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Analysis of adaptability estimation algorithms in dynamic systems

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Abstract. In this paper we show the adaptability analysis of estimation algorithms based on mobile average autoregressive models (ARMA) in an easy implementation system composed of a network of operational amplifiers (OpAmp) known as the Sallen- Key. Parameters are adjusted by means of projection algorithms, projection algorithm with covariance matrix, Recursive Least Squares (RLS) and Normalized RLS. The comparison of the performance of the estimation algorithms shows that those based on RLS and Normalized RLS quickly stabilize their values before disturbances in the input signals of the dynamic system.

1. Introduction

The implementation of control algorithms requires the construction of *a priori* mathematical models that describe the evolution of a system according to initial conditions and parameters. However, in the case of dynamic systems, developing a mathematical model that determines their behaviour, outside of ideal conditions, requires considering a large number of parameters whose behaviour against perturbations can't always be approximated to a linear and homogeneous relationship [1, 2], increasing the complexity of the model and the cost in computation time with implications in the control of processes in real time [3,4].

The algorithms of adaptive estimation in dynamic systems allow to establish conditions on the model that facilitate, before disturbances, adjust the parameters relatively quickly and stabilize the response of the system [5], thus decreasing the response time of the control algorithms [6, 7].

In this paper we show the adaptability analysis of estimation algorithms based on mobile average autoregressive models (ARMA) [8,9] in an easy implementation system composed of a network of operational amplifiers (OpAmp) known as the Sallen- Key.

2. Materials and methods

2.1. Dynamic system

The simulation of a second order system based on a network of operational amplifiers (OpAmp) known as the Sallen-Key network was performed, the topology is shown in figure 1. An easy-to-implement system composed of an acquisition card was designed ARDUINO ONE, and an interface in MATLAB (Figure 2).



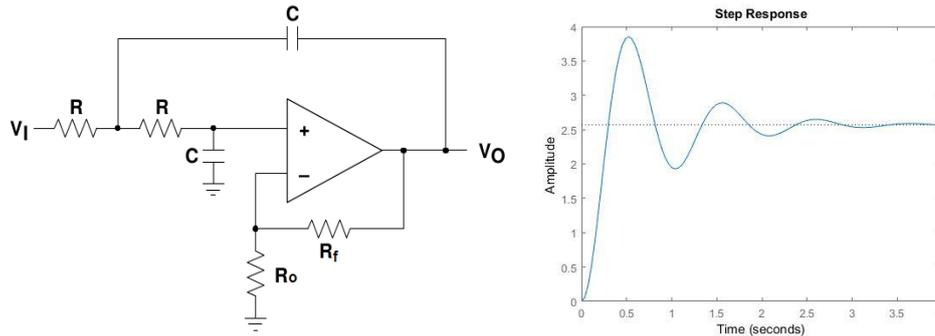


Figure 1. Topology of the Sallen-Key network and its step response design.

The design of the system is shown in Figure 1, which can be obtained the response of the dynamic system, was designed with the criterion of a maximum impulse (mp) of 50%, an establishment time (ts) of 3 seconds and a stable state gain μ of the Sallen-Key network (see equation 2) shown in Figure 1, its transfer function as a function of the circuit parameters (R and C) is shown in equation 1.

$$H(s) = \frac{\mu \left(\frac{1}{RC}\right)^2}{s^2 + \frac{(3-\mu)}{RC}s + \left(\frac{1}{RC}\right)^2} \quad (1)$$

$$\mu = 1 + \frac{R_f}{R_o} \quad (2)$$

Where, $R = 16.16k\Omega$, $C = 10\mu F$, $R_f = 15.69k\Omega$ and $R_o = 10k\Omega$

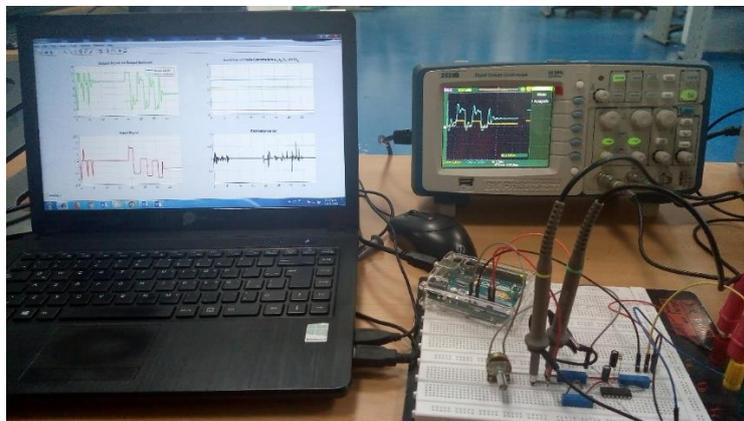


Figure 2. System assembly

2.2. Estimation algorithms

In this work an online estimation process is carried out. The estimation of parameters is based on autoregressive models of moving average (ARMA) [8]. This process can be done online if you have only one data at a time or offline if you have a data set. For the application of estimation algorithms in the identification of dynamic systems, it is necessary to capture the input signal $u(k)$ and output of the system $y(k)$ and construct a discrete linear model in the time domain (Eq.3) and in differences (Eq. 4). The system can be carried to a reduced model for regression (Eqs. 5 and 6).

$$H(z) = \frac{y(z)}{u(z)} = \frac{b_1z + b_2}{z^2 + a_1z + a_2} \quad (3)$$

$$y(k) = -a_1y(k-1) - a_2y(k-2) + b_1u(k-1) + b_2u(k-2) \quad (4)$$

$$y(k) = [-y(k-1) \quad -y(k-2) \quad u(k-1) \quad u(k-2)] \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix} \quad (5)$$

$$y(k) = \varphi^T \theta_o \quad (6)$$

The parameters to be estimated are a_1 , a_2 , b_1 and b_2 , which belong to the dynamic model of the system as shown in equation 1. These parameters are adjusted by means of projection algorithms, projection algorithm with covariance matrix, Recursive Least Squares (RLS) and Normalized RLS.

The estimation algorithms that are compared in the development of this document are the following:

a. Projection Algorithm (PA)

$$\theta_k = \theta_{k-1} + \frac{\varphi}{\varphi^T \varphi} (y - \varphi^T \theta_{k-1}) \quad (7)$$

b. Projection Algorithm with Covariance Matrix (PA CM)

$$P_k = P_{k-1} - \frac{P_{k-1} \varphi \varphi^T P_{k-1}}{1 + \varphi^T P_{k-1} \varphi} \quad (8)$$

$$\theta_k = \theta_{k-1} + \frac{P_k \varphi}{\varphi^T P_k \varphi} (y - \varphi^T \theta_{k-1}) \quad (9)$$

c. Recursive Least Squares (RLS)

$$\theta_k = \theta_{k-1} + P_k \varphi (y - \varphi^T \theta_{k-1}) \quad (10)$$

d. Recursive Least Squares Normalized (RLS N)

$$\theta_k = \theta_{k-1} + \frac{P_k \varphi}{1 + \varphi^T P_k \varphi} (y - \varphi^T \theta_{k-1}) \quad (11)$$

With the equation (8) the calculation of the covariance matrix is performed, and in the same way for the algorithms shown in equations 9, 10 and 11.

3. Methodology

The methodology proposed for the development of the experimentation was carried out in the following way:

- a. To the capture of the set of input and output signals of the dynamic system was performed, although an off-line estimate could be made because the whole set of samples is available, the process will be performed simulating the implementation recursively.
- b. During the calculation of the estimation recursively by testing the 4 proposed algorithms, the calculation of the RMS error will be made for each algorithm, thus identifying those with the least error, this procedure will be carried out by means of a Monte Carlo experiment with a stop criterion based on the calculation of the minimum variance.

$$E_{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \phi_i^T \theta_{i-1})^2} \quad (12)$$

- c. From the calculation of the estimate, we count on the signals generated from the parametric evolution during the estimation, to these signals obtained from the calculation of the average value and the minimum variance adjusting the RANSAC method to guarantee a constant as a result and the minimum variance this procedure was performed on the same Montecarlo experiment with the stopping criterion based on the minimum variance of the RMS error.
- d. The criterion for the identification of the adaptability and the estimation capacity is given by the algorithms that present the minimum mean squared error (RMS), and identifying those that present lower variance and therefore a static behavior in the parametric evolution during the process of estimation.

4. Results and discussions

For the estimation of the stable parameters of the dynamic system, the evolution of the parameters a_2 , a_1 , b_2 and b_1 was identified, and the parametric evolution of 4 recursive identification algorithms was compared. The comparison of the algorithms will be made using the same set of input and output signals (Figure 3).

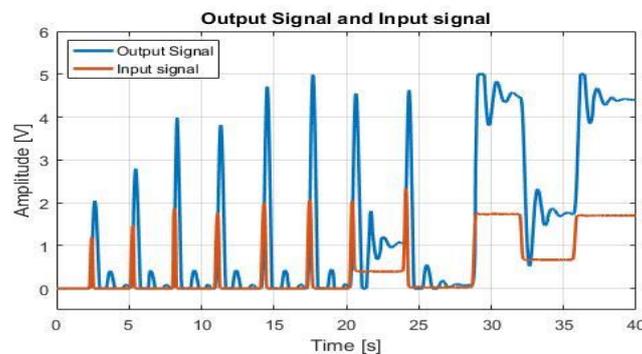


Figure 3. Set of input and output signals.

Figure 4 (Left) and (right) show the parametric evolution and the error, respectively, of the implementation of projection algorithms on the set of input and output signals shown in Figure 3. This implementation readjusts the parameters before changes of the input signal, however, does not allow to obtain a stable behavior.

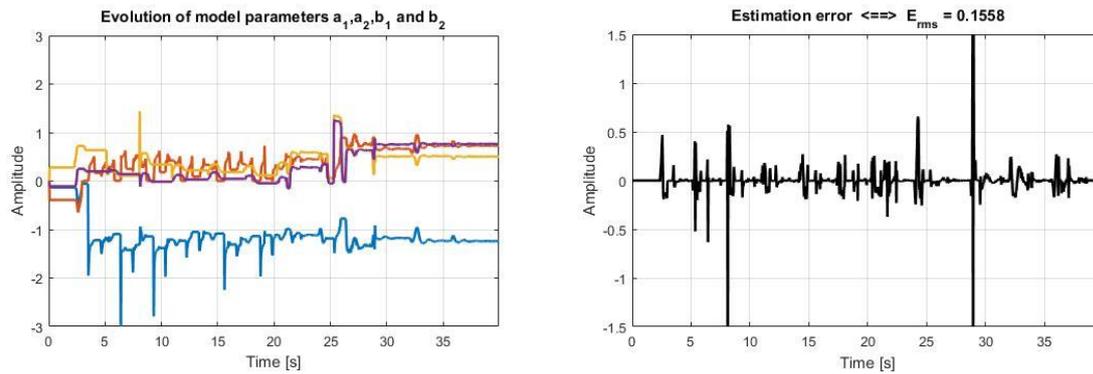


Figure 4. Parametric evolution projection algorithm

Figure 5 shows the parametric evolution obtained by the projection algorithm with covariance matrix (left) and the associated error (right). As in the case of projection algorithms, the parameters are sensitive to disturbances but do not exhibit stability.

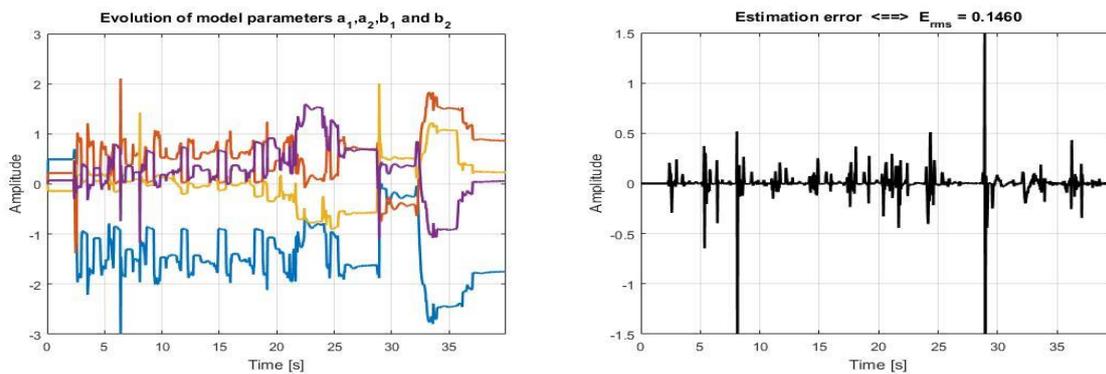


Figure 5. Projection algorithm with covariance matrix.

Figures 6 and 7 show the evolution of the parameters before disturbances of the input signal (left) and the estimated error (right). These algorithms present the most optimal performances, minimizing the error and exhibiting a stable readjustment of the parameters against variations of the input signal.

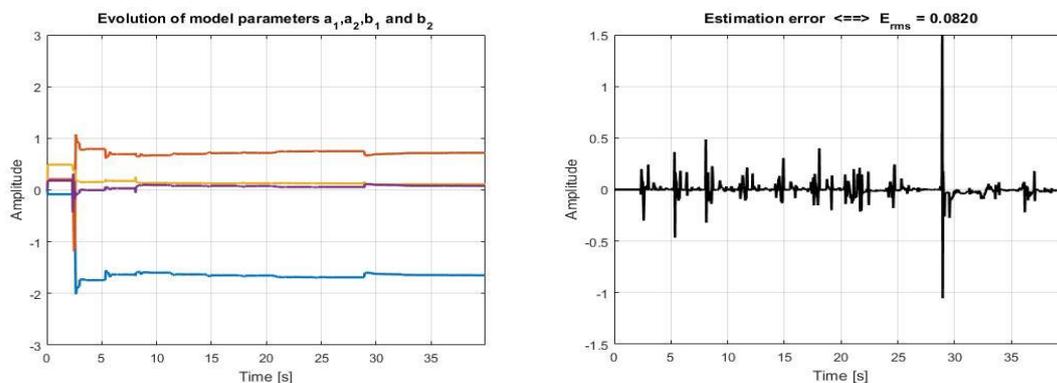


Figure 6. Evolution of RLS algorithms.

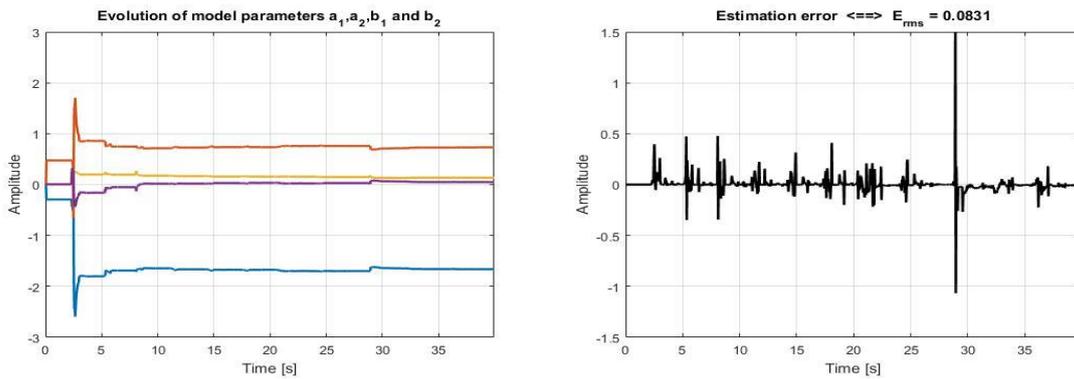


Figure 7. Evolution of Normalized RLS algorithms

The table 1 shows the relationship of the RMS errors obtained through the Montecarlo iterations, and in Figure 8 the behavior of the RMS error during the implementation of the Montecarlo experiment can be observed.

Table 1. Comparison of the RMS error of the estimation algorithms using statistical moments, Montecarlo experiments approx. 1000 iterations.

Estimation Algorithm	Error RMS mean	Error RMS deviation
Projection Algorithm (PA)	0.1591	2973.50×10^{-6}
PA with Covariance Matrix (PA CM)	0.1459	31.41×10^{-6}
Recursive Least Squares (RLS)	0.0819	56.09×10^{-6}
RLS Normalized (RLS N)	0.0885	5994.90×10^{-6}

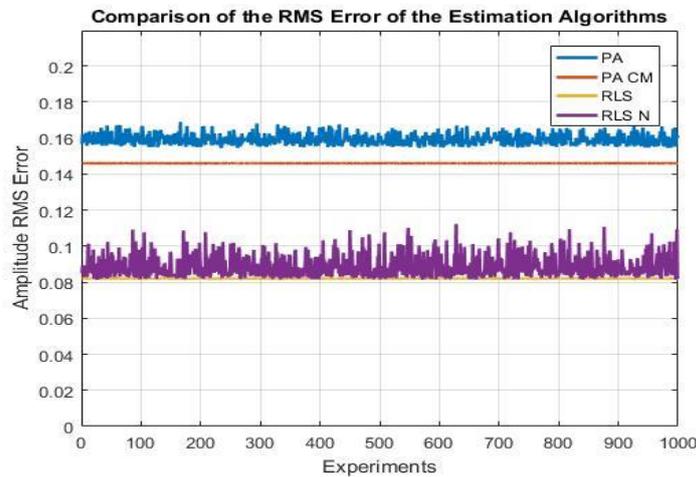


Figure 8. Comparison of the all algorithms using RMS error.

Table 2 shows the standard deviations obtained by the Montecarlo experiment, using in each iteration the RANSAC method to calculate the constant value and the minimum variance.

Table 2. Comparison of the standard deviations of the parameters Montecarlo experiments approx. 1000 iterations.

Estimation Algorithm	Parameter a_1 deviation	Parameter a_2 deviation	Parameter b_1 deviation	Parameter b_2 deviation
Projection Algorithm (PA)	0.1202	0.1175	0.0927	0.1685
PA with Covariance Matrix (PA CM)	0.2202	0.1669	0.1465	0.1660
Recursive Least Squares (RLS)	0.1458	0.0713	0.0284	0.0316
RLS Normalized (RLS N)	0.1554	0.0955	0.0360	0.0376

5. Conclusions

An easy-to-implement system was built that allowed analysing the performance of the estimation algorithms defined in terms of their response to variations in signals and ability to estimate and adapt.

The comparison of the performance of the estimation algorithms shows that those based on RLS and Normalized RLS quickly stabilize their values before disturbances in the input signals of the dynamic system, making it possible to approximate the transfer function in discrete time.

From Table 1, it can be identified that the RLS and RLS N algorithms have the lowest mean squared error of estimation in comparison with the projection algorithms. However, the RLS algorithm not only has the lowest RMS error, but also the minimum standard deviation. In Figure 8 you can observe the behavior of the errors during the evolution of the Monte Carlo experiment.

Finally, using table 2, we could identify that for each parameter of the estimation model the minimum possible variance was found, where for parameter a_1 , the PA algorithm showed the least deviation, however, the RLS algorithm is in second place, and for the rest of the parameters (a_2 , b_1 and b_2) the algorithm has the least deviation.

From the results obtained from the experimentation it can be concluded that the best estimation algorithm with parametric stability is the RLS algorithm, since it presents a low mean squared error of estimation, and the parametric evolution presents the minimum variance and stability in most of the estimated parameters.

6. References

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