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To cite this article: Wenke Gao *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **521** 012009

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A Nonlinear Preventive Maintenance Model with an Environmental Factor Based Weibull Distribution

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Abstract. Most of the Preventive maintenance (PM) reliability modeling research has focused on the relationship between two adjacent failure intensity functions, while few real case studies have demonstrated the application of these PM models. Thinking from this vision, a nonlinear PM model with scale and shape adjustment parameters is proposed based on a Weibull distribution, and two adjustment parameters can describe each PM effect. Meanwhile, the proposed model can separate the influence of an environmental factor from the failure intensity function of the new system when it operates under a new condition. The environmental parts in the failure intensity function can be improved or removed after several PM actions. Finally, one real case study is exhibited to illustrate the proposed model. The results indicate that the proposed model exhibits good fitting performance in reliability modeling and can describe the PM effect quantitatively as well as reveal the influence of the environmental factor on the system reliability.

Keywords: Nonlinear preventive maintenance model; Weibull distribution; shape adjustment parameter; maintenance effect; environmental factor.

1.Introduction

1.1 Motivation

Preventive maintenance (PM) research, which can increase equipment lifetime and decrease the frequency of in-service breakdowns, has attracted extensive attention since its introduction. According to the maintenance effect, PM can be generally classified into four categories as follows: better-than-perfect PM, perfect PM, imperfect PM and worse PM[1]. A better-than-perfect PM brings a system's operating condition to a state with a smaller failure rate or/and a slower failure process than a brand new identical system. This state can be due to technological advances, more reliable production or performance adjustment in a new environment via maintenance. A perfect PM action restores the system to an "as good as new" condition. Upon perfect maintenance, the failure intensity function of the item is the same as a new one. A worse PM action can increase the failure rate of the item. This type of maintenance might be due to various reasons such as using an inferior lubricant. An imperfect PM action does not lead the system to the "as good as new" or "as bad as old" condition, while it can bring a system to any condition between "as good as new" and "as bad as old" condition[2, 3]. Meanwhile, the environmental influence is a main factor for a new system when it is used in a new condition. This influence may result in some failures when a system operates in its several initial periods, while it can be improved or removed after an operating in the first several PM



intervals. Wu and Zuo[2] reviewed the PM policy and pointed out that the existing PM models present weaknesses.

Weibull model has a wide application and some good performances in reliability modeling. As a straightforward analysis approach, Weibull probability plot (WPP) can observe the change in the parameter of Weibull distribution. On the WPP, the monotonicity of the failure intensity function in each PM interval is the same value in the existing PM models, while the real case indicated that it may be different in different PM cycle[4]. Additionally, the environmental factor may play a leading role during the initial operating period when a new system operates in a new condition. These factors can be removed or improved by one or several times the PM after a period of time operating in a new condition, and yet the reliability model cannot be developed using the initial operating data for a new system in a new condition. Otherwise, the system reliability may be misleading. The current PM model was unable to meet these cases, and a few case studies in the literature have demonstrated the use of existing PM models. Based on this consideration, a nonlinear PM model is presented based on Weibull distribution.

1.2 Literature review

As can be seen, the failure intensity function is typically used to describe quantificationally some of these PM effects, as many maintenance actions may not realistically result in a perfect or minimal situation but in an intermediate one[5, 6]. Most research has focused on imperfect PM, whereas there have been few records aimed at better-than-perfect PM. Seo and Bai[7] introduced a periodic PM model. In this paper, the failure intensity function is $h_k(v_{k-1}(t))=h_{k-1}(w_{k-1}(x_{k-2}(t),T))$, where $v_{k-1}(\cdot)$ and $w_{k-1}(\cdot)$ are specified functions, and T is PM interval length. Wu and Derek[8] assumed that the quality of a PM action is a random variable following a probability distribution, and the failure intensity function is $h_k(t)=a^{k-1}h(t)$ after the k th PM, where a is the random value of $a \geq 1$. Kijima et al. [9, 10] introduced two types of virtual age PM models, and assumed that PM serves only to remove damage created during the last sojourn, and the virtual age at the start of working after PM is $v_k=t_{k-1}+\zeta_k(t_k-t_{k-1})$ and $v_k=\zeta_k(v_{k-1}+t_k-t_{k-1})$, where $0 < \zeta_k < 1$. Lam[11], Zhang and Wang[12] et al., have presented the geometric process maintenance. The failure intensity function is $h_k(t)=ah_{k-1}(at)$ after maintenance. Zequeira and Be'renguer[13] described a system with the following two types of failure mode: maintenance and non-maintenance. A failure intensity function defined as $h_k(t)=\lambda(t)+r(t-(k-1)T)+p_k(t)\lambda(t)$ is introduced, where $\lambda(t)$ is the failure rate of the non-maintenance failure modes, $r(t)$ is the failure rate of the maintenance failure modes and $p_k(t)$ is a function to model the dependence between the maintenance and non-maintenance failure models. Castro[14] also considered a system subject to two modes of failure: maintainable and non-maintainable, while the failure intensity function with definition $h_k(t)=\alpha^{N(kT)}h_0(t-kT)$ is related to each failure mode, where $\alpha > 1$. Clavareau and Labeau[15] presented a bi-Weibull expression $h(t)=h_0(t)+a(bt-c)^d$ to describe the system failure rate, which allows both the useful period and the ageing zone of a system to be covered with no consideration of infancy problems. Peng et al. [16] proposed a hybrid imperfect maintenance model with random adjustment-reduction parameters and claimed that this model is more realistic in real cases.

The influence of the environment on system reliability is a key factor, while it has been neglected in conventional PM. In practice, the influence of environment may be removed, weakened or improved by several PM actions until it can be ignored in the rest PM intervals. This situation can result in the monotonicity of the system failure intensity function displays different in the remaining PM interval. Wu and Scarf[17] regarded that the failure process of a system may be influenced by operational and environmental stress factors, and described the system failure intensity function with $\lambda_o(t)=\lambda_b(t)\phi(t)$, where $\lambda_o(t)$ is the baseline failure intensity function and $\phi(t)$ is a function of covariates that quantify such extrinsic factors. Xia et al,[18] introduced the effect of an environmental condition by using an environmental factor a_k , where the failure intensity function between the k th and $(k+1)$ th PM is $h_{k+1}(t)=a_k b_k h_k(t+c_k T)$. Tao et al,[19] improved Xia's failure intensity model and applied an increase in the hazard rate to describe the environmental factor, and then the model was used to model the reliability of a drilling machine. Xia et al,[20] applied an environment factor to reflect the external

environment effect on the machine reliability, and the developed model was used in the predictive maintenance of a manufacturing system. Lhorente et al.[4] provided a case study on wheel motor armatures of a fleet of Komatsu haul trucks in a mining application in Chile. Three successive PM reliability models obtained from real failure data showed that current PM models are unable to represent the real case because the failure intensity function of the 1st PM interval is decreased whereas it increased in the other two PM intervals. Wu and Zuo[2] suggested that PM models maybe consider more complex situations between $h_k(t)$ and $h_{k-1}(t)$.

1.3 Paper outline and definitions

The remaining parts of this paper are arranged as follows: a description of the nonlinear PM model is introduced in section 1, the parameter estimation is presented in section 2, a real case study is examined in section 3, model discussions are offered in section 4, and then a brief summary is given in the last section. Prior to providing a detailed description regarding maintenance policy, some terminologies and definitions used in the forthcoming sections are introduced as follows.

Definitions:

The k th PM restores the system to an "as good as new (AGAN)" state if $h_k(t)=h(t)$.

The k th PM restores the system to an "as bad as old (ABAO)" state if $h_k(t)=h_{k-1}(t_{k-1}+t)$.

The k th PM restores the system to a "better-than-new (BTN)" state if $h_k(t)<h(t)$ almost everywhere.

The k th PM restores the system to a "better-than-old (BTO)" state if $h_k(t)<h_{k-1}(t)$ almost everywhere.

2. Description of the nonlinear PM model

Herein, a general PM policy is considered. Without losing the generality, assume that a PM may be among the BTN, perfect and imperfect states. A minimal repair can be removed, which restores the system to its operating condition if it failed during each PM interval. PM actions are performed at time t_1, t_2, \dots, t_{N-1} , which can change the healthy condition of the system, and are described by the failure intensity function $h_k(t)$ ($k=1,2,\dots,N$). A replacement is employed after $(N-1)$ PMs at t_N , which can restore the system to the state of AGAN. The PM interval between the $(k-1)$ th and the k th PM is x_k that may be a periodic PM, a sequential PM, or a quasi-periodic PM if $x_i=T$, $x_i=T_i$ or x_i is a random variable valued in $(T, T+W)$, where T , T_i and the length of maintenance window W are constant[21]. On demonstration of the failure intensity function, the following three points are considered.

a) A non-negligible fact that a failure may be caused in a new system when it operates in a new environment and the environmental factor in the system reliability may be adjusted by some maintenance. This phenomenon, displayed in the failure intensity function, is a competing risk model that includes the influence of both the environmental and the baseline parts. The environmental part can be improved or removed by the early several PM intervals, and the baseline part is related to the PM effect.

b) Another fact is that some systems may experience a periodic PM policy, which may result in a failure intensity function that expresses only a decreasing part in the first several PM intervals. Users may be misled because they only can see the decreasing part of the failure intensity function, and are unable to find the increasing part. Specially, the influence of the environmental part may be weakened by several PM intervals and may be ignored in the remaining PM intervals. Thus, the monotonicity of the system failure intensity function displays differently in the remaining PM interval.

c) The PM effect not only exhibits a change in the scale parameters but also a change in the shape parameters of Weibull distribution in different PM intervals.

Based on the above consideration, the following assumptions are used to develop the proposed model.

1) The lifetime distribution of the system follows Weibull distribution, where the baseline failure intensity function $h_B(t)$ is a two-parameter Weibull distribution. The environmental influence on the system reliability decreases with the system operating time, and can be described by a decreasing function $h_Q(t)$ that is a two-parameter Weibull model in the failure intensity function. Thus, the failure intensity function for the new system is $h(t)$.

$$h(t) = h_b(t) + h_Q(t) \quad (1)$$

$$= \frac{b}{h} \left(\frac{t}{h}\right)^{b-1} + \frac{a}{q} \left(\frac{t}{q}\right)^{a-1}$$

where η and θ denote the scale parameters and $\beta(\beta > 1)$ and $\alpha(\alpha < 1)$ denote the shape parameter in a new system.

2) The failure intensity function of the system after the k th PM is $h_k(t)$, which meets the following relationship,

$$h_k(t) = A_k h_B(t^{B_k}) + C_k h_Q(t^{D_k}) \quad (2)$$

where $A_k = a_1 a_2 \dots a_k$, $B_k = b_1 b_2 \dots b_k$, and $t \geq 0$ is working time after the k th PM. The parameters a_k , c_k are the scale adjustment parameters after the k th PM, and $a_k, c_k > 0$. The parameters b_k , d_k are the shape adjustment parameters after the k th PM, and $b_k, d_k > 0$. The environmental part can be improved, removed or even ignored when the system experiences k PMs, and the failure intensity function can be stated as follows,

$$h_{k+m}(t) = A_{k+m} h_B(t^{B_{k+m}}) \quad (3)$$

3) All failures can be instantly detected and repaired. Each PM can bring the system to any state among the better-than-perfect, perfect or imperfect states. The system is restored to AGAN at replacement. The times for minimal repairs, PMs and replacements are negligible.

4) The mean cost of minor repair C_f is unrelated to the occurred failures and the total severed times of the system. The cost of PM C_p is not relevant to the total operational time of the system. The cost of a replacement is a constant C_r ($C_r > C_m$).

According to the above description of the maintenance process and model assumptions, an optimization model of the proposed imperfect sequential PM policy is presented. The system long-running cost rate is stated as below,

$$g(\{T_N\}, N) = \frac{C_p(N-1) + C_f \sum_{k=1}^N H_k(T_k) + C_r}{\sum_{k=1}^N T_k} \quad (4)$$

The optimal value ($\{T_N^*\}, N^*$) that can minimize the function $g(\{T_N\}; N)$ is given in Eq. (4). A unique optimal sequential PM policy with $\{T_N^*\}$ and N^* may be found under a consideration of A_k and B_k [21].

3. Parameter estimation

For a lifetime cycle, the models include a new system and preventively maintained system. Similarly, parameter estimations also involve α , θ , η , β , a_k , b_k , c_k and d_k , and then parameters A_k , B_k , C_k and D_k can be obtained according to definition of a_k , b_k , c_k and d_k . For failure data, complete data and censor data may exist in a PM interval, and thus, these two types of failure data are considered in the following reliability modeling.

Here, Let t_i and τ_i denote the failure time and the censored time for the observed individual i , $i = 1, \dots, n+m$. For complete data, $m=0$. If t_i and τ_i are independent random variables, then the log-likelihood function

$$L(t | X) = \prod_{j=1}^n f_k(t_j | X) \prod_{j=1}^m R_k(\tau_j | X) \quad (5)$$

$$L(t | X) = \prod_{j=1}^{n-m} (A_k h_Q(t_j^{B_k} | X) + C_k h_B(t_j^{D_k} | X)) e^{(-A_k H_Q(t_j^{B_k} | X) - C_k H_B(t_j^{D_k} | X))} \prod_{j=1}^m e^{(-A_k H_Q(\tau_j^{B_k} | X) - C_k H_B(\tau_j^{D_k} | X))} \quad (6)$$

If only the failed items are included, then the log-likelihood function $\ln L(t/X)$ can be described as,

$$\ln L(t | X) = \sum_{j=1}^{n-m} (\ln(A_k h_Q(t_j^{B_k} | X) + C_k h_B(t_j^{D_k} | X))) - \sum_{j=1}^n (A_k H_Q(t_j^{B_k} | X) + C_k H_B(t_j^{D_k} | X)) \quad (7)$$

where, $0 < \alpha < 1$ and $\beta > 1$

Setting the first partial derivatives of $\ln L(t/X)$ with respect to each parameter to zero. By solving the systems of the nonlinear likelihood equation, and then we can obtain the maximum likelihood estimates for the complete and censored data.

4. Real Case Study

A study in reference[4] explored wheel motor armatures of a fleet of Komatsu haul trucks in a mining application in Chile. In this real case, four years of maintenance data of these components were analysed. The failure data was separated into three groups according to PM frequencies: new armatures (before the first PM), after the first but before the second PM, and after the second PM. In the paper, two-parameter Weibull models are obtained for each PM cycle. The shape and scale parameters are: β_{1-3} (0.809, 1.247 and 1.235), η_{1-3} (36286, 6382 and 5264).

It can be found that the authors were unable to study the relationship among the three models, and the shape parameters from β_1 to β_3 increased with the PM frequencies. During the first operating interval, the system is new with $\beta_1 < 1$, which means that the system exhibited a decreasing failure intensity with operating time before the first PM is performed. This was not always the case and maybe caused by environmental or production quality, and the first PM was performed so early that the whole trend of the failure intensive function was unable to show completely. Obviously, the whole failure intensity function should be a “bathtub” type. This case is suitable to illustrate the proposed nonlinear PM model.

To develop the quantitative relation between adjacent PM cycles, two data sets are created from the presented reliability model of the investigation[4] because the failure data was not exhibited in the paper. One data set is a complete data with the 200 group data, and the other is a censor data also with the 200 group data. Consequently, the modeling process and maintenance optimization are given as follows.

4.1 Reliability Modeling

Step 1. Create $\{R_i(t_j)\}$

Twenty groups of random data are created within $[0.8, 1]$ and are viewed as reliability data sets for the new system using the new system parameters. For the complete data, each data set has 50 failure data, and it has failure data and non-failure data with the total number of 50 for the censor data. Similarly, 200 groups of random data are created within $[0, 1]$ and are viewed as reliability data sets for the second and third PM interval. Each group has 100 failure data, which are then marked as $\{R_i(t_j)\}$, where $i=1,2,3$ denotes the PM interval and $j=1,2,\dots,50$ or 100.

Step 2. Obtain the failure data $\{t_i\}$

Failure data sets $\{t_i\}$ are attained according to the created $\{R_i(t_j)\}$ and the two-parameter Weibull distribution function of each PM cycle.

Step 3. Estimate the parameters

Using the parameters estimation methods to obtain each parameter, and then compute their mean values as the system parameters. The parameters for complete data and censored data are obtained as bellows.

For the new system, $\alpha=0.7837$, $\theta=40629$, $\beta=1.9742$, $\eta=59695$. After the 1st PM, $a_1=30497$ and $b_1=0.2535$. After the second PM $a_2=1.3956$ and $b_2=0.9514$. The new system is a four-parameter Weibull model. After the two PM actions, the failure intensity is changed. It can be found that the environmental factors are removed after operating in the first PM interval. The parameters a_2 and b_2 can exhibit the second PM effect. β and η can reveal the system reliability in the new system, and θ and α display the environmental factors.

4.2 Maintenance optimization

In this case, a sequential PM policy is considered and takes $C_p=20$, $C_f=40$ and $C_r=60$, where $\alpha < 1$ and $\beta > 1$, that is $h(t)$ is a “bathtub” type function. According to Theorem 4, there are two solutions. The first solution is near zero and is ignored, and the optimal $\{T_i^*\}$ exists. Then, it can be found that the

optimal $N^*=1$ and $T_1=27414$. The results show that the sequential PM or PM policy is unnecessary for the system, while the periodic replacement policy is optimal.

4.3 Comparison with the linear PM model

Based on the obtained failure data, the differences between the proposed PM model and traditional linear PM model are analysed here, which only consider one linear PM model with failure intensity function $h_k(t)=a_k h(t)$. For the new system, the reliability function is the same as the above. After the 1st PM but before the second PM, the function has one unknown parameter a_1 that can be attained through the MLE method. Similarly, a_2 can also be obtained. The failure intensity function curves of the proposed nonlinear and linear PM model are shown in Fig.1.

From the Fig. 1, the proposed PM model has a better fit than the linear PM model. To evaluate the goodness-of-fit, the BIC value is usually utilized. Based on the estimated linear PM model and the proposed model, the BIC values are obtained. After the 1st PM, the BIC values of the proposed model and the linear PM model with the complete data are 1946 and 2290. After the 2nd PM, the BIC values are 1908 and 2333. For the censored data, the BIC values are 1946, 1970, and 1908, 1955 respectively. Obviously, the BIC value of the proposed model is smaller, and thus the proposed PM model has a greater goodness-of-fit than the linear PM model.

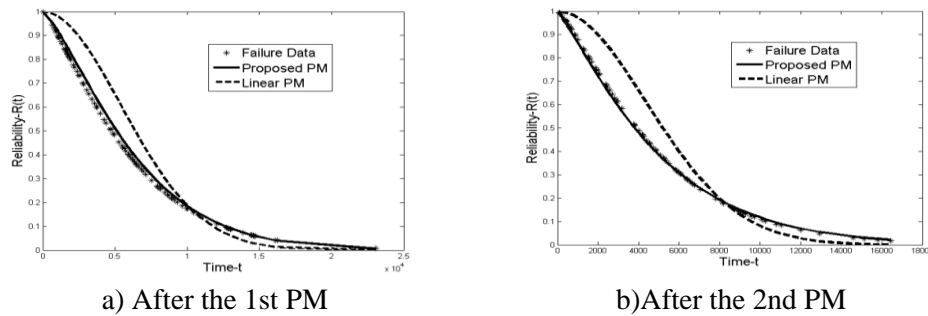
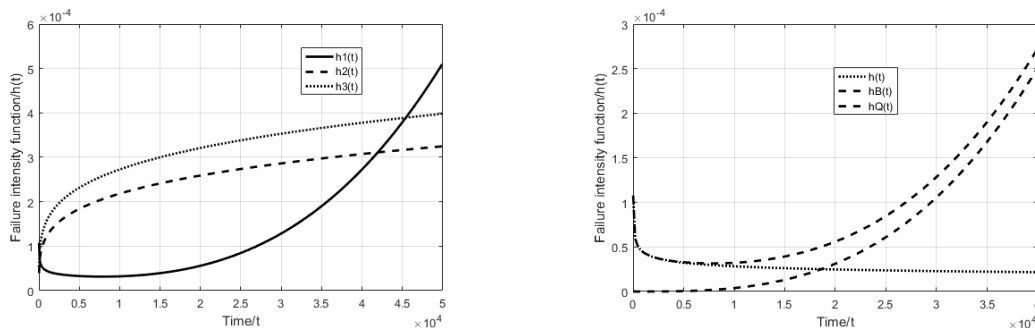


Fig. 1 $R(t)$ with different models

4.4 Environmental factors analysis

The influence of environmental factor on the system failure intensity requires more attention during the initial several PM intervals for a new system and can be removed by an operating adjustment or maintenance. Together with the system baseline failure intensive factor, the system reliability can be seen as a competing risk model. With the growth of the system operating time, the environmental factor can be weakened until it is ignored, while the system baseline failure intensive factor may be heightened. Herein, we illustrate this point using the proposed model.



a) Failure intensive function for 3 PM intervals b) Failure intensive function for the new system

Fig. 2 Failure intensive function

In the subplot of Fig. 2, $a_k > 1$ and $b_k < 1$. It can be found that the initial value of the environmental factor in the failure intensity function in the 2nd PM interval can be ignored because the time for a large value is transient and then approach zero.

5. Results

In this paper, a nonlinear PM model $A_k h_B(t^{B_k}) + C_k h_Q(t^{D_k})$ is proposed based on the Weibull distribution, and parameter estimation methods are given. The proposed model not only depicts the effect of some PM actions that may restore the system to any condition between AGAN and BTO, but also may bring the system to BTN or even to ABAO. Specially, the model can provide a goodness-of-fit for the failure data in reality. These properties exist in practice and are mainly described by parameter A_k and B_k . Additionally, the proposed model can describe the influence of the environmental factors on the system reliability in different PM interval via parameters C_k and D_k . The influence can be improved or removed by maintenance actions in the first several PM intervals. The property is proved by the real case study. The existence condition of the optimal periodic PM policy exhibited that the optimal PM policy does not always exist, which is decided by A_k , B_k , $h(t)$ and the maintenance cost of each item. A real case study is given to support the model's application and properties.

Acknowledgments

The research work is also supported jointly by the National Natural Science Foundation of China (grant number 71561016) and China Postdoctoral Science Foundation (grant number 2017M613297XB). The authors are grateful for valuable supports of Wu Centre of University of Michigan.

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