{m}}$, when $y = \delta_л$. If the condition of convergence and the thickness of the laminar sublayer are taken into account by expressing the term with dynamic velocity, the derivative on the wall can be defined as:

$$\frac{\partial}{\partial y} \left(\frac{T - T_0}{T_\delta - T_0} \right)_{y=0} = U^{\frac{m-1}{m+1}} \left(\frac{m}{\alpha_л^{m-1} \nu^{\frac{m-1}{2}} (m+1)(m+2)} \right)^{\frac{2}{m+1}} \frac{1}{\left(\delta_{t\varphi}^{**} \right)^{\frac{2}{m+1}}}.$$

The heat transfer law for the power profile (2) is:

$$St = \frac{\lambda}{\rho C_p U} \frac{\partial}{\partial y} \left(\frac{T - T_0}{T_\delta - T_0} \right) = \frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}} \left(\frac{m}{\alpha_л^{m-1} \nu^{\frac{m-1}{2}} (m+1)(m+2)} \right)^{\frac{2}{m+1}} \frac{1}{\left(\delta_{t\varphi}^{**} \right)^{\frac{2}{m+1}}}.$$

It should be noted that for the practical implementation of the law of heat transfer, it is necessary to determine the value $\alpha_л$, from the condition of closure of the laminar sublayer with a turbulent profile, with $y = \delta_л$, $m = 7$ which is defined

$$\frac{\partial}{\partial y} \left(\frac{T - T_0}{T_\delta - T_0} \right) = \frac{U^{0,75} \cdot 0,559}{\alpha^{1,5} \nu^{0,75} (\delta_{t\varphi}^{**})^{0,25}}, \tag{3}$$

the friction of a power turbulent profile law considering, with $m = 7$,

$$\left(U^*\right)^2=\frac{\tau_0}{\rho}=0,01256U^2\left(\frac{U\delta_{\varphi}^{**}}{\nu}\right)^{-0,25},$$

and it follows that:

$$\frac{\partial}{\partial y}\left(\frac{T-T_0}{T_\delta-T_0}\right)=\frac{\partial}{\partial y}\left(\frac{u}{U}\right)=\frac{\left(U^*\right)^2}{\nu U}=\frac{0,01256\cdot U^{0,75}}{\nu^{0,75}\left(\delta_{t\varphi}^{**}\right)^{0,25}}. \tag{4}$$

From equations (3) and (4) the laminar sublayer coefficient is determined as $\alpha_л = 12,559$.

The law of heat transfer and the integral relation of the energy equation considering (1) and the integral relation of the SBL equation of the straight-line flow for the power-law turbulent velocity distribution profile (2) are written as:

$$\frac{\partial}{\partial\varphi}\delta_{t\varphi}^{**}=\frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\left(\frac{m}{\alpha_л^{m-1}\nu^{\frac{m-1}{2}}(m+1)(m+2)}\right)^{\frac{2}{m+1}}\frac{1}{\left(\delta_{t\varphi}^{**}\right)^{\frac{2}{m+1}}}-\frac{\tau_{\varphi_0}\left(1+\varepsilon^2\right)}{\rho C_p\left(T_\delta-T_0\right)}, \tag{5}$$

The integral ratio of the SBL equation energy for the rotational flow of the power velocity distribution profile (2):

$$J\varepsilon\frac{\partial}{\partial R}\delta_{t\varphi}^{**}+\frac{J\varepsilon}{R}\delta_{t\varphi}^{**}=\frac{\lambda}{\rho C_p U^{\frac{2}{m+1}}}\left(\frac{m}{\alpha_л^{m-1}\nu^{\frac{m-1}{2}}(m+1)(m+2)}\right)^{\frac{2}{m+1}}\frac{1}{\left(\delta_{t\varphi}^{**}\right)^{\frac{2}{m+1}}}-\frac{\tau_{\varphi_0}\left(1+\varepsilon^2\right)}{\rho C_p\left(T_\delta-T_0\right)} \tag{6}$$

The obtained integral relations (5), (6) are integrated from zero to the current value $\delta_{t\varphi}^{**}$ and determined the loss of energy temperature thickness SBL for rectilinear and rotational flow according to the laws of "solid" $\frac{U}{R}=\omega=const$ and "free vortex" $UR=C=const$.

If we substitute the energy loss values of the temperature SBL thickness into the law of heat transfer, taking into account the neglect of the dissipative term, then the dimensionless heat transfer coefficient in the form of the Stanton criterion can be defined as:

- for straight turbulent flow

$$St=\frac{1}{\Pr^{\frac{m+1}{3+m}}}\left[\frac{m}{\alpha_л^{m-1}(m+2)(m+3)\mathrm{Re}_U}\right]^{\frac{2}{3+m}}$$

- for rotational flow according to the law of a solid

$$St=\frac{1}{\Pr^{\frac{m+1}{m+3}}}\left(\frac{2J\varepsilon\cdot m}{\alpha_л^{m-1}(m+2)(m+3)\mathrm{Re}_\omega}\right)^{\frac{2}{m+3}}$$

- for rotational flow according to the law of a free vortex

$$St=\frac{1}{\Pr^{\frac{m+1}{m+3}}}\left(\frac{2J\varepsilon}{\alpha_л^{m-1}(m+1)(m+2)\mathrm{Re}_\omega}\right)^{\frac{2}{m+3}}$$

The local heat transfer coefficient is defined as:

$$\alpha=\rho C_p U\cdot St.$$

Figure 1 presents a graph of the distribution of the dimensionless heat transfer coefficient in the form of the Nusselt criterion for turbulent rotational flow with the Prandtl criterion $\Pr=0,7$.

Analytical equations for heat transfer coefficients are in good agreement with the results of studies by other authors. The discrepancy between the theoretical dependence of a certain model with a convective component and the results of experimental studies does not exceed 3.5%. The discrepancy between the obtained formula for heat transfer coefficients and the dependence obtained by J.M. Owen [16] does not exceed 2.66%. And the discrepancy with the dependence obtained by I.V. Shevchuk [17] is 9.5%. The discrepancy between the obtained formula for heat transfer coefficients and the dependence obtained by L.A. Dorfman is 16.7% [18].

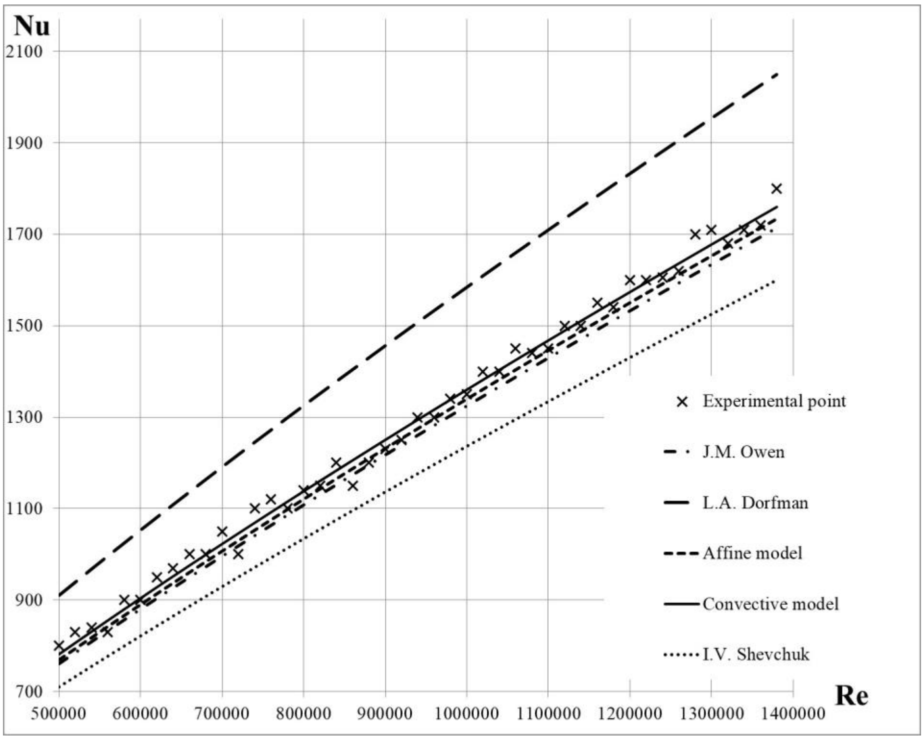

Figure 1. The dependence of the dimensionless heat transfer coefficient of turbulent rotational flow at $\mathrm{Pr} = 0,7$

5. Conclusion

The integral relation of the temperature spatial boundary layer energy equation, which allows integration over the surface of any shape, necessary for determining the thickness of the energy loss, was obtained. Equations for the energy loss of the temperature spatial boundary layer thickness calculating are needed to determine the local heat transfer coefficients for typical cases of flow, taking heat transfer into account.

Equations for determining the local heat transfer coefficient in the form of the Stanton criterion for straight uniform flow, rotational flow according to the law of a solid and rotational free vortex of a power profile flow of the dynamic and temperature boundary layer for Pr ≈ 1 parameters distribution were obtained analytically.

Analytical equations for heat transfer coefficients are consistent with experimental data and dependencies of other authors.

References

[1] Voiskunski JI, Faddeev, YI and Fedyaevsky KK 1982 *Hydromechanics* (Leningrad, Shipbuilding)
[2] Stoll J, Straub J 1988 Film cooling and heat transfer in nozzles, *J. Turbomach* **110** pp 57–64
[3] Dellimore K 2010 *Modeling and Simulation of Mixing Layer Flows for Rocket Engine Film*

Cooling (Ph.D. thesis) (University of Maryland at College Park)

[4] Jansson LS, Davidson L and Olsson E 1994 Calculation of steady and unsteady flows in a film-cooling arrangement using a two-layer algebraic stress model, *Numer. Heat. Transf., Part* **A** **25** 237–58

[5] Cruz C 2008 *Experimental and Numerical Characterization of Turbulent Slot Film Cooling (Ph.D. thesis)*, (University of Maryland at College Park)

[6] Cruz C, Marshall A 2007 Surface and gas measurements along a film cooled wall, *J. Thermophys. Heat. Transf.* **21** 181–9

[7] Knuth EL 1954 *The mechanism of film cooling, (Ph.D. thesis)* (California Institute of Technology)

[8] Rannie WD 1956 Heat transfer in turbulent shear flow, *J. Aeronaut. Sci.* **23** 485–9

[9] Turcotte DL 1960 A sublayer theory for fluid injection into the incompressible turbulent boundary layer, *J. Aeronaut. Sci.* **27** 675–8

[10] Stechman RC, Oberstone J and Howell JC 1969 Design criteria for film cooling for small liquid-propellant rocket engines, *J. Spacecr. Rocket* **6** 97–102

[11] Bartz DR 1957 A simple equation for rapid estimation of rocket nozzle convective heat transfer coefficients*, Jet. Propuls. - ARS J.* **27** 49–51

[12] Shine SR, Kumar SS and Suresh BN 2012 A new generalised model for liquid film cooling in rocket combustion chambers, *Int. J. Heat. Mass Transf.* **55** 5065–75

[13] Kishkin AA, Chernenko DV and Chernenko EV 2008 The equation of impulses of a three-dimensional boundary layer, *Izv. vuzov. Severo-Kavkazskiy region. Tekhnicheskiye nauki* **4** 35-41

[14] Romanenko PN1971 *Heat-mass transfer and friction under the gradient liquid current*(Moskow: Energy)

[15] Schliechting G1969 *Boundary layer theory*(Moscow: Nauka (Science))

[16] OwenJM, Haynes CM and Bayley FJ 1974 Heat Transfer From an Air Cooled Rotating Disk *Proceedings of the Royal Society of London* **336** 453-73

[17] Shevchuk IV 2009 *Convective Heat and Mass Transfer in Rotating Disk Systems* (Springer)

[18] Dorfman LA 1960 *Hydrodynamic Resistance and the Heat Loss of Rotating Bodies*(Moscow:Physmatgis)