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Analytical and numerical modeling of a clamped-clamped micro beam under electric actuation

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Abstract. This paper studies the deflection of a clamped-clamped micro beam actuator subjected to an electric voltage. Two approaches were applied to model the micro beam under study: the first one is analytical and the second is numerical. A solution to the analytical model was developed to investigate the mechanical behaviour of the micro beam-based MEMS device under electric actuation and simulations was done using Wolfram Mathematica® software. On the other hand, a finite element model (FEM) simulation was performed using COMSOL Multiphysics® for data validation for the micro beam model, and the results showed very good agreement. The effect of changing beam length, thickness and height from the ground on both the maximum beam displacement and pull-in voltage was also analyzed.

1. Introduction

Micro-electromechanical systems (MEMS) are those systems or devices that contains very small mechanical and electrical components in micro scale. These devices are used to sense, control, and actuate to generate effects on the micro scale. In general, actuators are devices that convert an electrical signal into a mechanical force to manipulate other mechanical devices to perform some desired functions.

The dynamics of electrically actuated micro beams having clamped-clamped boundary conditions have been studied by many researchers in literature. Many researchers developed models using different techniques to investigate the dynamic behaviour and response of micro beams under different conditions. Younis et al [1] proposed a reduced-order model to study the behaviour of micro electro mechanical beams under electric actuation. The model was based on Galerkin method to discretize the equations of motion and convert it to finite number of ordinary differential equations in time domain. Ghayesh et al [2] investigated the behaviour of a micro electro mechanical systems using the theory of modified couple stress and the technique of pseudo-arclength continuation. They examined the dynamic response of the micro beam under primary and super-harmonic excitations. Mobki et al [3] studied the global stability of a micro beam hanged between two parallel conductive fixed plates. Different values of electric voltages that lead to significant variations in the behaviour of micro beam were obtained through bifurcation diagrams for different values of the gap between the micro beam and the ground. Younesian et al [4] examined the harmonic vibration of a clamped-clamped micro beam using the method of multiple scales. They scrutinized both the primary and the secondary resonances of the micro beam. Liu and Wang [5] studied the dynamic vibration of a clamped-clamped micro beam subjected to electrical voltage and affected by air damping. The displacement of the micro



beam was studied under various actuating conditions and the effects of systems parameters on the center-point displacement of the micro beam were investigated. Bayat et al [6] used the Variational approach to analyze the vibration of an electrically actuated micro beam and provide analytical solutions for the significant vibrations of the micro beams.

The contribution of this paper is to develop a finite element model to find the deflection of a clamped-clamped micro beam using COMSOL Multiphysics software and compare the obtained numerical results with analytical solution available in literature. Moreover, the effect of the micro beam parameters on maximum beam's displacement and pull-in voltage will be investigated. This paper is organized as follows: Firstly, section 2 presents the governing equation of the clamped-clamped micro beam and the analytical solution of the model. Section 3 introduces the numerical validation using COMSOL Multiphysics software. Results will be presented and discussed in section 4, while section 5 reports a conclusion of the paper.

2. Mathematical model and solution

This study utilized the derived reduced order model presented by Younis et al [1] to study the static deflection of a clamped-clamped micro beam subjected to a constant DC voltage V loading as shown in figure (1).

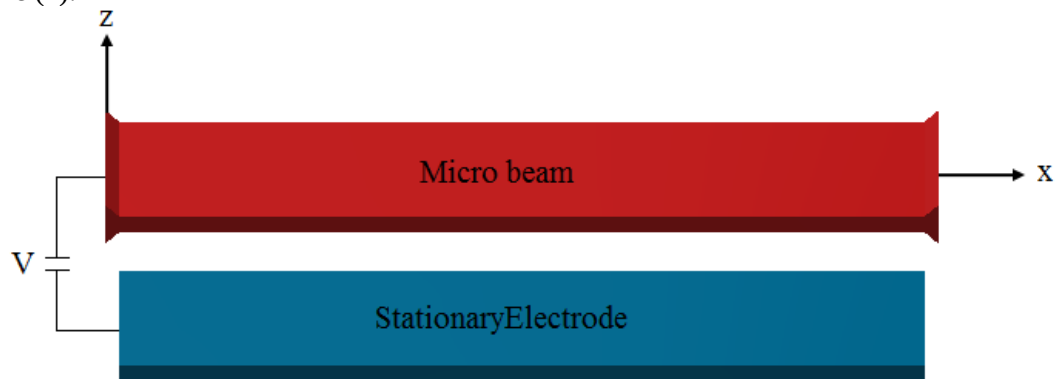


Figure 1. Electrically actuated clamped-clamped micro beam

The governing equation [1] of the static transverse displacement $w(x)$ is written as in equation (1)

$$EI \frac{d^4 w}{dx^4} = \left[\frac{EA}{2L} \int_0^L \left(\frac{dw}{dx} \right)^2 dx \right] \frac{d^2 w}{dx^2} + \frac{\epsilon b V^2}{2(d-w)^2} \quad (1)$$

Where x is the horizontal position across the beam length L , E is the modulus of elasticity, I is the second moment of inertia of the cross section, A is the cross section area of the micro beam, b is the micro beam's width, h is the micro beam thickness, ϵ is the dielectric constant of the gap medium and d is the gap width. For a clamped-clamped beam, the boundary conditions [1] are

$$\begin{aligned} w(0) &= w(L) = 0 \\ \frac{\partial w}{\partial x}(0) &= \frac{\partial w}{\partial x}(L) = 0 \end{aligned} \quad (2)$$

In order to solve these equations, one can introduce $w^* = \frac{w}{d}$, $x^* = \frac{x}{L}$ as a non-dimensional parameters to normalize the equation. Substituting the defined non-dimensional parameters into equation (1), dropping the star notations, and expanding the electric voltage term in a Taylor series up to fifth order, one can obtain

$$\frac{d^4 w}{dx^4} = \left[\alpha_1 \int_0^L \left(\frac{dw}{dx} \right)^2 dx \right] \frac{d^2 w}{dx^2} + \alpha_2 V^2 (1 + 2w + 3w^2 + 4w^3 + 5w^4 + 6w^5) \quad (3)$$

Where the parameters α_1 and α_2 are given by $\alpha_1 = 6\left(\frac{d}{h}\right)$, $\alpha_2 = \frac{6 \epsilon l^4}{E h^3 d^3}$. One can solve for the transverse displacement $w(x)$ of the micro beam by discretizing equation (3), using the first mode shape $\phi_1(x)$ of a clamped-clamped micro beam [7] as

$$\phi_1(x) = \cosh(4.73 x) - \cos(4.73 x) + 0.9825[\sin(4.73 x) - \sinh(4.73 x)] \quad (4)$$

$$w(x) = K \phi_1(x) \quad (5)$$

3. Model Validation with COMSOL

The finite element analysis software, COMSOL Multiphysics 4.4 [8] was used to develop a model of the micro beam to study its deflection. Figure (2-a) displays the view of the micro beam model in COMSOL Multiphysics, while figure (2-b) shows the deflection along the length of the micro beam after running the simulation. When comparing the analytical results and the numerical results obtained from COMSOL Multiphysics, there is a very good agreement between the two models as shown in figure (3).

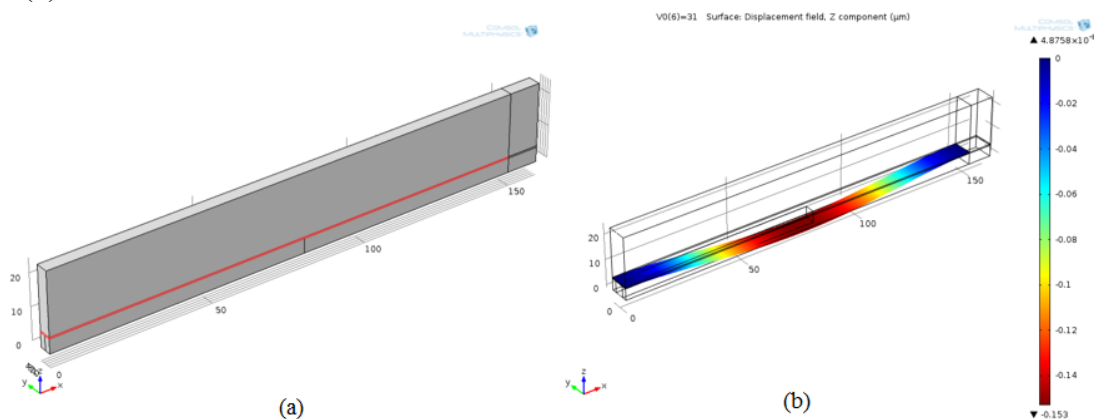


Figure 2. (a) micro beam model in COMSOL Multiphysics (b) displacement of the micro beam

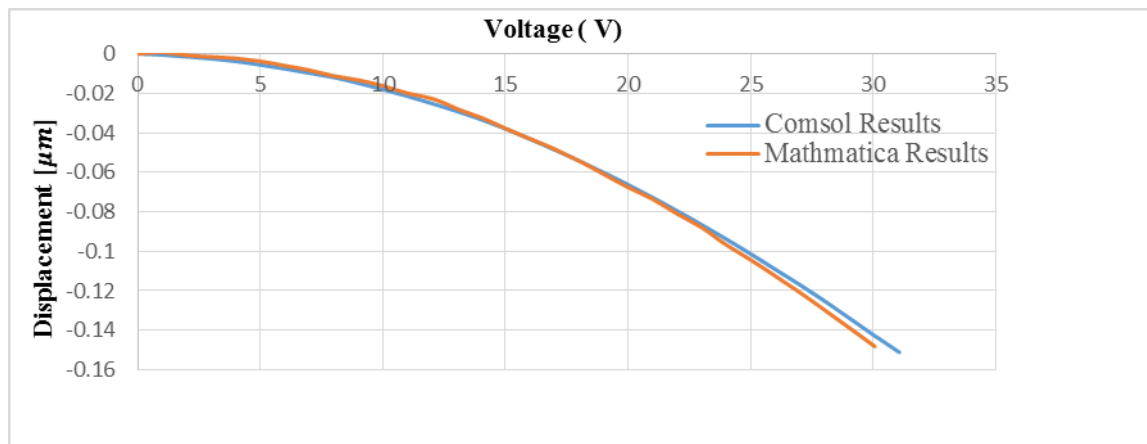


Figure 3. micro beam's maximum displacement under different voltages, Results Validation

4. Results and discussion

Consider the displacement of a micro beam under electric voltage. The micro beam under study is assumed to have the properties shown in table 1.

Table 1. Dimensions and material properties

Parameter	Value
d	$4.5 \mu m$
b	$8 \mu m$
h	$0.5 \mu m$
L	$150 \mu m$
ϵ	$8.854 * 10^{-12}$
E	$169 * 10^9 \text{ Pa}$
ρ	2332 kg/m^3
I	$\frac{bh^3}{12}$

4.1. Maximum displacement of the micro beam

To investigate the effect of the micro beam's parameters (length, thickness, height from the ground) on maximum beam displacement under different values of voltages, a parametric study is done by changing the parameter under study and fixing all other parameters values.

Figure (4) shows the effect of voltage applied to the micro beam on maximum beam's deflection for different values of beam's lengths. It can be seen that increasing the applied voltage produces a significant increase in the maximum beam's displacement. In case of small voltages, i.e., up to 10 volts, changing beam's length affects the maximum beam's displacement very slightly. In details, when applying voltage of 10 V, changing beam's length from $80 \mu m$ to $150 \mu m$ produces a maximum displacement less than $0.02 \mu m$ for all three different lengths shown in figure (4), while at voltage of 30 V, changing beam's length from $80 \mu m$ to $150 \mu m$ increases the maximum displacement from $0.01 \mu m$ to $0.15 \mu m$.

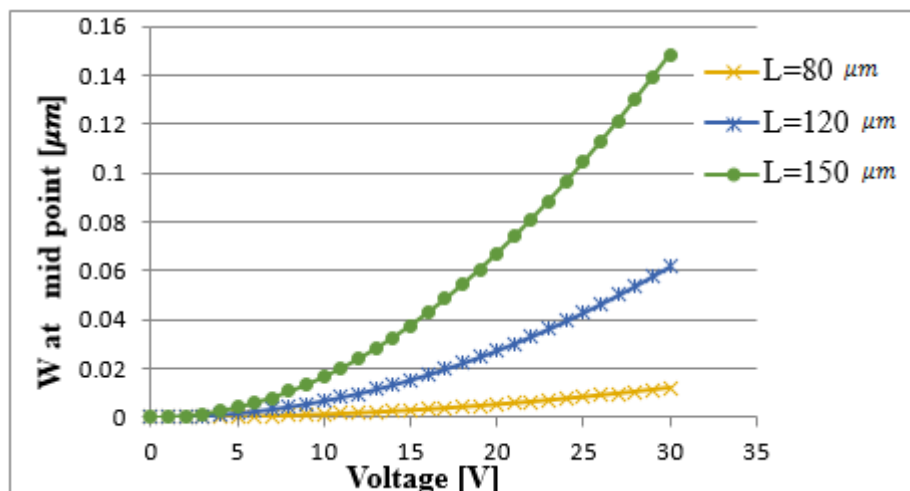
**Figure 4.** Effect of beam length on maximum beam displacement

Figure (5) demonstrates the effect of the micro beam thickness on its maximum displacement for different values of electric voltage, V . It can be seen that increasing the thickness of the micro beam reduces the maximum displacement, especially when higher voltages are applied. For a voltage of 30 V, a reduction in thickness from $1\ \mu\text{m}$ to $0.5\ \mu\text{m}$, increases the maximum displacement from $0.002\ \mu\text{m}$ to $0.15\ \mu\text{m}$. In case of small voltages, the increase in micro beam's thickness affects the maximum displacement slightly.

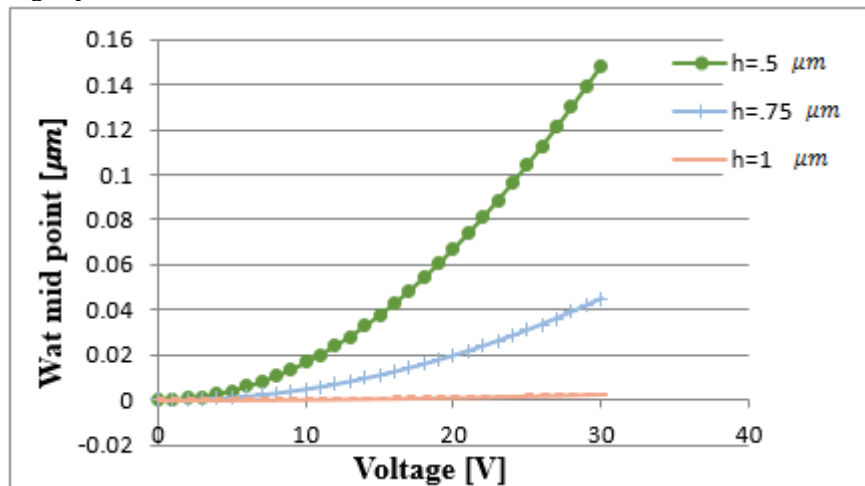


Figure 5. Effect of beam thickness on maximum beam displacement

Figure (6) shows the effect of the micro beam's height from the ground on its maximum displacement for various values of electric voltage, V . It can be seen that increasing the height from the ground reduces the maximum displacement of the micro beam, especially when higher voltages are applied. For a voltage of 30 V, a reduction in thickness from $5\ \mu\text{m}$ to $2\ \mu\text{m}$, increases the maximum displacement from $0.1\ \mu\text{m}$ to $0.7\ \mu\text{m}$. In case of small voltages, the increase in micro beam's thickness affects the maximum displacement slightly.

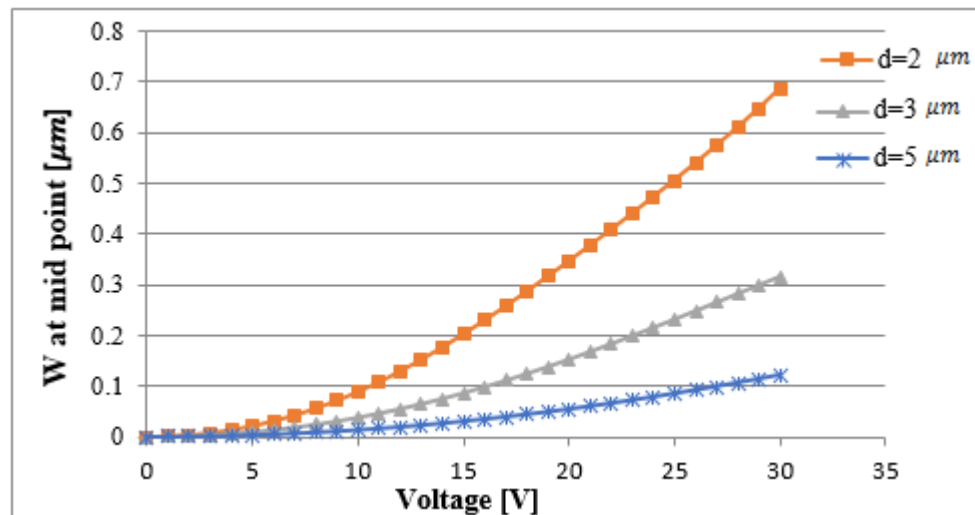


Figure 6. Effect of beam height form the ground on maximum beam displacement

4.2. Pull-in voltage

The term Pull-in voltage has been used widely in literature. The phenomenon of pull-in occurs when the deflection of the micro beam increases to the limit that the micro beam contacts the bottom electrode [9]. There are many studies published to investigate the pull-in phenomenon and its relation with parameters of MEMS devices. Chao et al [9] developed a correlation that predict the pull-in voltage of a clamped-clamped micro-beam using analytical solution of the partial differential

equations of the dynamic model by plotting bifurcation and phase portrait diagrams. Pamidighantam et al [10] derived an expression to determine the pull-in voltage of clamped-clamped micro beams based on a lumped spring-mass system model. Farokhi et al [11] inspected the pull-in phenomenon occurrence in a curved clamped-clamped micro beam under actuation of an electrical voltage and investigated the effect of different system's parameters on pull-in voltage. Chen and Meguid [12] investigated the vibration of a micro beam electrically actuated by various values of DC and AC voltages. The governing equations of motion were developed and solved using the multiple scales method, and the frequency response of the micro beam was obtained.

In This section, authors will investigate the occurrence of the pull-in phenomenon for different values of beam's parameters. Figure (7a) presents the pull-in voltage for different values of beam's lengths. The pull-in voltage decreases with increasing the beam length, i.e., for a micro beam length of 150 μm the pull-in phenomenon will occur at voltage of 75 V, while if length of the beam increased to 300 μm the pull-in phenomenon will occur at lower voltage of 20 V. Figure (7b) shows that pull-in phenomenon occurs at low voltages for micro beams with low thicknesses. The pull-in voltage will increase from 20 V to 90 V as a result of increasing the thickness from 0.1 μm to 0.6 μm . Figure (7c) shows that pull-in phenomenon occurs at low voltages for micro beams close to the ground. The pull-in voltage will increase from 5 V to 90 V as a result of increasing the gap between the micro beam and its ground from 1 μm to 5 μm .

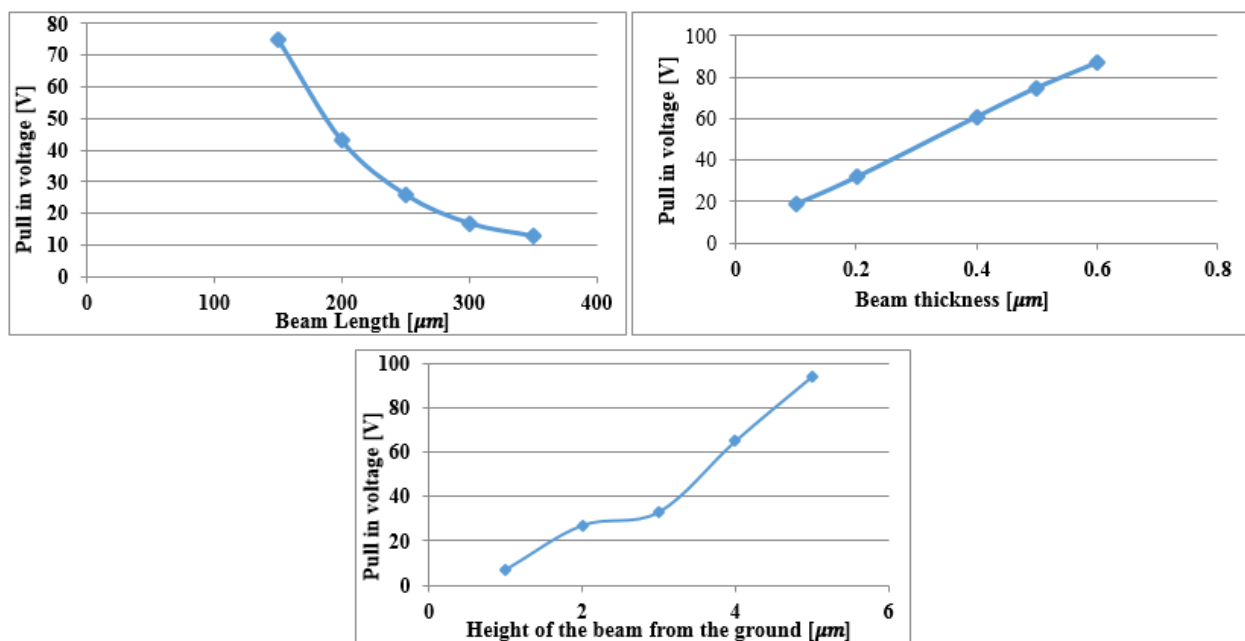


Figure 7. Effect of beam's parameters on pull-in voltage: (a) Length (b) Thickness (c) Height

5. Conclusion

The clamped-clamped micro beam actuator was modeled analytically and then a FEM simulation was performed for results validation and the obtained results showed a very good agreement. The effect of changing beam's parameters on its maximum displacement was analyzed by changing one parameters and keep other parameters at nominal values. Maximum beam's displacement increases with increasing beam's length while increasing beam's thickness and height from the ground reduce the maximum beam's displacement for the same applied voltage. Pull-in phenomenon was observed and the effect of beams parameters on the pull-in voltages was investigated.

Acknowledgments

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