# Mathematical Model and Simulation Analysis of a New Underwater Launching System

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# Mathematical Model and Simulation Analysis of a New Underwater Launching System

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**Abstract.** In order to solve the problems of air turbines about complex structure, difficult processing and material requirements, a new type of vaneless underwater launch prime mover scheme is proposed. After simplification and assumption of working conditions, the launching system of the prime mover and pump is modeled and simulated by matlab. The results show that the change of chamber pressure in launcher tube is reasonable, the head and the flow of the pump is enough to provide the power for pushing the torpedo, and the other parameters such as power and speed of the prime mover scheme meet the requirements of each index, which provides theoretical support for practical application.

## 1. Introduction

Currently, the most advanced hydrocushion type deep-water launcher equipped on navy submarine is turbine pump launch system, which has the advantage of deep depth (600m), small volume, light weight, strong generality, high utilization ratio of launch energy and low noise, etc., making it suitable for all types of submarine[1]. Both the America and Britain have applied it to the submarine of 1990s. For the rotary pump launch system, other aerodynamic device could also be used as prime motor to drive the pump despite the air turbine[2]. Air jet rotary engine is one choice of high feasibility. This paper establishes the mathematic model of air jet rotary engine on the foundation of introducing the structure, operating principle of it, and then builds the simulation software to study the launch process of this launch system.

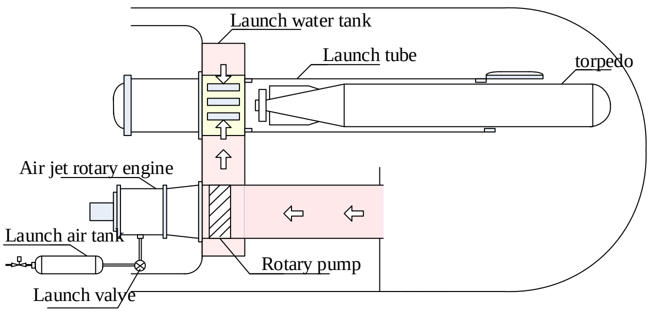



**Figure.1** Schematic of AJRP launch system

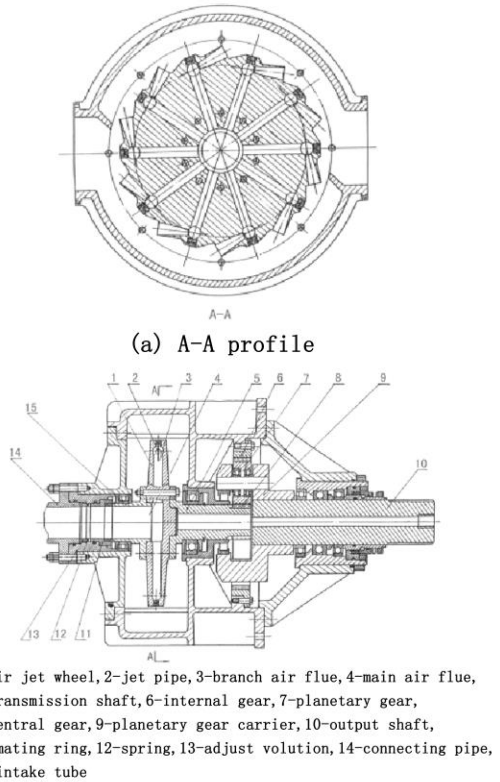
## 2. The structure and basic operation principle of AJRP

AJRP lunch system mainly consists of launch air bottle, launch valve, air jet rotary engine, rotary pump and launch tube, as is shown in Fig.1. The operation principle of AJRP is: when launching, the valve opens at a certain rule, then the high pressure gas goes into air jet rotary engine through the valve and drive the engine, and the engine drives the pump through moderator, at last the pump pumps the water into launch tube and launch the weapon.

As is shown in figure.2, air jet rotary engine is a rotary engine device with high pressure air as working medium, which takes advantage of the counter-acting force of high speed jet airstream to generate rotating torque and power. It consists of shell, air jet wheel and moderator, and the output shaft of moderator is connected to the load.

## 3. Basic assumption of launch system modeling

The launch process of real launch system is really complicated. To make the problem easier to study, assumption is made that: the phase of air flows through launch valve is a throttling process of adiabatic expansion; the decompression throttling process of air flows in the air jet engine pipe is adiabatic process; the wear-stress of weapon in launch pipe is under simplified analysis, which ignores the elements like additional mass, which have small influence on the weapon movement; the water in the launch pipe is considered to only make work on weapon, ignoring the working to the out flowing water through the interval of weapon and pipe.



(b) Axial profile

**Figure.2** Schematic of Air-jet Rotary

## 4. Modeling of launch system

### *4.1 Mathematic model of launch gas tank gas state*

This model is used to describe the variation of gas pressure, temperature and mass during the launch process.

Assuming that the gas tank air-supply process during the launching is an adiabatic process, then through the law of thermodynamics for gas we can get:

$$\frac{\mathrm{d}p_\mathrm{B}}{\mathrm{d}t}=\kappa\frac{p_\mathrm{B}}{m_\mathrm{B}}\frac{\mathrm{d}m_\mathrm{B}}{\mathrm{d}t} \tag{1}$$

$$\frac{\mathrm{d}T_\mathrm{B}}{\mathrm{d}t}=(\kappa-1)\frac{T_\mathrm{B}}{m_\mathrm{B}}\frac{\mathrm{d}m_\mathrm{B}}{\mathrm{d}t} \tag{2}$$

In which $p_\mathrm{B}$ is the air pressure in the launch gas tank, $T_\mathrm{B}$ is the temperature of the tank, $m_\mathrm{B}$ is the mass of the tank, $\kappa$ is the adiabatic index.

*4.2 Mathematic model of launch valve*

The amount of gas flow in and out of the engine is adjusted by the launch valve, which is realized through the opening area changing with the time. In the simulation the changing rule is set to $S_\mathrm{v}=f(t)$.

*4.3 Mathematic model of air jet rotary engine*

The model of air jet rotary engine includes three parts: the model of jet pipe, calculation model of rotate torque of pipe utter and kinematic model of air jet rotary engine.

*4.3.1 Mathematic model of jet pipe* This model mainly solves the gas flow speed of pipe utter.Assuming that the flow of the pipe is isentropic adiabatic process, obeying the flow basic equations with one unknown[3], energy conservation equation can be obtained through equation (3)-(5):

$$c_\mathrm{p}T_\mathrm{no}+\frac{w_\mathrm{no}{}^2}{2}-\frac{u_\mathrm{no}{}^2}{2}=c_\mathrm{p}T_0 \tag{3}$$

$T_\mathrm{no}$, $w_\mathrm{no}$ and $u_\mathrm{no}$ represent the gas temperature, relative speed and circular velocity of pipe utter section respectively, $T_0$ is relative stagnation temperature. Assuming that high pressure gas flows through valve without loss and the process being adiabatic, generally $T_0=T_\mathrm{B}$, $T_\mathrm{B}$ is the temperature of launch gas tank.

Flow conservation equation of pipe flow process is as follow:

$$q_\mathrm{nm}=q_\mathrm{no} \tag{4}$$

In which $q_\mathrm{nm}$ is the flow of smallest section of pipe, $q_\mathrm{no}$ is the flow of utter section.

On the smallest section, the pressure is critical pressure ratio, and then the flow $q_\mathrm{nm}$ is:

$$q_\mathrm{nm}=\varphi_\mathrm{n}S_\mathrm{nm}\varepsilon_1\bullet\frac{p_\mathrm{ni}}{\sqrt{RT_\mathrm{B}}} \tag{5}$$

In which $\varphi_\mathrm{n}$ is the flow coefficient of pipe, $p_\mathrm{ni}$ is the pressure of the entrance of pipe, $S_\mathrm{nm}$ is the area of the smallest section of pipe, $R$ is the air gas constant. $\varepsilon_1=\left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}}\cdot\sqrt{\frac{2\kappa}{\kappa+1}}$

The flow $q_\mathrm{no}$ of utter section can be calculated as follow:

$$q_\mathrm{no}=\varphi_\mathrm{no}S_\mathrm{no}\rho_\mathrm{no}w_\mathrm{no} \tag{6}$$

In which $\varphi_\mathrm{no}$ is the flow coefficient of utter section, $S_\mathrm{no}$ is the area of utter section, $\rho_\mathrm{no}$ is the air density of utter section, $w_\mathrm{no}$ is the flow velocity of utter section.

With gas adiabatic equation[4] we can get that:

$$\frac{\rho_\mathrm{no}}{\rho_0}=\left(\frac{T_\mathrm{no}}{T_0}\right)^{\frac{1}{\kappa-1}} \tag{7}$$

$$\frac{p_{\mathrm{no}}}{p_0}=\left(\frac{T_{\mathrm{no}}}{T_0}\right)^{\frac{\kappa}{\kappa-1}} \tag{8}$$

In which $\rho_0$ , $p_0$ is the air density and pressure when the temperature is $T_0$ .

Substitute equation (5), (6) into (4) and combine them with (7) we can get that:

$$T_{\mathrm{no}}=\left(\frac{A_{\mathrm{n1}}}{w_{\mathrm{no}}}\right)^{\kappa-1} \tag{9}$$

$$A_{\mathrm{n1}}=\varphi_{\mathrm{n}}S_{\mathrm{nm}}\varepsilon_1\bullet\frac{p_{\mathrm{ni}}}{\varphi_{\mathrm{no}}S_{\mathrm{no}}\rho_0\sqrt{RT_{\mathrm{B}}}}T_0^{\frac{1}{\kappa-1}} \tag{10}$$

Substitute equation (9) into (5) and solve the equation with numerical methods, then the relative velocity of air in pipe utter section when the air jet wheel speed is known[4].

*4.3.2 Calculation model of air jet rotary engine rotate torque* To calculate the torque and power when the engine is running, single pipe should be analyzed firstly (as is shown in Fig.3).

The rotate torque produced by the work of single jet pipe is made up of two parts: the moment of momentum produced by gas ejection and the moment produced by the pressure differential of the ejector.

(1) Calculation of the moment of momentum produced by gas ejection

When the air flows through the center pipe, the entrance velocity is $\mathbf{v_1}$ , the relative velocity of pipe utter section is $\mathbf{w_{no}}$ , which keeps an angle of $\beta$ with the direction of wheel tangent. Assuming that the flow in the pipe is isoenergetic isentropic flow. Take the gas in AABB as the object, after the time of $\mathrm{d}t$ ,it moves to the position A'A'B'B' , then AAA'A' is mass of the part that flows into pipe, marked as $\mathrm{d}m_1$ ; BBB'B' is the mass of the part that flows out of the pipe, marked as $\mathrm{d}m_2$ . Assuming that the parameter of A'A'BB during $\mathrm{d}t$ remain unchanged, then:

$\mathrm{d}m_1=\mathrm{d}m_2$

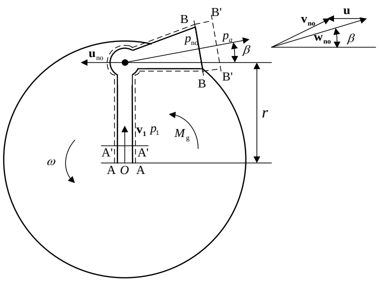



**Figure.3** Schematic of jet pipe

Take point O as the rotate center, then according to the theorem of moment of momentum:

$$M_{\mathrm{T1}}dt=dm_2\cdot\mathbf{r}_2\times(\mathbf{w_{no}}+\mathbf{u_{no}})-dm_1\cdot\mathbf{r}_1\times\mathbf{v_1} \tag{11}$$

$M_{\mathrm{T1}}$ is the joint momental vector of the air mass; $\mathbf{r}_1$ is the radius vector of entrance; $\mathbf{r}_2$ is the radius vector of the utter; $\mathbf{u_{no}}$ is the peripheral speed of the utter.

Assuming that anticlockwise on the plane where air jet wheel lays is positive, since $\mathbf{r}_1=0$ and the module of $\mathbf{r}_2$ is $r_2=r$ , and $\mathrm{d}m_1=\mathrm{d}m_2=\mathrm{d}m$ , then the scalar quantity after trimming is as follow:

$$M_{T1} = r(u_{no} - w_{no}\cos\beta)\frac{dm}{dt} = r(u_{no} - w_{no}\cos\beta)\dot{m} \tag{12}$$

In which $\dot{m}$ is the air mass flow of single pipe, the peripheral speed of pipe utter is $u_{no} = \omega r = 2\pi n_{tur} r$ , and $\omega$ is the air wheel angular velocity, $n_{jet}$ is the air wheel rotating velocity.

Since the direction of the joint moment on the air mass in opposite to that of moment it cast on the wheel and the size of two moment is the same, the kinetic moment produced of single pipe ejection is: $M_{g1} = -M_{T1}$

(2) The moment produced by pressure differential of utter

The utter of pipe is connected to the outside, so the pressure moment on the air jet wheel is imbalanced. Assuming that the outside air pressure is $p_a$ , the air pressure of utter is $p_{no}$ , then the moment produced by pressure differential of utter is:

$$\begin{aligned} M_{g2} &= rp_{no}A_2\cos\beta - rp_a A_2\cos\beta \\ &= (p_{no} - p_a) r A_2 \cos\beta \end{aligned} \tag{13}$$

(3) The joint momentum on the air wheel

When the number of pipe is $n_j$ , according to the analysis above, the joint momentum is:

$$M_{jet} = n_j(M_{g1} + M_{g2}) \tag{14}$$

*4.3.3 The motion model of air jet rotary engine* The motion equation of the procession of the engine driving rotary pump[5-6] is shown as:

$$\frac{\pi}{30} I_{jet} \frac{d n_{jet}}{d t} = M_{jet} - i_r \left( M_p + \frac{\pi}{30} I_p \frac{d n_p}{d t} \right) \tag{15}$$

In which the rotational inertia of rotation pump is $I_p$ , that of the air rotary engine is $I_{jet}$ , the torque of rotation pump is $M_p$ , and the initiative torque of air jet rotary engine is $M_{jet}$ , and the rotation velocity of rotation pump is $n_p$ , that of the air jet rotary engine is $n_{jet}$ ,and the reduction ratio of the moderator is $i_r = n_p / n_{jet}$ .

After summing up, the angular accelerated velocity of air wheel is:

$$\frac{d n_{jet}}{d t} = \frac{30(M_{jet} - i_r M_p)}{\pi(I_{jet} + i_r^2 \bullet I_p)} \tag{16}$$

After solving the equation (14) the impeller rotation velocity $n_{jet}$ is obtained, thus the periphery velocity $u$ is:

$$u = \frac{\pi}{30} r \bullet n_{jet} \tag{17}$$

$r$ is the radius of the pipe utter.

The output torque and rotation velocity of air jet rotary engine is known, and then the output power is:

$$N_{jet} = \frac{\pi}{30} M_{jet} n_{jet} \tag{18}$$

*4.4 Model of rotation pump*

The mathematic model of rotation pump is main to describe the relationship of pump and the parameters like output lift, flow and power, etc.

Combining with the lift curve of pump, the equation of lift curve of pump under any rotation velocity is:

$$\begin{aligned} H_P &= H(n_P, Q_P) \\ &= 6.1583\times10^{-5} n_P^2 + 5.217\times10^{-4} Q_P n_P - 3.9660 Q_P{}^2 \end{aligned} \tag{19}$$

$Q_P$ is the flow of pump.

The life is known, and the drag torque of pump to the air jet rotary engine principal axis is:

$$M_p = \frac{30N_{pe}}{\pi n_P} \tag{20}$$

In which $N_{Pe}$ is the valid power of pump, $N_{pe} = \eta_p p_o Q_P$ , $\eta_p$ is the power of pump, $p_o$ is the output water pressure of pump, $p_o = 1.02\times10^4 H_P$ (Pa).

*4.5 Kinematic model of weapon launch tube*

During the launch process, the force cast on the weapon in the launch tube is(as is shown in Fig.4): gravity $G$ , buoyancy $F$ , head resistance $A_x(v_T + v_{sub})^2$ , friction $f_g$ between the weapon and the tube, water pressure $p_t$ of weapon stern and the external water pressure $p_H$ ,the motion equation of which is[7]:

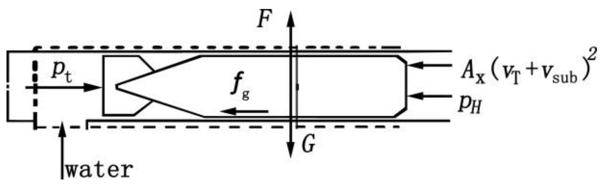



**Figure.4** Schematic of the force acting on the torpedo in tube

$$\frac{dv_T}{dt} = a_T = \frac{1}{m}\left[p_c S_T - f_g - A_x(v_T + v_{sub})^2 - S_T \rho v_T^2\right] \tag{21}$$

$$\frac{dl}{dt} = v_T \tag{22}$$

$$m = m_W + m_T = m_{W0} + \rho S_T l + m_T \tag{23}$$

$$p_c = p_t - p_H;\ \ p_t = p_o - \Delta p \tag{24}$$

$S_T$ is the area of weapon section; $v_T$ , $a_T$ is the velocity and the acceleration velocity of weapon; $l$ is weapon displacement inside the tube; $p_c$ is the chamber pressure of the launch tube; $\Delta p$ is the pressure loss during the pump to the tube; $m_T$ is the mass of weapon; $m_W$ the mass of water that move with the weapon; $m_{W0}$ is the initial mass of the water moves with the weapon. $f_g = \mu(G - F)$ , $\mu$ is the friction coefficient; $v_{sub}$ the voyage velocity when the weapon is launched; $A_x$ is the head on fluid resistance coefficient; $\rho$ is the water density.

## 5. Calculation simulation of launch process and the analysis of simulation result

After building simulation software according to the models above, simulation can be conducted to the process of AJRP launching weapon.

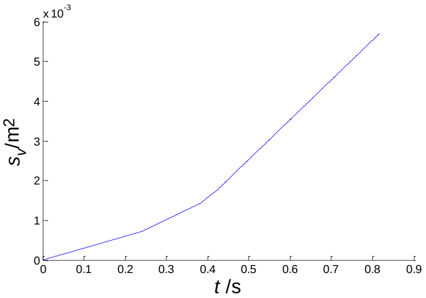



**Figure.5** The area variation of launching valve

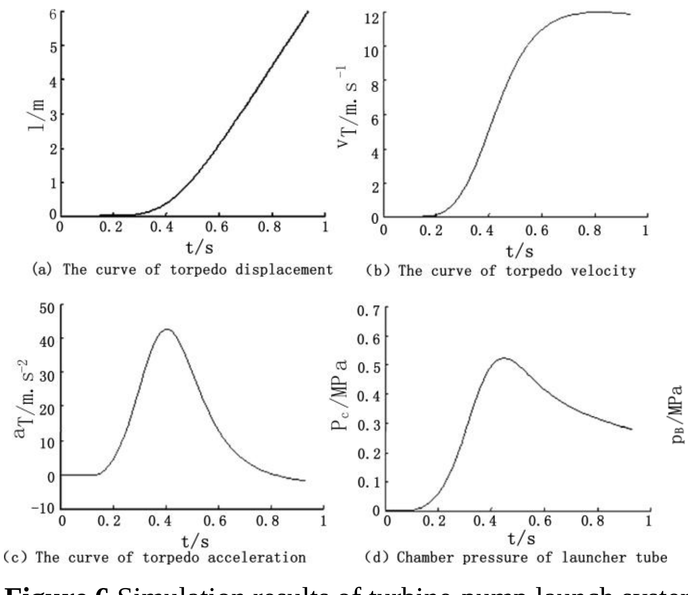
*5.1 main parameters of simulation*

Initial pressure of gas tank:15MPa; initial temperature :300.0K; volume :300L; torpedo CSA: 0.2254m$^2$; weight: 1850.0kg; water density: 1040.0kg/m$^3$; displacement: 6.0m; head on fluid resistance coefficient :16N.s$^2$/m$^2$;air wheel rotational inertia $I_t$ : 1.0 kg.m$^2$; air wheel pipe utter: 0.25m; number of pipe :9, position angle of pipe utter r: 0.25m;reduction ratio of moderator ;pump blade rotational inertial :13.0kg.m$^2$; voyage velocity when the torpedo is launched: 5m/s.
The changing law of launching valve opening area is shown in Fig.5.

*5.2 Simulation result*

The simulation result of air jet rotary engine is shown as follows.

(1) Simulation result of basic parameters



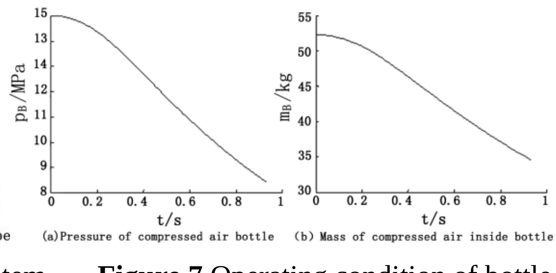




**Figure.6** Simulation results of turbine-pump launch system **Figure.7** Operating condition of bottle

*5.3 Analysis of simulation result*

It can be learnt from the simulation result that: with fixed parameters, AJRP can push the torpedo to the utter in 0.9s (as is shown in Fig.6a), of which the speed hits 12.0m/s (as is shown in Fig.6b), the max acceleration velocity of torpedo is 42.6m/s$^2$ (as is shown in Fig.6c), which meets the demand of submarine for velocity and acceleration velocity. Launching chamber pressure is 0.52MPa (as is shown in Fig.6d), which meets the demand of security.

The required air power can be obtained through simulation. When the launch is over, launch bottle pressure of AJRP has reduced to 8.4MPa from 15MPa, the mass of gas has reduced to 34.56kg from 52.26kg, the total loss of gas is 17.70kg (as is shown in Fig.7).

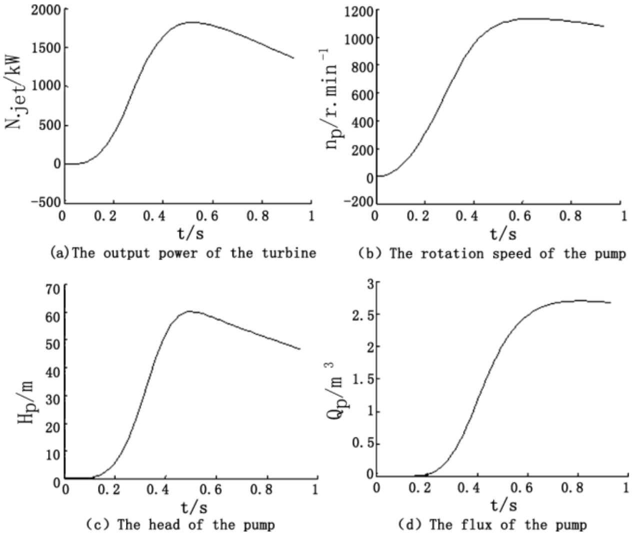



**Figure.8** Dynamic characteristics of turbine and pump

The dynamic character of air jet rotary engine can also be obtained. Air jet rotary engine can provide the maximum power of 1823KW (as is shown in Fig.8a), when the rotation speed of pump is increasing to 1130r/min from 0r/min (as is shown in Fig.8b), the head increases to 60m/s from 0r/s (as is shown in Fig.8c), and the flow of pump is increasing to 2.7 $m^3/s$ (as is shown in Fig.8d). From the simulation result, the character of air jet rotary engine meets the demand of AJRP launch system.
According to the simulation model, the influences of main parameters of launch system like rotational inertia, size of flow area and initial pressure of bottle to the system can also be studied.

## 6. Conclusion

The simulation result of AJRP launch system shows that, the established model for this system is basically correct. The simulation software based on these models can not only be used to simulate the system and study its character, but can also be used to provide theory foundation for the design of the system and the key module like air jet rotary engine, launch valve and rotation pump.

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