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## Design Advanced Algorithm of the Single Dimension for Resolve the Electrostatic problem by Using the MoM Method

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# Design Advanced Algorithm of the Single Dimension for Resolve the Electrostatic problem by Using the MoM Method

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**Abstract.** In this paper, we are updating a new technique for the solve of electrostatic problems so as to evaluation a numerical solution via usage the Method of Moments (MoM). In this paper, the MoM is helpful to simulate the electrostatic problem of a thin conductive rod with length  $L$  and radius  $a$ . The segment of the rod with  $1m$  of length was kept at a constant voltage of  $V_0$ . This problem was indicated as an effective numerical method for resolving the singularity of a matrix which advantageous a Cauchy Principal Integral and Error Estimation. The Cauchy Principal Integral (privacy dissolve) represents  $\int_a^b [f(x) (x-x_m)^{-1}] dx$  ( $a < x_m < b$ ) has been solved singularity and attain a reliable estimation of the approximation error in order to tolerate was trimmed at the safe level, while the computation has been got by separated the integral and put that numeric bury into the matrix. The exact formulation for the matrix element was set a function  $f(x)$  on the numerator of the integral and we are gainful symmetric matrix (square matrix) which was getting the numeric value not infinity number. At this point are interesting the examples were included to illustrate the procedure. The Electrostatic problem was processing details, and illustrative computations have been given in some cases. This way could be generalized to be utilized the functions such as unit pulse and delta function as a basis and testing, respectively, is applied for analyses.

## 1. Introduction

Method of Moments is one of the widely exercises, numerical techniques were employed for the solution of Integral equations [1] into two parts of equation integral which has been getting the efficiency results and exact [2]. In proposed method has been improved run time for matrix and safely in efficient track [3]. MoM is an efficient way that is used to solve the integral equations that interest applicable a big role in numerical techniques. MoM is useful when the problem becomes simpler especially when it has been taken than one dimension. The integral equation is the unique mathematical problem that is used to resolve the electrostatic problems [4]. Our suggestions are utilizing a modern algorithm called Cauchy principal value integral to solving the puzzle singularity in order to split the singularity equation [5]. MoM is a suitable numerical method for the solution of specific problems that has been getting rigorous and efficient results [6]. We are using some power series of processing has been set the singularity, but regrettably do not give us true value. MoM dose expansion to find the unknown function into the equation integral in terms of known basis functions with unknown coefficient factors to will be locate it and evaluation compute it. A favored technique or another mechanism commonly which has been read in a MoM survey results that concern to a system of a linear equation is equal in number to that of unknown coefficients therefore the matrix has been mathematically resolved which was so called the “moment” matrix and the unknown coefficients could be acquired via to that matrix inversion. For more details in our paper are make equations and integrals that will be discussed in the next sections. The target of this process is collecting the unknown charge distribution on a thin wire with that the radius ( $a \ll L$ ) and this conductor its handheld with steady potential voltage  $V_0$  so as the strategy of the MoM



techniques to discretization the conductor rod in spite of growing to sub domains that has been getting one dimension of that problem [7]. The proposed option of the right paradigm for the treatise to that published in subject of the charge distribution conductor effects is very ticklish as computational decreases with boost of model and has been getting purity bulk.

## 2. Generation of the Cofactors Matrix

We are obtaining the guaranteed to discover a unique solution for all a set of values of the Eq.(3.1), in matrix  $A$  subsequently, we are lighting to simulate a matrix vector equation with that the  $[x = z \times b]$  whereas, the correspond points  $x'_1, x'_2, x'_3, x'_4, x'_N$  are set at the center of each segment, figure 1 was explained their slice of partitions to the thin conductor [9]. Eq. (3.1) can be graphed as in Eq. (2.2), [10] is the matrix whose elements are unknown. We can be located and determined [10] from the Eq. (2.2) using Cauchy Principal Integral, matrix inversion, or Gaussian Elimination Technique, using matrix inversion. This equation as shown its illustrative to how run the matrix and obtained as shown.

$$x = z b \quad (2.1)$$

$$[Vx] = [Zmn] [Xn] \quad (2.2)$$

$$z = inv(A) \quad (2.3)$$

Where:

$z$  is the inverse of matrix  $A$  in equation (2.3) and it is evaluating the diagonal elements of the matrix in Eq. (2.2) or Eq. (3.1), caution must be applied. And fetch us a unique solution (real value).

## 3. Simulation of the Work

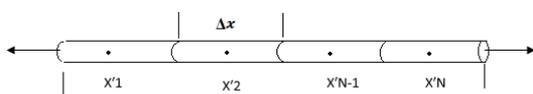
The output voltage across the wire is shown in Eq. (3.1)

$$V(x) = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\rho(x')}{|x-x'|} dx' \quad (3.1)$$

Where:

$x'$ -The centre point of each segment as it was seen in figure 1. The rod has been sliced up in to series subsection each with a width  $\Delta x$  and  $xn'$  as a mid-point of sub section  $xn' = n \Delta x / 2$ . The basis function (rectangle function) [4].

$$un = \begin{cases} 1 = |x - x'| \leq \frac{\Delta x}{2} \\ 0, \text{ otherwise.} \end{cases} \quad (3.2)$$



**Figure 1.** The charged conductor segmented into a series of subdomains with length  $\Delta x$

## 4. Description and Mathematically Analyses

MoM methods are useful for obtaining the solution of the many problems involve a new process of integration technique to be solved analytically. The rules of integration typically work very well for that one-dimensional integral can be extended to higher dimensions so as to get the next computations of the numerical method increases when the problems dimensions will be increases in future. This paper was discovered a new technique of integration which modified to separate the singularity that was set up in. This does not seek any extra stress to remedy the singularity snag as in demand condition can be embed immediately in a new technique oneself in a not one of several statements of the MATLAB code in work for the study in simulation. The test formula as shown beneath:

Let us start by Cauchy Principal Integral.

$$I_{a,b,\tau}(f) = \int_a^b \frac{f(x)}{x-\tau} dx \quad \lim_{\mu \rightarrow 0} \left( \int_a^{\tau-\mu} + \int_{\tau+\mu}^b \right) \frac{f(x)}{x-\tau} dx \quad (4.1)$$

$$I_{\tau}(f) \equiv I_{\tau-1,1}(f) = \int_a^b \frac{f(x)}{x-\tau} dx, \text{ where } a,b = 1 \quad (4.2)$$

for simplicity to modification the variables were usage

$$\delta = \min \{1+\tau, 1-\tau\}.$$

wherever we are employed the convenient equations

$$\int_{|x-\tau| \geq \delta} \equiv \int_{\tau+\delta}^1 \text{ if } \delta = 1+\tau \quad (4.2a)$$

and when we are changing the limitations of integration

$$\int_{|x-\tau| \geq \delta} \equiv \int_{-1}^{\tau-\delta} \text{ if } \delta = 1-\tau \quad (4.2b)$$

The integration becomes as it follows:

$$I_{\tau-1,1}(f) = \int_{-1}^1 \frac{f(x)}{x-\tau} dx = \int_{-1}^1 \frac{u(x'-xn')}{|x-x'|} dx' \quad (4.3a)$$

After when we are using up the unit pulse function, the integral equation becomes as follow:

$$I_{\tau}(f) = \int_{\Delta x} \frac{1}{|x-x'|} dx', \quad x = xm \text{ where } xm \text{ is a mid of subdomain}$$

$$I_{\tau}(f) = \int_{\Delta x} \frac{1}{|xm-x'|} dx', \quad xn = xm \text{ (diagonal element)}$$

While when we are using a logical decision for  $\tau = x_m$  (start using diagonal of matrix)

$$A(n,m) = \int_{\Delta x} \frac{1}{|xm-x'|} dx', \quad xm = xn \quad (4.4)$$

Therefore, the integration of Cauchy Principal Integral it was seen bellow:

$$\text{If } (f) = f(\tau) \text{Log} \frac{1-\tau}{1+\tau} + \int_{|x-\tau| \geq \delta} g(x) dx + \int_0^{\delta} h(x) dx \quad (4.5)$$

$$g(x) = \frac{f(x) - f(\tau)}{x - \tau}, \quad h(x) = \frac{f(\tau+x) - f(\tau-x)}{x} \quad (4.5a)$$

We are necessity to find the calculate of sub equations (4.5a)

$$f(x_m) = 1, \text{ since } x_m \in \Delta x$$

where

$$g(x) = \frac{f(x') - f(xm)}{x' - xm} = \frac{f(x') - 1}{x' - xm} \quad (4.6)$$

The limits of integrations are to that our aim as shown:

$$\delta = \min \{1 + x_m, 1 - x_m\} = 1 - x_m \text{ since } x_m > 0$$

$$\int_{-1}^{2x_m-1} \frac{f(x')-1}{x'-x_m} dx' = \int_{-1}^{x_m-\Delta x/2} \frac{-1}{x'-x_m} dx' + \int_{x_m+\Delta x/2}^{2x_m-1} \frac{-1}{x'-x_m} dx' \quad (4.7)$$

Then the limit of the integration after separation it's making up as shown:

$$\int_{-1}^{2x_m-1} \frac{f(x')-1}{x'-x_m} dx' = \begin{cases} 0, & x' \in \Delta x \\ \int_{-1}^{2x_m-1} \frac{f(x')-1}{x'-x_m} dx', & \text{otherwise} \end{cases} \quad (4.8)$$

The part of the following integral was becoming

$$\int_0^{\delta} h(x') dx' = \int_0^{1-x_m} h(x') dx' \quad (4.9)$$

After we are place the Eq. (4.9) it's showing as

$$h(x') \frac{f(x_m+x')-f(x_m-x')}{x'} \quad (4.9a)$$

Figure 2 is segmented rod to subsections with haulage distribution and applied voltage  $V_0$  Since  $x_m$  is putting the midpoint of  $\Delta x$ .

$$f(x_m + x') - f(x_m - x') = 1 - 1 = 0$$

Then the integration was appearing as follows:

$$\int_{-1}^{2x_m-1} \frac{f(x')-1}{x'-x_m} dx' = \int_{-1}^{x_m-\Delta x/2} \frac{-1}{x'-x_m} dx' + \int_{x_m+\Delta x/2}^{2x_m-1} \frac{-1}{x'-x_m} dx' \quad (4.10)$$

$$\int_{-1}^{2x_m-1} \frac{f(x')-1}{x'-x_m} dx' = \int_{x_m-\frac{\Delta x}{2}}^1 \frac{-1}{x'-x_m} dx' + \int_{2x_m-1}^{x_m+\frac{\Delta x}{2}} \frac{-1}{x'-x_m} dx' \quad (4.11)$$

When we have the following integral of matrix  $A$  becomes as shown

$$A(m,n) = \int_{-1}^1 \frac{1}{x'-0} dx' \quad (4.12)$$

since  $x_n - he/2 = -1$  then  $x_n = (0.5he) - 1$

$$\int_{x_n - he/2}^{x_n + he/2} \frac{1}{|x_m - x'|} dx' = \int_{x_m - he/2}^{x_m + he/2} \frac{1}{|x_m - x'|} dx' = \int_{-1}^1 \frac{1}{|x'|} dx' \quad (4.12a)$$

By using  $he$  is delineated by different segmentation:

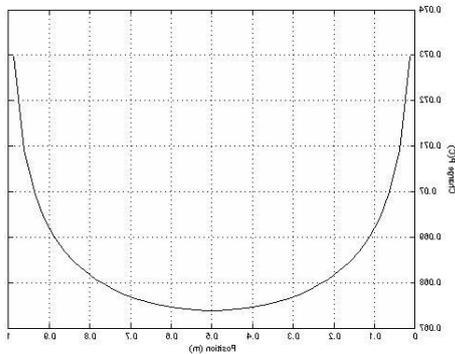
$$\begin{aligned} x_m - he/2 = -1 & \quad \text{then} \quad x_m = he/2 - 1 \\ x_m + he/2 = 1 & \quad \text{then} \quad x_m = 1 - he/2 \quad \text{that's lead} \quad 2x_m = 0 \quad \text{since} \quad x_m = 0 \\ \therefore he/2 = x_m + 1 = 1 & \quad \text{that's lead} \quad he = 2 \end{aligned}$$

To this end our new paper is gaining a novel algorithm which has been split the singularity function into more than one part of integration as shown in Eqs. (4.2 - 4.5) to avert singularity with each part of integration after we are getting not infinity numeral.

## 5. Error Estimation

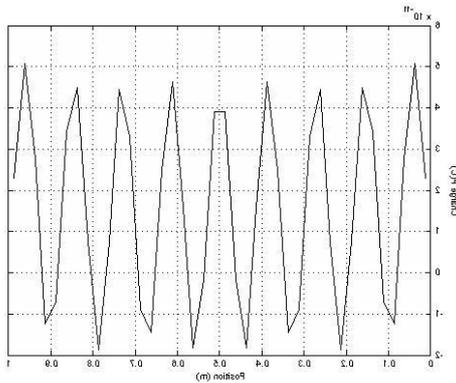
The results of error estimation in giving undefined value of the matrix is applied method that's related with reference as shown in figure 2. This method has been taken less running time and less accuracy as compare with other algorithm, despite of Eq. (3.1) we are obtaining the oscillation shape when we are select a value of  $\mu$  very far from zero but when we are selecting values of  $\mu$  approach to zero the

figures dose take the natural state of distribution charge on the thin rod as shown below that's corresponding to the values of  $\mu$ . The table (1) of equation (3.1) shows coincidence with the result of Eq. (3.5) at  $\mu$  equal (0.001) which are represented the best value. So, as the accuracy and period time is trade-off, our paper takes more time for run and high precision.

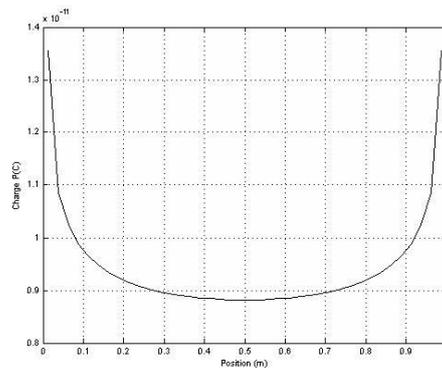


**Figure 2.** Charge distribution of singularity function with  $N$  points of interest on a bar with (40)

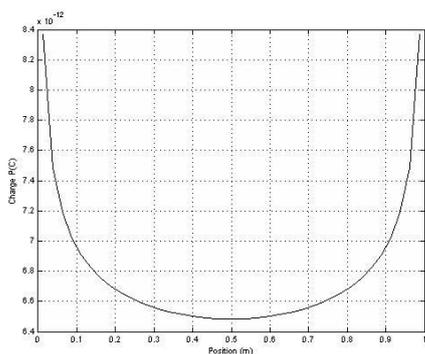
Our option in Eq. (4.1) have got to us oscillated shape when we have been chosen a values of  $\mu(\mu)$  far away from zero as shown in figure 4.



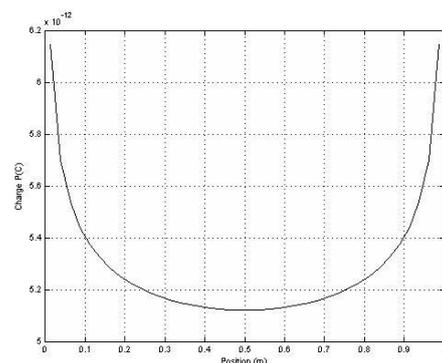
**Figure 3.** Charge distribution at  $\mu = 0.01$



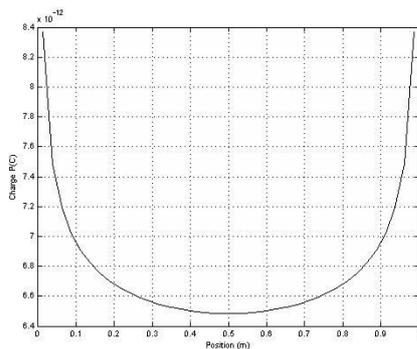
**Figure 4.** Charge distribution at  $\mu = 0.001$



**Figure 5.** Charge distribution at  $\mu = 0.0001$



**Figure 6.** Charge distribution at  $\mu = 0.00001$



**Figure 7.** Charge distribution of Eq. (4.5)

**Table 1.** Tried and tested numerical values of Eq. (4.5)

No	$\mu$	Result	Figures
1	0.01	6.3466e-10	Figure 3
2	0.001	3.7879e-10	Figure 4
3	0.0001	2.7156e-10	Figure 5
4	0.00001	2.1185e-10	Figure 6
Tested numerical value of Eq.(4.7)			
1	-----	2.7156e-10	Figure (7)

## 6. Conclusion

The current style is a second hand in this paper when numerical MoM to numeration unknown charge distribution density on a thin wire. Moreover, it's used to overcome the singularity by a subdivision of domain that has been divided the whole rang. This job demonstrates an efficient reliable way for fix this problem with error estimation is acceptable which has been taken the precision and time period of computations are trade-off.

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