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Tree etude problems on composite fuel gas tank

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Abstract. As an illustration of the design features of composite winding pressure gas tanks, the article provides three illustrative simple examples:
1. calculation of the critical pressure in composite cylinders according to the layer-by-layer analysis scheme for pairs of symmetrically wound layers;
2. assessment of the permissible wall thickness by a rational design based on the “thread analogy”;
3. analysis of the effectiveness of multi-cavity spherical gas tank, which are located in each other - box in a box model ("nested doll").

1. Calculation of bearing capacity by criteria for pairs of layers

The cylindrical part of the gas tank is wind with a unidirectional tape with a symmetrical alternation of orientation angles, so that the reinforcement system can be broken down into orthotropic pairs of layers ($\pm\alpha$). If the stacking symmetry were not observed, under internal pressure, not only the axial and circumferential stresses would arise, but also shear ones, leading to a curvature of the cylinder axis, to its twisting, which is clearly undesirable.

We illustrate the use of a layer-by-layer method [1, 3] for pairs of layers using the example of a cylindrical section of thin-walled tank. Axial $\sigma_z = pR/(2h)$ and circumferential $\sigma_\theta = pR/h$ stresses are created by the internal pressure p. Where R, h are the average cylinder radius and wall thickness. If we denote E_{ij} – four constants of elasticity modules matrix of a monolayer, then for each pair of layers the Young’s modulus in the axial direction is expressed in the following form:

$$E_z(\alpha) = \frac{\left(E_{11}E_{22} - E_{12}{}^2\right)\cos^2 2\alpha + E_{66}\left(E_{11} + E_{22} + 2E_{12}\right)\sin^2 2\alpha}{E_{11}\sin^4\alpha + E_{22}\cos^4\alpha + \left(2E_{12} + 4E_{66}\right)\sin^2\alpha\cos^2\alpha} \tag{1}$$

The Young’s modulus $E_\theta(\alpha)$ is expressed by the Eq. (1) with $\cos\alpha$ replaced by $\sin\alpha$. Under Voigt’s assumption of equal deformation of all layers and the composite as a whole, the effective elasticity modules of the composite are found by averaging over all pairs of layers. Knowing the elastic modules and average stresses, we can determine the average circumferential and axial strains, multiplying which by the corresponding elastic modules (1), we find the circumferential and axial stresses for each pair of layers. Further, for each pair of layers under the conditions of the known biaxial stress state we apply the “tilting rhombus type” strength criteria [1, 2, 3] (Figure. 1):

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$$\left| \frac{\sigma_z}{\sigma_z^*(45)} \operatorname{tg}\alpha - \frac{\sigma_\theta}{\sigma_\theta^*(45)} \operatorname{ctg}\alpha \right| = 1. \qquad (2)$$

where $\sigma_z^*(45)$ and $\sigma_\theta^*(45)$ **-** axial and circumferential strengths of the composite tube wound with one pair of layers at angles $\pm 45^{\mathrm{O}}$.

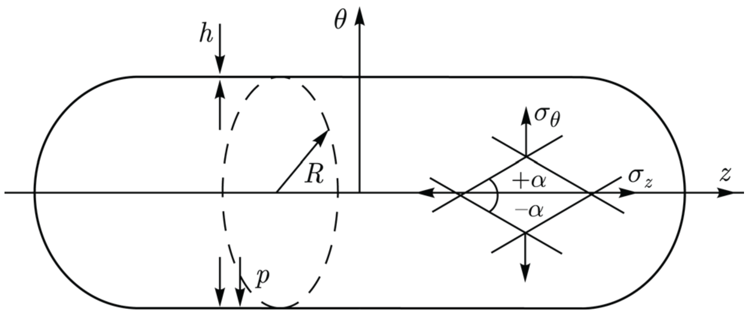

Figure 1. The cylinder scheme and the rhombus model of inextensible threads.

The fiber rupture condition (straight line 3 in Fig. 2) takes the form:

$$\sigma_z = \sigma_0 \cos^2\alpha;\ \sigma_\theta = \sigma_0 \sin^2\alpha, \qquad (3)$$

where in rupture moment $\sigma_0 = \sigma_z^*(0)$ – critical tensile stress along the fibers.

The highest strength according to (2) corresponds to the optimal winding angle α^* (or the optimum ratio of applied stresses $tg\beta = \sigma_\theta/\sigma_z$) [4, 5, 6]:

$$\alpha^* = arctg\sqrt{tg\beta\ \frac{\sigma_z^*(45)}{\sigma_\theta^*(45)}}. \qquad (4)$$

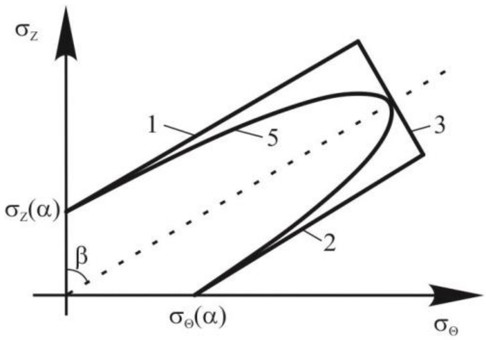

Figure 2. Limiting surfaces in the stress space for a tube with winding (±α) under biaxial tension: 1, 2 – the conditions for tilting rhombus type (2), 3 – fiber rupture condition (3), 5 – limiting ellipse (5).

The use of piecewise linear approximation (2) - (3) in computer calculations is more difficult than using a single expression for the strength criterion. In the biaxial tension of the tubes under stresses σ_z and σ_θ, the experimental data in the σ_z–σ_θ coordinates are well described by the ellipse section (*5* in Fig. 2). The ellipse passing through points on the axes ($\sigma_z(\alpha)$ and $\sigma_\theta(\alpha)$) corresponding to the strengths under uniaxial loading, and through a point on the optimal loading line (4) from the origin to the length associated with the strength (3) along the fibers $\sigma_z^*(0)$:

$$\sigma_z{}^2t^4 \ +\sigma_\theta{}^2-\sigma_z\sigma_\theta\left(2t^2-\frac{\sigma_z^{*2}\left(45\right)\left(1+t^2\right)^2}{\sigma_z^{*2}\left(0\right)}\right)=t^2\sigma_z^{*2}\left(45\right), \tag{5}$$

where $\sigma_z{}^*(0)$, $\sigma_z{}^*(45)$ are the tensile strengths of tubes with windings (0) and ($\pm 45^{O}$); $t=tg\alpha$.

2. Rational design of the cylinder is a thread analogy

A simpler scheme for calculating the winding gas tanks is to use three assumptions, the so-called 'thread analogy' [5]:

1. The fibers work only on tension condition and carry the entire load, and the matrix is not loaded at all.
2. All fibers are equally stressed.
3. The structural failure occurs as a result of the limit stress $\sigma_0 = \sigma_z{}^*(0)$ reaching in all fibers (at the same time, according to the second assumption).

Assumption 2 is very 'strong', it restricts the considered winding systems only to the class of equally stressed fibers: there are no superfluous fibers, all fibers work "at the end of tether". Such constructions in reality do not exist, and should not exist. The failure of them is like an explosion – all fibers are break simultaneously, but this model can be used for engineering estimates. Using the example of the 'thread analogy' it is easy to explain the concept of '*rational design*'.

The often used term '*optimal design*' – in the broadest sense of the word means the creation of the best structure. In a narrower sense, the optimal design is traditionally called the process of finding in the design space bounded by force, kinematic, criterial, technological and other conditions, a multidimensional vector of design parameters that implements a minimum of some goal function (for example, weight, cost or some convolution of these or other criteria). Usually, optimization problems are solved by methods of non-linear programming with the use of 'penalty' functions.

Designing is called rational when certain parameter relationships are predetermined, for example, the equal strength and equal stress in all fibers. This greatly simplifies the search for an optimum. In fact, this finding of a conditional optimum is not the "top of the mountain", but the highest point on a certain section "of the mountain". In the simple case of equally stressed fibers, we get a certain maximum optimum: no fiber can be added – it will be underloaded and no fiber can be eliminated: the stresses in the remaining fibers will exceed the ultimate strength.

As an example, consider the scheme of reinforcement of the cylindrical part of a gas tank by two families of fibers: 1 – with orientation $\pm\alpha_1$ and with layer thickness h_1 and 2 – with orientation $\pm\alpha_2$, with thickness h_2. The total wall thickness $h=h_1+h_2$. The average axial and circumferential stresses

$$\sigma_z=\frac{pR}{2h},\ \ \sigma_\theta=\frac{pR}{h} \tag{6}$$

in a critical state can be expressed through the limiting stress along the fibers σ_0, summing up the forces created in the two families of layers,

$$\begin{gathered}\frac{A_0}{2}=h_1\cos^2\alpha_1+h_2\cos^2\alpha_2\\ A_0=h_1\sin^2\alpha_1+h_2\sin^2\alpha_2\end{gathered} \tag{7}$$

where $A_0 = pR/\sigma_0$.

Two equations (7) contain four design parameters, two of which can be found after setting the other two arbitrarily (?). The question-mark (?) means that we want to make sure that we can really specify any values of the angles. Suppose that h_2=0. There remain two parameters that are found from (7):

$$h=h_1=\frac{3A_0}{2};\ \ \alpha_1^*=\alpha^*=54^044';\ \left(\mathrm{tg}^2\,\alpha^*=2\right).$$

Choosing angles α_1, α_2 equal to (±30/90), (±45/90), (0/±60), we will see that the total thickness of the wall remains the same, $3A_0/2$. The same will remain for the algebraic sum of the thicknesses and for a combination of angles (±60/90), (0/±45) or (0/±30), only one of the thicknesses will be negative, which means that it is impossible to provide equilibria in two stresses if two families of fibers have an orientation angle both smaller or both larger than α^*. In fact, not the thickness is negative, and the stresses in these, ‘unsuccessfully’ selected families, should become compressive to ensure the stress uniformity of the fibers (7).

This example shows that any rational project of equally stressed families of fibers provides the same weight of the structure under given loading conditions; choosing the best remains for technological or constructive reasons.

3. Multi-cavity spherical gas tank

Multi-cavity gas tanks (of the type “nested doll”) - (Fig. 3) are of interest to reduce the overall dimensions, since for a given external radius of the increasing number of the inner tanks to increase their allowable pressure, and hence can increase the total mass of injected gas.

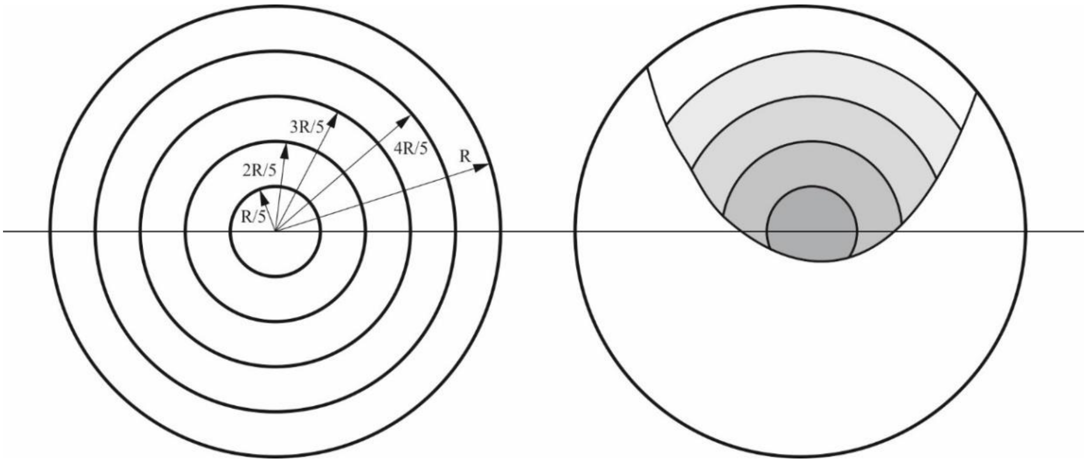

Figure 3. Multi-cavity gas tank in section.

As a model, consider ***n*** equidistant spherical gas tank put one into another (box in a box model or "russian nested doll") with a constant wall thickness *h*. The outer cylinder with the number *n* has a radius *R*, the inner, the smallest with the number 1 – *R/n* accordingly, the *i*-th cylinder has a radius *iR/n*.

1) The total mass of the gas tank, which naturally increases with the number of tanks:

$$M = \sum_{i=1}^{n} m_i = 4\pi\rho h R^2 \sum_{i=1}^{n} \frac{i^2}{n^2} = B\frac{n(n+1)(2n+1)}{6n^2} = \frac{B}{6}\left(1+\frac{1}{n}\right)(2n+1); \tag{8}$$

where $B=4\pi\rho hR^2$ is the mass of the outer (largest) sphere;
ρ **-** the density of the tank material.

2) The volume of the cavities between the spheres:

$$v_1 = \frac{4}{3}\pi\left(\frac{R}{n}\right)^3 = \frac{A}{n^3}; \quad v_i = A\frac{i^3-(i-1)^3}{n^3}; \; i=1,...,n; \tag{9}$$

where $A=4\pi R^3/3$ **-** is the volume of the external spherical gas tank.

3) Calculate the allowable pressure in each cavity. This is analytically - the most complex, iterative procedure, not solvable in algebraic form, since there is no general formula for partial sums of a harmonic series. The permissible stress $[\sigma]$ for each cavity is expressed in terms of the external p_{i+1} and internal p_i pressure, which, for convenience, we refer to 0.1 *MPa* = 1 *atm* to estimate how many times the volume of accumulated gas exceeds the volume of the cavity v_i:

$$[\sigma]=(p_i-p_{i+1})\frac{R_i}{2h};\ N_i=\frac{p_i}{p^0};\ \sigma^*=\frac{[\sigma]}{p^0}\Rightarrow$$
$$N_n=C;\ N_i=C\frac{n}{i}+N_{i+1}=Cn\left(\frac{1}{n}+\frac{1}{n-1}+...+\frac{1}{i}\right)=Cn\sum_{k=0}^{n-i}\frac{1}{n-k}=Cn\sum_{j=i}^{n}\frac{1}{j},\tag{10}$$

where C=2$h\sigma^*$/R the permissible gas pressure (in atm.) in the outer (largest) spherical gas tank.

4) Calculate from (9), (10) the total volume of gas in all cavities in terms of the normal pressure of 1 atm. The volume of accumulated gas increases with the number of cavities with the former external dimensions, i.e. with the same radius of the external sphere:

$$V=\sum_{i=1}^{n}(v_iN_i)=\sum_{i=1}^{n}\left\{\frac{AC}{n^2}\left[i^3-(i-1)^3\right]\sum_{j=i}^{n}\frac{1}{j}\right\}.\tag{11}$$

5) We find from (8), (11) the efficiency factor K, as the ratio of the largest possible volume of gas in a multi-cavity tank to the mass of the walls:

$$K=\frac{V}{M}=\frac{AC}{B}=\frac{2\sigma^*}{3\rho}.\tag{12}$$

As can be seen from equality (12), obtained on the basis of numerical calculations, the efficiency factor K (in the 1st approximation, without taking into account the effect of wall thickness on cavity volumes) does not depend on the gas tank radius, nor on the wall thickness, nor on the number of embedded tanks. This is, in fact, a property of the gas tank material: metal, composite. You can compare, for example, the effectiveness of steel and aluminum cylinders. The result is not obvious and ambiguous. It is necessary to check the validity of formula (12) by direct arithmetic calculations for n = 1, 2, 3, 4, 5, etc.

Table 1. The results of calculations by formulas (8) - (12) for small number of cavities.

	1 cavity	2 cavities	3 cavities	4 cavities	5 cavities
The mass of gas tank M (Eq. 8) kg	$4\pi\rho R^2 h\sigma^*=B$	$\frac{5}{4}B$	$\frac{14}{9}B$	$\frac{15}{8}B$	$\frac{11}{5}B$
The volume of gas tanks v_1 (Eq. 9) m^3	$\frac{4}{3}\pi R^3=A$	$v_1=A/8$; $v_2=7A/8$	$v_1=A/27$; $v_2=7A/27$; $v_3=19A/27$.	$v_1=A/64$; $v_2=7A/64$; $v_3=19A/64$; $v_4=37A/64$.	$v_1=A/125$; $v_2=7A/125$; $v_3=19A/125$; $v_4=37A/125$; $v_5=61A/125$.
Permissible pressure in each tank N_n (Eq. 10)MPa	$\frac{2h\sigma^*}{R}=C$	$N_1=3C$; $N_2=C$;	$N_3=C$; $N_2=5C/2$; $N_1=11C/2$;	$N_4=C$; $N_3=7C/3$; $N_2=5C/2$; $N_1=11C/2$;	$N_5=C$; $N_4=9C/4$; $N_3=47C/12$; $N_2=77C/12$; $N_1=137C/12$.
Bulk-volume gas V (Eq. 11) m^3	$v_iN=AC$	$\frac{5}{4}AC$	$\frac{14}{9}AC$	$\frac{15}{8}AC$	$\frac{11}{5}AC$
Efficiency factor $K=\frac{V}{M}$	$\frac{2\sigma^*}{3\rho}=\frac{AC}{B}$	$\frac{AC}{B}$	$\frac{AC}{B}$	$\frac{AC}{B}$	$\frac{AC}{B}$

To obtain an analytical expression for the partial sums of the harmonic series, no one else (since the time of Euler) has succeeded.

Two words about the features of the harmonic series. The name “harmonic” itself is connected with the fact that when the length of a string (violin) decreases by a factor of **i**, its tone corresponds to the **i**–th harmonic with respect to the total length of the string. Euler proposed the following approximation:

$$S_n = \sum_{i=1}^{n} \frac{1}{i} = \ln n + \gamma + \varepsilon_n; \;\; \varepsilon_n \xrightarrow[n\to\infty]{} 0;$$

where γ=0.5772 - Euler-Macheroni constant.

Table 2. Partial sums of the harmonic series

n	1	2	3	4	5	6	7	8	∞
S_n	1	3/2	11/6	25/12	137/60	49/20	263/140	761/280	∞

Another surprising property of the harmonic series (apart from the absence of an analytical expression for partial sums) is that these sums grow very slowly. Despite the fact that the series diverges, that is, its partial sum tends to infinity, in order to achieve, for example, a value of 100, it is necessary to sum up an insanely large number of members of the order of 10^{43}. The computer with a speed of a billion operations per second will calculate over millions of years such a number members. But this fact, apart from curiosity, has nothing to do with the task at hand: it’s just that - a surprising number.

4. Conclusion

1. Using the cylindrical part of the gas tank as an example, the easiest way to illustrate the possibility of optimizing the structure is by choosing the winding angles.
2. Any rational project of equally stressed families of fibers provides the same mass of structure under given loading conditions; choose the best is from technological or design considerations.
3. In the first approximation, without taking into account the wall thickness, the ratio of the largest amount of gas stored in a multi-cavity gas tank to the mass of its walls does not depend on the tank radius, nor on the wall thickness, nor on the number of built-in tanks.

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