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Computation of vertical bearing capacity factor N_c of strip footing by FEM

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Abstract. Thispaper presents the computation of vertical bearing capacity factor N_c of the smooth and rough strip footings using the finite element algorithm based elasto-viscoplasticstrain method. The soil behaviour is idealized as elastic perfectly plastic satisfying the Mohr Coulomb yield criterion with only non-associated flow rules because it tends to treat the actual behaviour of soil in which value of its frictional angle is always different from that of dilative angle. The bearing resistance is computed using the superposition assumption in Terzaghi classical equation in the way that the surcharge pressure and the unit weight of soil are neglected. Consequently, N_c is determined. The comparison of the present solutions with the previous published works including experimental and numerical studies have been presented and discussed. The results reveal in close agreement with the existing published data. In the nutshell, by applying the non-associated soil flow rule representing the real behaviour of soil, the conservation of the superposition assumption is confirmed. Therefore, it could conclude that the results obtaining from these techniques may be confidently applied to a wide range of soil bearing resistance problems.The magnitudes of the factors for a rough footing base provide the same values as the smooth footing for the entire range of soil friction angles (phi).

1. Introduction

The computation of the bearing capacity of footing, such as strip, circular, ring and conical footing, is one of the most extremely important area in geotechnical engineering for both researchers and practising engineers. Generally, the calculation of bearing capacity is based on the superposition assumption proposed by Terzaghi, in which the contribution of different loading and soil strength parameter such as cohesion, friction angle, surface surcharge and self-weight, expressed in the form of non-dimensional bearing capacity factors N_c, N_q and N_γ are added [1]. Based on the used solution techniques, both analytical and numerical solutions to bearing capacity problem can be classified into the following categories: the limit equilibrium method, the upper bound plastic limit analysis method, method of characteristics, finite difference method and finite element method. In recent year, finite element method has been widely used to compute the bearing capacity of strip, circular, ring and conical footings and its pre- failure behaviour because it is capable of combining all the parameters into a single problem. It has seen that analytical expressions derived from the slip line method [2], upper bound of limit analysis [3] and limit equilibrium method [1] provide the bearing capacity factors N_c that remain unchanged from the variation in the roughness of the footing-soil interface. Therefore, this paper also aims to illustrate the no difference between the smooth and rough strip base in term of bearing capacity factor N_c.

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Finite elements algorithm based elasto-viscoplastic solutions have been successfully employed to evaluate the bearing capacity [4-6]. The current note presents the computation of bearing capacity factor Nγ of strip footing, which is generally representative of the footings of buildings, using finite element algorithm based elasto-viscoplastic solution. The soil behavior is idealized as elastic perfectly plastic satisfying the Mohr Coulomb yield criterion with only non-associated flow rules because it tends to treat the actual behaviour of soil as the value of its frictional angle is always different from that of dilative angle. The total bearing resistance is computed using the superposition assumption in Terzaghi classical equation of the three basic components. The comparison of the authors' results with previous available published works, including experimental and numerical studies, have been presented and discussed. The results reveal a close agreement with existing published data.

2. Solution techniques

The technique of the solution is generally divided into two: analytical technique and numerical technique. Moreover, there are several methods are embedded inside these two techniques. In this paper, finite element algorithm based viscoplastic technique has proven to be an efficient way of solving the plasticity problem in geomechanics is employed. Likewise, this technique has been suggested by [7,8] and subsequently successfully used by [4-6]. This method is also known as the initial strain solution techniques. The plasticity problem is solved by iterating using the equivalent elastic solutions until any stresses which temporarily violated yield have returned back to the failure surface by obeying the fairly strict tolerances. To verify the change in body's forces from iteration to one another, the convergence criterion plays a vital role in doing so. As the stresses violate the yield, the balancing forces of the body (self-equilibrating body forces) are incremented at each iteration by the amount related to the magnitudes of violated stresses. As the increment in body forces diminished, the stresses also return to the yield surface. For this present work, convergence is allowed as long as the change in body forces with respect to the maximum absolute value, nowhere is more than 0.1%. Yet, [7, 8] have also reported a failure of the algorithm to converge at collapse. Throughout this study, 500 is set to be the maximum restricted value for the number of steps of time integration. The displacement control finite element is employed satisfying the equilibrium and continuity. Eight-node quadrilateral isoparametric element with reduction of integration technique using a (2x2) Gaussian quadrature is used for all types of foundations to compute their stiffness. To define the bearing pressure, the average of the vertical stress component in the first row of integration points below the displaced nodes is calculated.

3. Definition of the problem

To complete the simulation in this paper, the plane strain solutions are raised to compute the bearing capacity factors for strip footing underlying the general cohesive-frictional soil. The ground surface is horizontal and besides the footing, the soil surface is subjected to the uniform surcharge pressure q, while the footing is subjected to vertical downward load Q without any shear stress (eccentricity). Smooth footing base is considered assuming there is no mobilized shear stress over the base, while rough footing base has a perfect bounding with the ground underneath. Table 1 shows the summary of the calculation terms for the bearing capacity of strip footings. The footing is placed on a general cohesive-frictional soil with thickness (h). The ultimate bearing pressure (q_{ult}) is evaluated based on the equation (1). The notation $Q_u, p \; and \; b$ used in equation (1) represent the ultimate applied load, the vertical pressure at any point along the footing base and a half-width of strip footing respectively. The mesh of the present analysis is illustrated in Figure 1. It has 1307 and 408 representing the node number and the element number, respectively.

Table 1. A summary of the calculation terms for strip footing.

Calculation terms	Strip footing
Plane strain	✓
Element type: 8 node quadrilateral element	✓
Ultimate pressure load	Equation (1) $Q_u = \int_0^b p dy$ $q_{ult} = Q_u / b$
Perfectly smooth and perfectly rough bases	✓
Soil friction angles, $\phi = 5^o, 10^o, 15^o, 20^o, 25^o, 30^o, 35^o, 40^o, 45^o$	✓

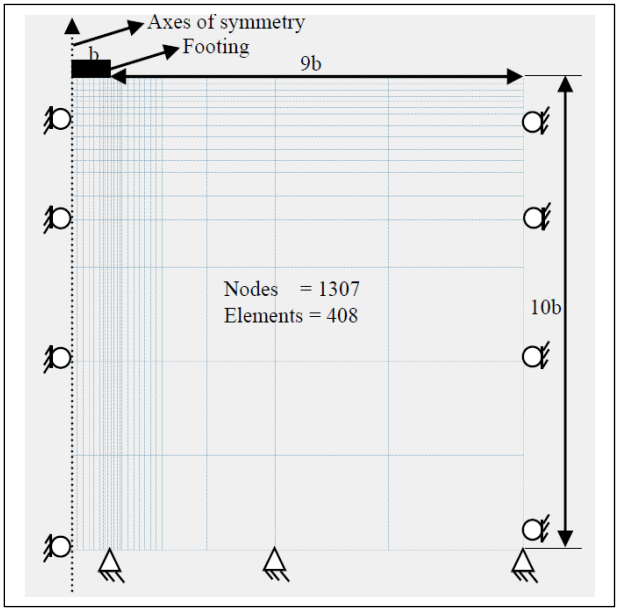

Figure 1. Mesh used in the present analysis

4. Soil constitutive relation

Soil behaviour satisfying the Mohr Coulomb yield criterion with only non-associated flow rules because it tends to treat the actual behaviour of soil as value of its frictional angle is always different from that of angle dilator. Regarding the determination of expansion angle, [10] reported that soil dilation angle varies between 0° and 20° and is at least 20° lower than angle of soil friction. Moreover, [11] utilized the light microscopy (LM) and scanning electron microscopy (SEM) imaging techniques for his experimental works to study the influence of grain shape on dilatancy using 4 different types of sand such as Silica 40 sand; Ottawa 20 sand; Rillito River sand and Santa Cruz River Sand. Melissa found that the limit of soil expansion angle is between 0° to 15°. Hence, the present analysis is conducting by using the non-associated soil with the maximum dilation angle value of 15° if the internal friction angle of soil is greater than 35°.

The material properties of soil throughout the analysis in this paper are assumed such as Young's modulus E = 2e5kN/m^2; Poisson's ratio ν=0.3, while cohesion c = 20kN/m^2; unit weight of soil γ=20kN/m^3 and surface surcharge q = 25kN/m^2. All these parameters are taken into account, if necessary. The friction angles of soil varied from 5° to 45° for the parametric study of bearing capacity factors.

5. Results and discussion

The bearing capacity of shallow strip footing based Terzaghi's superimpose assumption is expressed as:

$$q_{ult} = c \times N_c + q_s \times N_q + \gamma \times b \times N_\gamma / 2 \quad (2)$$

These three dimensionless bearing capacity factors N_c, N_q and N_γ are assumed to be a function of the soil friction angle. Finite element method has been utilized together with plasticity theory to predict bearing capacity factors. Griffiths used finite elements to compute each of the bearing capacity factors of strip footing in turn as a function of the soil friction angle by assuming (1) weightless soil with cohesion and friction to determine the N_c, (2) weightless, cohesion-less soil with friction receiving a uniform surface surcharge to evaluate N_q and (3) cohesion-less soil with friction and self-weight to compute N_γ [4]. The present analysis has been conducted for both perfectly smooth and rough footings with the soil friction angle ranging from ϕ=5° to 45°. The vertical displacements applied to the footing are carried out continuously downwards with the constant rate of velocity until soil failure is reached. The soil resistance is based on parameters ϕ and c representing the angle of friction and cohesion of soil, respectively.

5.1. Computation of N_c

To define solely the contribution of the N_c term to the total bearing capacity of a footing, the assumption of soil to be weightless with no surcharge acting is made by vanishing the last two terms of the equation (5). With the ultimate bearing capacity, the Nc value corresponding to the particular friction angle computed in the analysis is obtained using:

$$N_c = q_{ult}/c \quad (3)$$

Where q_{ult} is computed from equation (1).

Figure 5 shows the comparison of the computed bearing capacity factor N_c from present analysis (both smooth and rough footings are illustrated with cyan and blue color, respectively) with the published solution available in the literature. The numerical results of N_c are summarized and shown in Table 4. The values with and without parenthesis in Table 2 respectively demonstrate the solutions of smooth and rough footings analysis. The average solutions of this analysis are in excellent agreement at low and medium friction angles for ϕ<=40° with the solution provided by Prandtl (1921) (used slip line method) [4] and with [3] (based on upper bound analysis), but it is significantly lower than theirs for ϕ>40°. In similar way, Terzaghi used limit equilibrium to solve for N_c. His results are in close agreement the present analysis at low-friction angles ϕ<25°. However, his solutions are considerably greater than the current solution for high friction angles ϕ>25°.

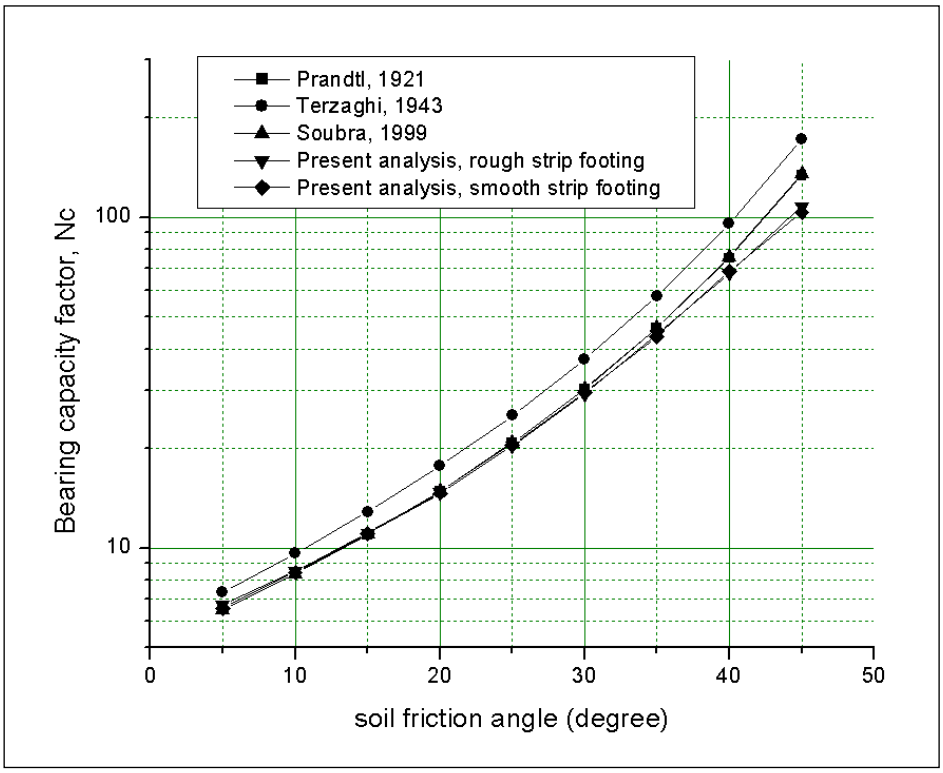

Figure 2. Comparison of bearing capacity factor N_c of smooth and rough strip footing from the represent analysis with published solutions

Table 2. A summary of the calculation terms for strip footing.

Authors	Methods	Soil friction angle ϕ (degree)								
		5	10	15	20	25	30	35	40	45
Prandtl, 1921 [4]	Slip line method	6.49	8.35	10.98	14.83	20.72	30.14	46.12	75.31	133.88
Terzaghi, 1943 [1]	Limit equilibruim	7.34	9.61	12.86	17.69	25.13	37.16	57.75	95.66	172.28
Soubra, 1999 [3]	Upper bound analysis	6.5	8.36	10.99	14.86	20.77	30.24	46.33	75.77	134.99
Present analysis	Finite element	6.57 (6.73)	8.43 (8.52)	11.03 (11.10)	14.64 (14.69)	20.39 (20.32)	29.44 (29.14)	43.64 (44.28)	68.67 (67.91)	103.73 (107.59)

6. Conclusions

Bearing capacity factor N_c was computed for a smooth and rough strip footing using 2D finite element method. The analysis was executed by considering the actual properties of soil in which the dilation angle is generally less than 15°. The results from the present analysis reveal in close agreement with the existing published data. The assumption of additive contribution of soil cohesion, surcharge and weight was proposed by Terzaghi provides bearing capacity predictions sufficiently accurate. Bearing capacity factors depend on the angle of dilation, especially for higher values of the friction angle. The

magnitudes of the factors for a rough footing base provide the same values as the smooth footing for the entire range of soil friction angles (phi).

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