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Model of fuzzy estimation of mechanical stress concentration for aerospace and industrial flat structures with polygonal holes of uncertain curvature at rounded corner points

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Abstract. The problem of uncertainty factor management in the description of mechanical stress nearby hatchways of multilateral form with rounded corners of fuzzy curvature in stretch thin plate is considered. Numerical and analytic methodology based on heuristical principle application of extension to transition to fuzzy set arguments in special analytical representations for estimated factors of stress concentration is proposed. Algorithm of fuzzy interval parameter spread of curve in corner points is described. Usage of proposed methodology in theoretical and applied researches on integrity problems of multiply connected thin-walled structure elements is characterized.

1. Introduction

Estimation of mechanical stress concentration level nearby technologic holes and insertions in thin-walled contracture elements is an important part of its integrity analyze [1-4]. It is necessary to take into account uncertainty factor for different exogenous parameters of utilizable deformation model upon receipt of these estimations. In particular uncertainty management in some parameter points of geometric forms of holes can be realized with stochastic modeling [5]. However, in many cases nature of original information about exogenous parameter spread of deformation models fundamentally makes confusing usage of probability theory method in these questions. For solving the problem of taking into account of one from uncertainty factors of stress concentration geometric in thin-walled constructions the usage of alternative fuzzy set methodology [6-11], is proposed to use based on heuristic principle application of extension [8-12] to accurate design ratio of deterministic versions of proper models [13, 14].

2. The problem of determining indicators of stress concentration in rounded top holes in the shape of regular polygons

In the calculation of index stress concentration estimation nearby multangular holes in thin-walled structures, in particular in stretching thin isotropic plates, one of the determining factors of the model

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is curvature indicators in rounded top polygon contour. One of the effective numerical and analytical approaches used in phenomena mechanic stress concentration research nearby multangular holes in thin-walled plates are methods of complex-variable functions (methods of complex potential) [1, 2, 4, 15]. Where by number of variants of complex potential method which are elaborated for stress concentration analysis near corner points of holes with polygonal shape methodology based on usage of conformal representation $z = \omega(\zeta)$ functions of view of analyzed multangular hole in complex area $z = x_1 + ix_2$ on interior rounded contour of singular radius in reference complex plane $\zeta = r\exp(i\theta)$. Effective design ratio of this methodology is received from works [1, 4] using conformal representation functions which are simple by structure a power approximation of Christoffel- Schwartz integrals. Such functions of conformal representation describe polygon holes' geometry with curves in corner points which have [4] curvature

$$k(\theta) = |\omega'(\sigma)|^{-3}\,\mathrm{Re}\{\overline{\omega'(\sigma)}[\sigma\omega''(\sigma) + \omega'(\sigma)]\} \tag{1}$$

In formula (1) $\sigma = \exp(i\theta)$ – values of complex variable ζ on rounded contour of singular radius. In particular during research of mechanic stress concentration near square holes with centers at the beginning of complex plane z and corner points lying on quadrant diagonals in this plane, approximate functions of conformal representation are used

$$z = \omega(\zeta) = R(\zeta^{-1} - m\zeta^3), \tag{2}$$

via use of which contours of rounded in corner points $\theta_n^{(c)} = \pi/4 + (n-1)\pi/2 \quad (n = \overline{1,4})$ square holes with changeable along borders curvature

$$k(\theta) = (1 - 6m\cdot\cos 4\theta - 27m^2)(1 + 6m\cdot\cos 4\theta + 9m^2)^{-3/2} \qquad \theta \in [0, 2\pi] \tag{3}$$

are set on contour circle of singular radius in the area of complex value substitution ζ. Curvature k_c of such a contour in corner points $\theta_n^{(c)}$ is defined by formula

$$k_c = (1 + 6m - 27m^2)(1 - 6m + 9m^2)^{-3/2}, \tag{4}$$

and parameter point m at specified curvature contour k_c in corner points of hole is determined from cubic equation

$$27k_c m^3 - 27(k_c + 1)m^2 + (9k_c + 6)m + (1 - k_c) = 0 \tag{5}$$

and is described by arising from formula Cardano formula.

$$m = -(\alpha + \beta)/2 + i\sqrt{3}(\alpha - \beta)/2 + (k+1)/(3k), \tag{6}$$

$$\alpha = (-q/2 + \delta^{1/2})^{1/3}, \ \ \beta = (-q/2 - \delta^{1/2})^{1/3}, \ \delta = (p/3)^3 + (q/2)^2,$$

$$p = -(4k+3)/(9k^2), \ \ q = -2(k+1)^3/k^3 + (9k^2 + 15k + 6)/(81k^2) + (1-k)/(27k).$$

Contour view of square hole which is described by comfort expression (2) and has curvature in rounded top points $k = 4.5 \ \ (m = 0.44)$, is presented in figure 1.

Received by Cauchy integrals method expression for complex potential $\varphi(\zeta)$ under extension by proportional force intensification P of thin isotropic plate with hole of visible form along direction Ox_1 [3] is as follows

$$\varphi(\zeta) = (PR/4)(\zeta^{-1} + (2/(m+1))\zeta + m\zeta^3) \tag{7}$$

with the usage of expression (7) and formula for contour stress

$$\sigma_{\theta\theta} = 4\,\mathrm{Re}[\varphi'(\sigma)/\omega'(\sigma)] \tag{8}$$

representation for distribution function of normalized contour stress $\sigma_{\theta\theta}$ along hole border is set down

$$F_1(k,\theta) = (\sigma_{\theta\theta}/P) = -((3m+1)(3m-1) + (2(3m+1)/(m+1))\cos 2\theta)/(1 + 9m^2 + 6m\cos 4\theta) \tag{9}$$

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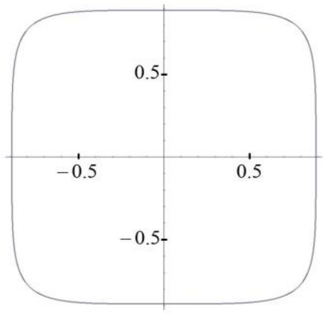

Figure 1. Shape of square hole contour described with usage of reflecting function (2) where $m = 0.44$.

Among other factors, correlation of concentration stress link indicators $(\sigma^*_{\theta\theta} / P)$ runs out from formula (9), in corner contour points $\theta_n^{(c)}$ from curvature parameter k_c of its curving

$$(\sigma^*_{\theta\theta} / P) = F_{1c}(k_c), \tag{10}$$

$$F_{1c}(k_c) = (3m+1)/(1-3m). \tag{11}$$

Hole represented in figure 2 in the shape of equilateral triangle with curvature in rounded top points $k_c = 8$ is described with the help of conformal representation function

$$z = \omega(\zeta) = R(\zeta^{-1} - m\zeta^2) \tag{12}$$

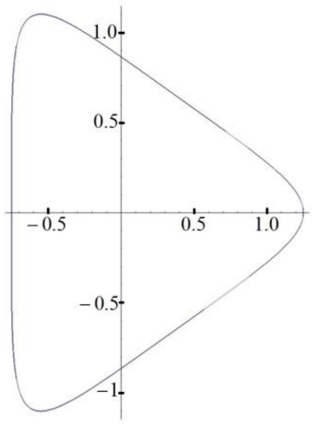

Figure 2. The shape of equilateral triangle hole described with the help of mapping function (12) where $m = -0.25$.

In case of $m = -0.25$. So curvature change along contour hole of such form is described by formula

$$k(\theta) = (1 - 2m \cdot \cos 3\theta - 8m^2)(1 + 4m \cdot \cos 3\theta + 4m^2)^{-3/2} \qquad \theta \in [0, 2\pi]. \tag{13}$$

Contour curvature in corner points $\theta_n^{(c)} = 2n\pi / 3 \;\; (n = \overline{0,2})$ has value

$$k_c = (1-4m)/(1+2m)^2, \tag{14}$$

And parameter value m where contour curvature k_c in corner points of hole is determined by

$$m = ((3k_c + 1)^{1/2} - k_c - 1)/(2k_c). \tag{15}$$

Formula for complex potential $\varphi(\zeta)$ in case of spread by proportional intensive efforts P of thin isotropic plane with a hole of visible shape along coordinate direction Ox_1 has a view of

$$\varphi(\zeta) = (PR/4)(\zeta^{-1} + 2\zeta + (4m+1)\zeta^2). \tag{16}$$

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With usage of (8), (12), (16) representation for function of standard contour stress distribution is written $\sigma_{\theta\theta}$ along border of triangle hole with curves in top points

$$F_2(k,\theta) = (\sigma_{\theta\theta} / P) = \tag{17}$$

$$= (1 - 2m(8m+2) - 4m\cos\theta - 2\cos 2\theta - (6m+2)\cos 3\theta)/(1 + 4m^2 + 4m\cos 3\theta)$$

So, correlation of link of stress concentration indicator $(\sigma^{*}_{\theta\theta} / P)$ in corner point $\theta^{(c)} = 0$ contour of this hole from parameter k_c of curvature of rounding has a view of

$$(\sigma^{*}_{\theta\theta} / P) = F_{2c}(k_c), \tag{18}$$

$$F_{2c}(k_c) = -(16m^2 + 14m + 3)/(2m+1)^2.$$

3. Methodology of getting uncertain estimations for stress concentration index in top points of polygon holes with rounding of uncertain curvature

Taken into account corners rounding of square holes in thin-walled constructions in the current model is a real technique process of undesirable effect reduction of appearance in them of higher level of mechanic stress, determined by integrity level reduction of constructions. With that usage of current approach by way of element of pre-project modeling requires getting factor scores of influences of possible technological spread in exogenous parameters of contour curvature hole in corner point on corresponding indicators of maximum stress concentration. Fuzzy plural methodology is created for solving this problem which is based on usage of alfa-level form of modified heuristic generalization principle [8-12] to exact analytical functional correlations (9), (11), (17), (18) of link of given stress concentration indicators near corner contour points with indicators of rounding curvature.

Hypothesis is entered about non-contrast value of exogenous parameter of contour curvature k_c in rounded top points of polygon hole in the shape of regular polygon by normal trapezoidal fuzzy interval $\tilde{k}_c$ with the list of rap points (k_1, k_2, k_3, k_4) and membership function $\mu_{\tilde{k}_c}(k_c)$. Normal trapezoidal fuzzy interval $\tilde{k}_c$ is presented in the form of range expansion α - slices

$$\tilde{k}_c = \bigcup_{\alpha\in[0,1]} [\underline{k}_\alpha, \overline{k}_\alpha], \ \underline{k}_\alpha = (1-\alpha)k_1 + \alpha k_2, \ \overline{k}_\alpha = \alpha k_3 + (1-\alpha)k_4. \tag{19}$$

For uncertain estimation of stress concentration index in the point of contour hole, corresponded to corner coordinate θ, formula is written

$$\tilde{F}_j(\tilde{k},\theta) = \bigcup_{\alpha\in[0,1]} [\underline{F}_{j\alpha}(\theta), \overline{F}_{j\alpha}(\theta)], \tag{20}$$

where

$$\underline{F}_{j\alpha}(\theta) = \inf_{k\in[(1-\alpha)k_1+\alpha k_2,(1-\alpha)k_4+\alpha k_3]} F_j(k,\theta), \tag{21}$$

$$\overline{F}_{j\alpha}(\theta) = \sup_{k\in[(1-\alpha)k_1+\alpha k_2,(1-\alpha)k_4+\alpha k_3]} F_j(k,\theta).$$

In particular index $\tilde{F}_{1c}(\tilde{k}_c)$ of uncertain estimation of contour stress concentration in rounded top point of uncertain curvature for square hole and index $\tilde{F}_{2c}(\tilde{k}_c)$ of uncertain estimation of contour stress concentration in rounded top point $\theta_0^{(c)}$ of uncertain curvature for triangle hole have the following view

$$\tilde{F}_{jc}(\tilde{k}_c) = \bigcup_{\alpha\in[0,1]} [\underline{F}_{jc\alpha}, \overline{F}_{jc\alpha}], \tag{22}$$

$$\underline{F}_{jc\alpha} = \inf_{k_c\in[(1-\alpha)k_1+\alpha k_2,(1-\alpha)k_4+\alpha k_3]} F_{jc}(k_c), \quad \overline{F}_{jc\alpha} = \sup_{k_c\in[(1-\alpha)k_1+\alpha k_2,(1-\alpha)k_4+\alpha k_3]} F_{jc}(k_c). \tag{23}$$

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4. Numerical analysis results

As an example, the case of getting uncertain estimation for stress concentration where contour curvature index of square hole in corner point by uncertain interval with list of rap points $\tilde{k}_c : (9.4;\ 9.9;\ 10.2;\ 10.5)$ is considered

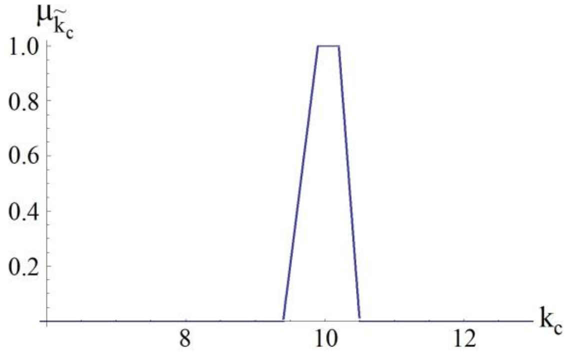

Figure 3. Membership function for $\tilde{k}_c$.

Profile line of membership function $\mu_{\tilde{k}_c}(k_c)$ for fuzzy interval $\tilde{k}_c$ is presented in figure 3. Calculations results of membership function for fuzzy multiplex index characteristic $\tilde{F}_{1c}$ of stress concentration in corner point of square hole with uncertain curvature of rounding, which are received with the help of correlations (6), (11), (22), (23) for option under consider of task of uncertain parameter curvature $\tilde{k}_c$, are presented in figure 4.

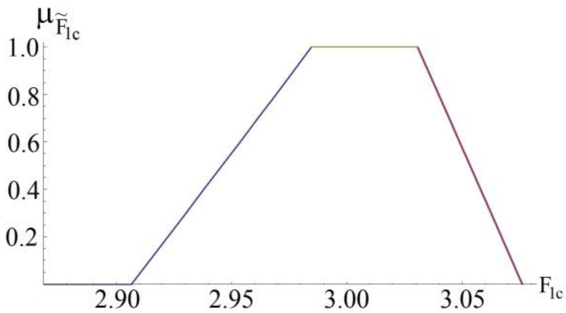

Figure 4. Membership function for $\tilde{F}_{1c}$.

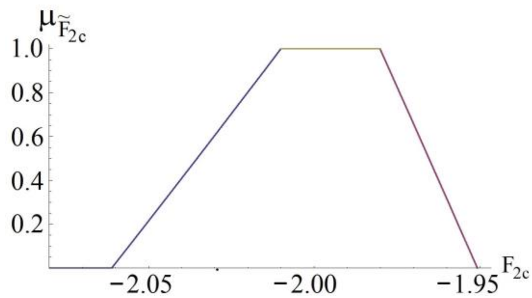

Figure 5. Membership function for $\tilde{F}_{2c}$.

Analogical results for fuzzy plural index characteristic $\tilde{F}_{2c}$ of stress concentration in corner point $\theta_0^{(c)}$ of triangle hole with uncertain curvature of rounding which are received from correlation usage (6), (11), (22), (23) for option of the task with uncertain parameter of curvature $\tilde{k}_c : (7.4;\ 7.9;\ 8.2;\ 8.5)$, are presented in figure 5.

Presented in figure 4, figure 5 disposals testify that degrees of uncertainty indicators of contour stress concentration in rounding corner points of holes under consideration have lower relative level in comparison with spread index for exogenous parameters of contour curvature in corner points.

Derived estimations provide valuable and reasonable conclusions about the most accurate deviation range in values of analyzed stress concentration indicator values in considered curvature spread of rounded top points holes and also about bounders of possible concentration indicator values on the lowest confidence level.

5. Conclusion

For solving the taking account problem of uncertainty factor of geometry characteristics of concentrators of mechanic stress in thin-walled construction elements of plate type with rounded top points holes of multangular shape numerical and analytical methodology is proposed which is based on machinery of fuzzy calculations theory. Approach under consideration consists in usage of alfa-level form of heuristic principle of spread during transaction to fuzzy plural arguments in received in special form of analytical representations for estimated indefinite indicators of mechanic stress concentration in rounded corner points under spread of values of curvature parameters. Calculations results are presented which testify efficiency of proposed methodology.

References

[1] Murakami Y 2017 *Theory of Elasticity and Stress Concentration* (Chichester: John Wiley & Sons)
[2] Momčilović N, Motok M and Maneski T 2013 Stress concentration on the contour of a plate opening: Analytical, numerical and experimental approach *J. Theoret. Appl. Mech.* **51(4)** 1003-12
[3] Kharat A and Kulkarni V V 2013 Stress Concentration at Openings in Pressure Vessels – A Review *Int. J. Innovative Res. Sci. Eng. Technol.* **2(3)** 670-8
[4] Savin G 1961 *Stress concentration around the holes* (Oxford: Pergamon Press)
[5] Lomakin V A 1970 *Statistical problems of the mechanics of solid deformable bodies* (Moscow: Nauka)
[6] Nikolova-Poceva S and Iliev A 2016 Hybrid Fuzzy Regression Model for Determining Specific Active Power Generation Characteristic of Hydro Power Plants *International Journal on Information Technologies and Security* **8(1)** 55-68
[7] Kandasamy W B V, Smarandache F and Ilanthenral K 2009 *Special set linear algebra and special set fuzzy linear algebra* (Slatina, Judetul Olt, Romania: Editura CuArt)
[8] Bede B 2013 *Mathematics of Fuzzy Sets and Fuzzy Logic* (Berlin Heidelberg: Springer-Verlag)
[9] Sonbol A H and Fadali M S 2012 TSK Fuzzy Function Approximators: Design and Accuracy Analysis *IEEE Trans. Syst. Man and Cybern*. **42** 702-12
[10] Hanss M 2005 *Applied Fuzzy Arithmetic. An introduction with Engineering Application* (Berlin Heidelberg: Springer-Verlag)
[11] Anastassiou G A 2010 *Fuzzy Mathematics: Approximation Theory* (Berlin Heidelberg: Springer-Verlag)
[12] Rotshtein A P, Shtovba S D and Kozachko A N 2007 *Simulation and optimization of reliability of multidimensional algorithmic processes* (Vinnitsa: Universum)
[13] Nombre S B, Priymenko S A and Storozhev S V 2017 Estimates of the influence of the fuzziness of geometric exogenous parameters in the model of the stretching of an orthotropic plate with an elliptical hole *J. Theoret. Appl. Mech.* **58** 19-26
[14] Bolnokin V E, Nombre S B and Storozhev S V 2017 Estimations of influence of uncertainty of exogenous parameters in the model of generalized plane stressed state of isotropic plate with elliptic elastic inclusion *Mekh. Tverd. Tela* **47** 122-36
[15] Malkov V M and Malkova Yu V 2015 Deformation of a plate with elliptic elastic inclusion *Vestnik St.Petersburg University Ser. 1* **60(4)** 617-32
[16] Grzegorzewski P and Mr´owka E 2005 Trapezoidal approximations of fuzzy numbers *Fuzzy Sets Syst.***153** 115-35
[17] Ban A I, Coroianu L C and Grzegorzewski P 2011 Trapezoidal approximation and Aggregation *Fuzzy Sets Syst.* **177** 45-59