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## Identification and viewing of singularities for a new type of 6R<sub>SS</sub> parallel mechanism

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# Identification and viewing of singularities for a new type of 6RSS parallel mechanism

L Milica<sup>1</sup>, A Năstase<sup>1</sup> and G Andrei<sup>1</sup>

<sup>1</sup> Mechanical Engineering Department, “Dunarea de Jos University” of Galati, Galati, Romania

E-mail: milica.lucian@ugal.ro

**Abstract.** This paper presents a method for determining the singular configurations of a 6RSS parallel mechanism based on its kinematic model. The parallel mechanism consists of a fixed plate and a mobile platform connected through six independent kinematic chains. Each of the six kinematic chains has an actuating rotational joint and two spherical joints. The kinematic analysis is based on the kinematic screw method. The resultant motion can thus be represented by composing the kinematic screws of the component motions. Starting from the two Jacobian matrices of the kinematic model, the two types of singularities encountered in parallel mechanisms are identified. Singular configurations are determined by an algorithm which is further used to produce three-dimensional representations of the two matrices determinants. Also, with the same program, it is possible to accurately establish the existence or inexistence of a mechanism for a given position and orientation.

## 1. Introduction

Parallel mechanisms have attracted the attention of many researchers due to the high degree of structural resistance that makes them less susceptible to input errors than their serial counterparts [1-5]. These qualities make parallel mechanisms find their applicability in various fields: flight simulators and fine positioning devices and fast packing in high-speed milling machines [6, 7].

Because of the kinematic complexity of these mechanisms, numerous researches have been undertaken on this issue [8-13].

An important role in the kinematic control of the parallel mechanisms is the determination of the singular configurations associated with the direct Jacobian matrix and the inverse one, respectively. Determining the singular configurations is a central issue for the kinematic model of the robot .

Identifying them means finding those critical configurations that cause important changes in the kinematic performance of a manipulator. The study of singularities is the main tool in order to avoid these critical positions.

The development of the robotic industry and the emergence of parallel mechanisms with increasingly complex movements imply the development of general methods that contribute to the analysis of singularities, thus improving the process of designing and programming the robot. Planning robot movements, taking into account the identification of all singular configurations, is the major objective to be pursued.

In these particular positions, the degree of spatial mobility of the effector element decreases therefore some of the input parameters no longer have an effect on its movement. The issue of critical positions is subject to special studies on parallel mechanisms [14-16].

In order to avoid singularities the researches undertaken so far have revealed different means in



their analysis. Some researchers have developed a specific technique based on treating the geometric place determined by singularities as a set of obstacles [17]. Using three-dimensional representations, other researchers have determined the singular configurations for a Stewart parallel mechanism, for a certain orientation and position of the mobile platform [18].

## 2. The kinematics of the 6RSS parallel mechanism

The kinematic analysis of a mechanism establishes the implicit relationships between the infinitesimal motion of the actuated kinematic joints and the effector element. The present paper performs the kinematic analysis of a six-degree parallel manipulator 6RSS (rotational joint, spherical joint, spherical joint) from figure 1, using the kinematic screw method.

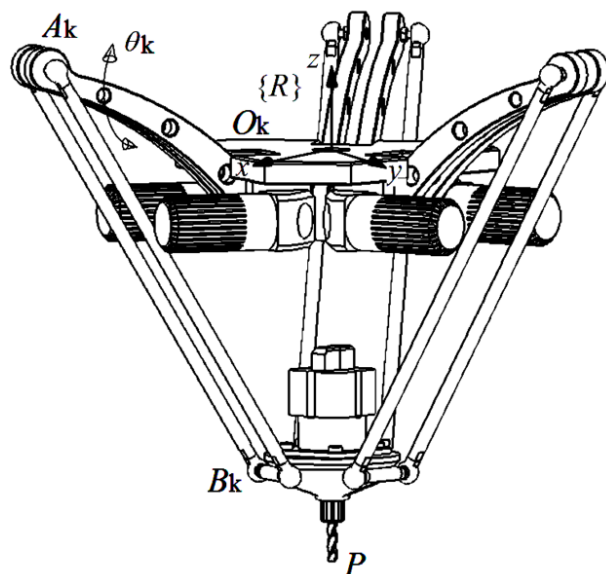
The parallel mechanism is represented in figure 1 in 3D view. The 6RSS parallel manipulator consists of a fixed plate to which six actuators are fitted, and a mobile platform on which is fitted the seventh actuator to drive a milling tool. One of the six kinematic  $k$  chain of the 6RSS parallel manipulator has the kinematic rotational joints at points  $O_k$  and the kinematic spherical joints at points  $A_k$  and  $B_k$  respectively. The drive arms  $O_k A_k$  are rotated at an angle  $\theta$  around the axis of the vertex  $\mathbf{u}_k$  passing through the point  $O_k$ .

The mechanism has the following features:

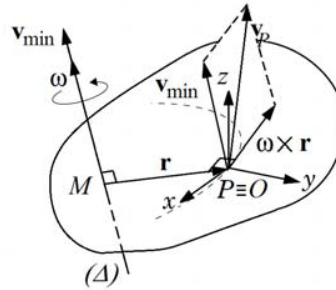
- axis of rotational active joints that generate circular trajectories has an  $\mathbf{u}_k$  unit vector and are coplanar, two-by-two coincident and central-symmetrically disposed ;
- the actuating arms  $O_k A_k$  are of equal length;
- $A_k B_k$  rods are of equal length;
- spherical joints of the mobile platform are coplanar and their centres are located on the vertices of a regular hexagon.

Determining the kinematics of parallel mechanisms involves establishing the equations of the resultant motion of a rigid body having a relative motion in relation to a mobile coordinates system. The resultant movement can be represented through the kinematic screws by composing the screws of all the components in motion.

In figure 2 we have attached to a rigid body a coordinate system with center in point  $P$  (identical to the origin of the system  $O$ ) which belongs to this.



**Figure 1.** The new 6RSS parallel manipulator.



**Figure 2.** Elements of the kinematic screw.

Consider the rigid body in an instantaneous motion relative to a roto-translational axis ( $\Delta$ ).  $\omega$  is the angular velocity of the body,  $r$  is the distance from point  $P$  relative to the roto-translational axis ( $\Delta$ ) and  $\lambda$  is the screw parameter given by the relation:

$$\lambda = \frac{v_{\min}}{\omega} \quad (1)$$

The screw parameter  $\lambda$  has the length dimension and characterizes the movement geometrically. We have thus expressed two particular cases:

- when  $\lambda = 0$  results a pure rotation;
- when  $\lambda = \infty$  results a pure translation.

We have the following expressions:

$$\omega = \omega \cdot \mathbf{u} \quad (2)$$

$$\mathbf{k} = \mathbf{r} \times \mathbf{u} + \lambda \cdot \mathbf{u} \quad (3)$$

where  $\mathbf{u}$  is the unit vector of the instantaneous axis of rotation. When this axis is coincident with the axes of the reference system, the expressions of the unit vector  $\mathbf{u}_x$ ,  $\mathbf{u}_y$ ,  $\mathbf{u}_z$  of the homologous axes can be determinate easily.

Relationships (2) and (3) define the geometric screw  $\$$  whose general formula is:

$$\$ = [\mathbf{u} \cdot \mathbf{k}]^T \quad (4)$$

If for the expression (3) we make the notation:

$$\mathbf{w}_j = \mathbf{r}_j \times \mathbf{u}_j \text{ și } \mathbf{r}_j = \mathbf{M}_j \mathbf{O}_j \quad (5)$$

we have the following two cases:

- for the rotational joint:

$$\$ = [\mathbf{u}_j \cdot \mathbf{w}_j]^T \quad (6)$$

- for the translational joint:

$$\$ = [\mathbf{0} \cdot \mathbf{u}_j]^T \quad (7)$$

By notating the kinematic screw with the  $\Omega$ , the expression becomes:

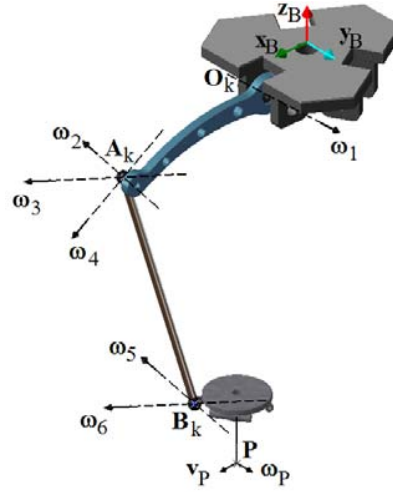
$$\Omega = \omega \cdot \$ \quad (8)$$

in which  $\omega = |\omega|$ .

In the case of the parallel mechanism analyzed, by composing a number of kinematic screws of a kinematic chain, the resulting motion will be described by the kinematic screw obtained by summing up the  $n$  kinematic screws:

$$\Omega = \Omega_1 + \Omega_2 + \dots + \Omega_n \quad (9)$$

Let us examine one of the kinematic chains  $k$  of the parallel mechanism  $6R_{SS}$  in figure 3. Each actuated arm has a local mobility given by the rotation around the axis  $A_k B_k$ , independent movement relative to the other possible movements of the mechanism. To simplify the calculations, we eliminated these six isolated nobilities and replaced the spherical joint from point  $B$  with a fourth-class joint that does not affect the degree of mobility of the mechanism.



**Figure 3.** Angular velocities of kinematic joints belonging to a  $k$ -chain.

We have attached to a rigid body a coordinate system with center in point  $P$  (identical to the origin  $O$  of the system) which belongs to this.

At each angular velocity  $\omega_1, \dots, \omega_P$  corresponds a kinematic screw  $\Omega_1, \dots, \Omega_P$  so that for the six kinematic chains of the mechanism will result the system of equations:

$$S = \begin{cases} \Omega_1^{(1)} + \Omega_2^{(1)} + \dots + \Omega_6^{(1)} = \Omega_P \\ \Omega_1^{(2)} + \Omega_2^{(2)} + \dots + \Omega_6^{(2)} = \Omega_P \\ \vdots \\ \Omega_1^{(6)} + \Omega_2^{(6)} + \dots + \Omega_6^{(6)} = \Omega_P \end{cases} \quad (10)$$

in which it is known that:

$$\Omega_P = [\Omega_1^P \quad \Omega_2^P \quad \Omega_3^P \quad \Omega_4^P \quad \Omega_5^P \quad \Omega_6^P]^T \quad (11)$$

Taking into account the relation (8) above system of equations is:

$$S = \begin{cases} \omega_1^{(1)} \cdot \mathcal{S}_1^{(1)} + \omega_2^{(1)} \cdot \mathcal{S}_2^{(1)} + \omega_3^{(1)} \cdot \mathcal{S}_3^{(1)} + \omega_4^{(1)} \cdot \mathcal{S}_4^{(1)} + \omega_5^{(1)} \cdot \mathcal{S}_5^{(1)} + \omega_6^{(1)} \cdot \mathcal{S}_6^{(1)} = \Omega_P \\ \omega_1^{(2)} \cdot \mathcal{S}_1^{(2)} + \omega_2^{(2)} \cdot \mathcal{S}_2^{(2)} + \omega_3^{(2)} \cdot \mathcal{S}_3^{(2)} + \omega_4^{(2)} \cdot \mathcal{S}_4^{(2)} + \omega_5^{(2)} \cdot \mathcal{S}_5^{(2)} + \omega_6^{(2)} \cdot \mathcal{S}_6^{(2)} = \Omega_P \\ \vdots \\ \omega_1^{(6)} \cdot \mathcal{S}_1^{(6)} + \omega_2^{(6)} \cdot \mathcal{S}_2^{(6)} + \omega_3^{(6)} \cdot \mathcal{S}_3^{(6)} + \omega_4^{(6)} \cdot \mathcal{S}_4^{(6)} + \omega_5^{(6)} \cdot \mathcal{S}_5^{(6)} + \omega_6^{(6)} \cdot \mathcal{S}_6^{(6)} = \Omega_P \end{cases} \quad (12)$$

Taking into account expression (6), the expression of screws for this joint is:

$$\mathcal{S}_i^{(k)} = [\mathbf{u}_i^{(k)} \cdot \mathbf{r}_i^{(k)} \times \mathbf{u}_i^{(k)}]^T \quad (13)$$

It is known that for any five geometric screws  $\mathcal{S}_j$ , ( $j = 1 \dots 5$ ) there is a reciprocal screw  $\mathcal{S}_r$ , so that the product reciprocally between it and any of the five screws is 0:

$$\begin{aligned} \mathcal{S}_1 \circ \mathcal{S}_r &= 0 \\ \vdots \\ \mathcal{S}_5 \circ \mathcal{S}_r &= 0 \end{aligned} \quad (14)$$

Considering two geometric screws  $\mathcal{S}_1 = [\mathbf{u}_1 \cdot \mathbf{w}_1]^T$  and  $\mathcal{S}_2 = [\mathbf{u}_2 \cdot \mathbf{w}_2]^T$  we have the relationship (15) that defines the expression of their reciprocal product:

$$\mathcal{S}_1 \circ \mathcal{S}_2 = \mathbf{u}_1 \cdot \mathbf{w}_2 + \mathbf{u}_2 \cdot \mathbf{w}_1 \quad (15)$$

The components of the reciprocal screws  $\mathbf{S}_{2,6}^{r(k)} = [\mathbf{u}_r^{(k)} \cdot \mathbf{w}_r^{(k)}]^T$  for the  $k$  kinematic chains are determined with condition (14) and through expression (15), the six systems will have the form:

$$S = \begin{cases} \mathbf{S}_2^{(k)} \circ \mathbf{S}_{2,6}^{r(k)} = 0 \\ \mathbf{S}_3^{(k)} \circ \mathbf{S}_{2,6}^{r(k)} = 0 \\ \mathbf{S}_4^{(k)} \circ \mathbf{S}_{2,6}^{r(k)} = 0 \\ \mathbf{S}_5^{(k)} \circ \mathbf{S}_{2,6}^{r(k)} = 0 \\ \mathbf{S}_6^{(k)} \circ \mathbf{S}_{2,6}^{r(k)} = 0 \\ u_{r_x}^2 + u_{r_y}^2 + u_{r_z}^2 = 1 \end{cases} \rightarrow S = \begin{cases} \mathbf{u}_2^{(k)} \cdot \mathbf{w}_r^{(k)} + \mathbf{u}_r^{(k)} \cdot \mathbf{w}_2^{(k)} = 0 \\ \mathbf{u}_3^{(k)} \cdot \mathbf{w}_r^{(k)} + \mathbf{u}_r^{(k)} \cdot \mathbf{w}_3^{(k)} = 0 \\ \mathbf{u}_4^{(k)} \cdot \mathbf{w}_r^{(k)} + \mathbf{u}_r^{(k)} \cdot \mathbf{w}_4^{(k)} = 0 \\ \mathbf{u}_5^{(k)} \cdot \mathbf{w}_r^{(k)} + \mathbf{u}_r^{(k)} \cdot \mathbf{w}_5^{(k)} = 0 \\ \mathbf{u}_6^{(k)} \cdot \mathbf{w}_r^{(k)} + \mathbf{u}_r^{(k)} \cdot \mathbf{w}_6^{(k)} = 0 \\ u_{r_x}^2 + u_{r_y}^2 + u_{r_z}^2 = 1 \end{cases} \quad (16)$$

where  $u_{r_x}^2, u_{r_y}^2, u_{r_z}^2$  are the components on the three directions of the vector  $\mathbf{u}_r^{(k)}$ . Solving each of the six systems (16) results in the components of the six reciprocal screw  $\mathbf{S}_{2,6}^{r(k)}$ .

Multiplying each system equation (16) by the reciprocal screw, it results:

$$S = \begin{cases} \omega_1^{(1)} \cdot \mathbf{S}_1^{(1)} \circ \mathbf{S}_{2,6}^{r(1)} = \Omega_P \circ \mathbf{S}_{2,6}^{r(1)} \\ \omega_1^{(2)} \cdot \mathbf{S}_1^{(2)} \circ \mathbf{S}_{2,6}^{r(2)} = \Omega_P \circ \mathbf{S}_{2,6}^{r(2)} \\ \vdots \\ \omega_1^{(6)} \cdot \mathbf{S}_1^{(6)} \circ \mathbf{S}_{2,6}^{r(6)} = \Omega_P \circ \mathbf{S}_{2,6}^{r(6)} \end{cases} \quad (17)$$

The reciprocal products  $\Omega_P \circ \mathbf{S}_{2,6}^{r(k)}$  will have the following expressions:

$$\Omega_P \circ \mathbf{S}_{2,6}^{r(k)} = [\Omega_1^P \quad \Omega_2^P \quad \Omega_3^P]^T \cdot \mathbf{w}_{2,6}^{(k)} + \mathbf{u}_{2,6}^{(k)} \cdot [\Omega_4^P \quad \Omega_5^P \quad \Omega_6^P]^T \quad (18)$$

The system (17) becomes:

$$S = \begin{cases} \omega_1^{(1)} \cdot \mathbf{S}_1^{(1)} \circ \mathbf{S}_{2,6}^{r(1)} = [\Omega_1^P \quad \Omega_2^P \quad \Omega_3^P]^T \cdot \mathbf{w}_{2,6}^{(1)} + \mathbf{u}_{2,6}^{(1)} \cdot [\Omega_4^P \quad \Omega_5^P \quad \Omega_6^P]^T \\ \omega_1^{(2)} \cdot \mathbf{S}_1^{(2)} \circ \mathbf{S}_{2,6}^{r(2)} = [\Omega_1^P \quad \Omega_2^P \quad \Omega_3^P]^T \cdot \mathbf{w}_{2,6}^{(2)} + \mathbf{u}_{2,6}^{(2)} \cdot [\Omega_4^P \quad \Omega_5^P \quad \Omega_6^P]^T \\ \vdots \\ \omega_1^{(6)} \cdot \mathbf{S}_1^{(6)} \circ \mathbf{S}_{2,6}^{r(6)} = [\Omega_1^P \quad \Omega_2^P \quad \Omega_3^P]^T \cdot \mathbf{w}_{2,6}^{(6)} + \mathbf{u}_{2,6}^{(6)} \cdot [\Omega_4^P \quad \Omega_5^P \quad \Omega_6^P]^T \end{cases} \quad (19)$$

By denoting with  $r_1^{(k)}$  the scalar of the reciprocal product  $\mathbf{S}_1^{(k)} \circ \mathbf{S}_{2,6}^{r(k)}$ , ( $k = 1, \dots, 6$ ) from (19), its matrix expression becomes:

$$\begin{bmatrix} r_1^{(1)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r_1^{(6)} \end{bmatrix}_{(6 \times 6)} \cdot \begin{bmatrix} \omega_1^{(1)} \\ \vdots \\ \omega_1^{(6)} \end{bmatrix}_{(6 \times 1)} = \begin{bmatrix} \mathbf{w}_{2,6}^{(1)} & \cdots & \mathbf{w}_{2,6}^{(1)} & \mathbf{u}_{2,6}^{(1)} & \cdots & \mathbf{u}_{2,6}^{(1)} \\ \vdots & & \vdots & \vdots & & \vdots \\ \mathbf{w}_{2,6}^{(6)} & \cdots & \mathbf{w}_{2,6}^{(6)} & \mathbf{u}_{2,6}^{(6)} & \cdots & \mathbf{u}_{2,6}^{(6)} \end{bmatrix}_{(6 \times 6)} \cdot \begin{bmatrix} \Omega_1^P \\ \vdots \\ \Omega_6^P \end{bmatrix}_{(6 \times 1)} \quad (20)$$

The left-components  $r_1^{(k)}$  of matrix determine the direct Jacobian [B] and the right-components  $\mathbf{w}_{2,6}^{(k)}, \mathbf{u}_{2,6}^{(k)}$  of the matrix determine the inverse Jacobian [A].

The equation (20) written in matrix form becomes:

$$[B] \cdot \begin{bmatrix} \omega_1^{(1)} \\ \vdots \\ \omega_1^{(6)} \end{bmatrix} = [A] \cdot \begin{bmatrix} \Omega_1^P \\ \vdots \\ \Omega_6^P \end{bmatrix} \quad (21)$$

The expression of the kinematic screw of the platform is:

$$\Omega_P = \omega_P \cdot \mathbf{S}_P = \omega_P \cdot [\mathbf{u}_P \quad \mathbf{r}_P \times \mathbf{u}_P + \lambda_P \cdot \mathbf{u}_P]^T = [\omega_P \cdot \mathbf{u}_P \quad \omega_P \cdot (\mathbf{r}_P \times \mathbf{u}_P) + \lambda_P \cdot \mathbf{u}_P \cdot \omega_P]^T \quad (22)$$

Where, based on the relationships below:

$$\lambda_p = \mathbf{u} \cdot \mathbf{w} \quad (23)$$

$$\mathbf{r}_p = \mathbf{u} \times \mathbf{w} \quad (24)$$

we get the expressions of  $\omega_p$ ,  $\lambda_p$  and  $\mathbf{r}_p$  that fully describe the movement of the platform:

$$\omega_p = \sqrt{(\Omega_1^p)^2 + (\Omega_2^p)^2 + (\Omega_3^p)^2} \quad (25)$$

$$\lambda_p = \Omega_1^p \cdot \Omega_4^p + \Omega_2^p \cdot \Omega_5^p + \Omega_3^p \cdot \Omega_6^p \quad (26)$$

$$\mathbf{r}_p = (\Omega_1^p \cdot \mathbf{i} + \Omega_2^p \cdot \mathbf{j} + \Omega_3^p \cdot \mathbf{k}) \times (\Omega_4^p \cdot \mathbf{i} + \Omega_5^p \cdot \mathbf{j} + \Omega_6^p \cdot \mathbf{k}) \quad (27)$$

### 3. Determining the singular configurations of the 6RSS parallel manipulator

Based on the two Jacobian matrices, we can define the two types of singularities encountered in parallel mechanisms. These singularities interfere when the mechanism loses one or more degrees of freedom in certain configurations. Thus, when the determinant of the matrix  $[A]$  tends to infinitely we have type I singularity:

$$\det[A] = \pm\infty \text{ or } \det[B] = 0 \quad (28)$$

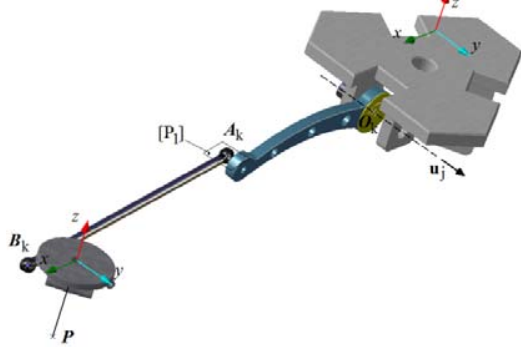
In the case of this type of singularity, the mobile platform remains motionless while the actuated arms move. When matrix determinant  $[A]$  is zero, we have singularity of type II:

$$\det[A] = 0 \quad (29)$$

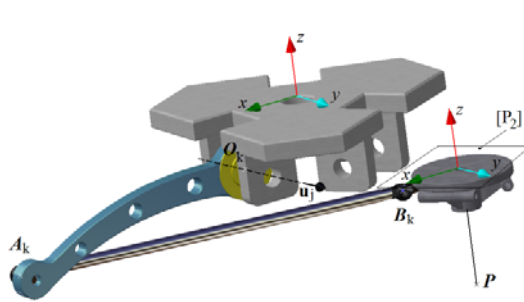
This type of singularity is characterized by the fact that the mobile platform continues to move while the actuated arms are locked.

In the case of the 6RSS parallel manipulator, these singular positions can be geometrically expressed by the following two conditions:

- when point  $B$  belongs to the plane  $[P_1]$  determined by the unit vector  $\mathbf{u}$  of the rotational joint and point  $A$  (figure 4);
- when point  $A$  belongs to the plan  $[P_2]$  of the mobile platform (figure 5).



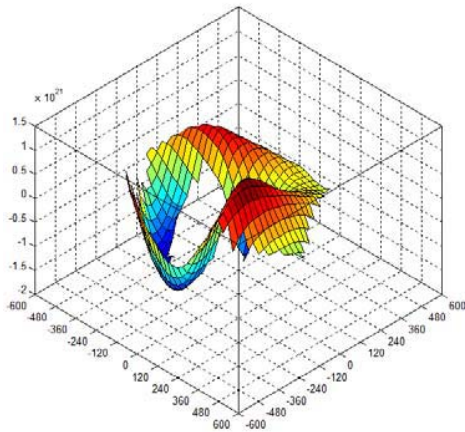
**Figure 4.** Singularity of type I, point  $B$  belongs to the plane  $[P_1]$



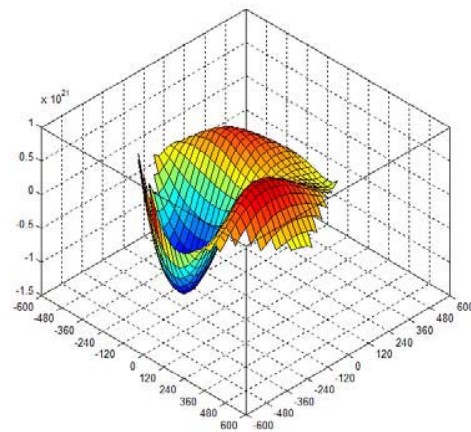
**Figure 5.** Singularity of type II, point  $A$  belongs to the plane  $[P_2]$

To determine these singular configurations, a program was created to produce three-dimensional representations of the functions of determinants  $\det[B]$  and  $\det[A]$  of the direct and inverse Jacobian matrix from equation (21). The two Jacobian matrices determine the kinematic model of the parallel mechanism. The analysis of singular configurations has been achieved for cases where the mobile platform operates at a certain  $z$ -position relative to the fixed reference system attached to the base (figures 6-13).

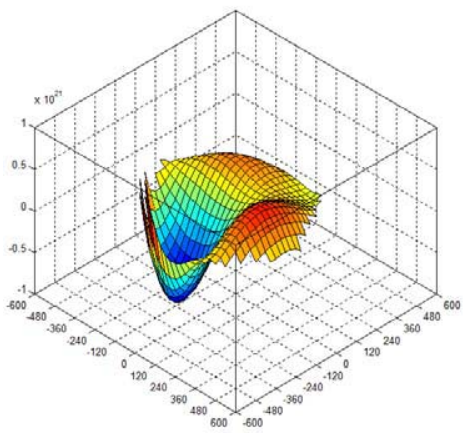




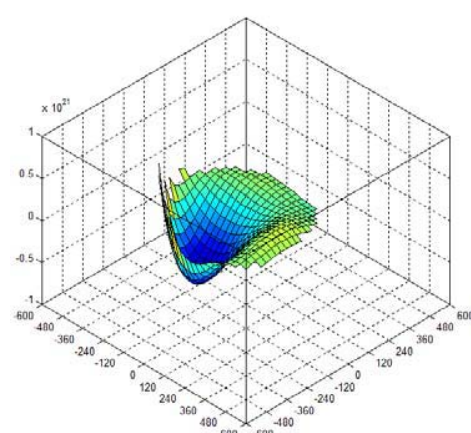
**Figure 6.** Function  $\det[A]$ ,  $z = -200\text{mm}$



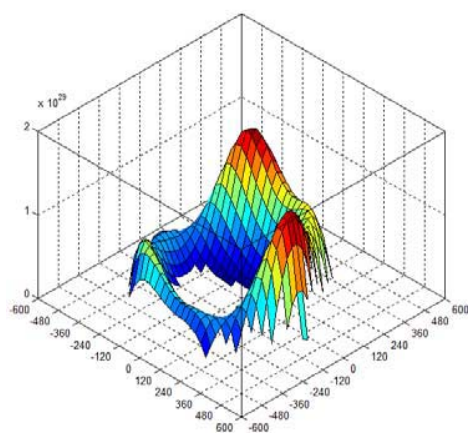
**Figure 7.** Function  $\det[A]$ ,  $z = -315\text{mm}$



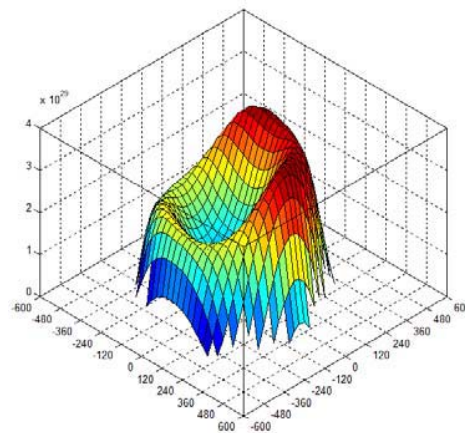
**Figure 8.** Function  $\det[A]$ ,  $z = -400\text{mm}$



**Figure 9.** Function  $\det[A]$ ,  $z = -495\text{mm}$

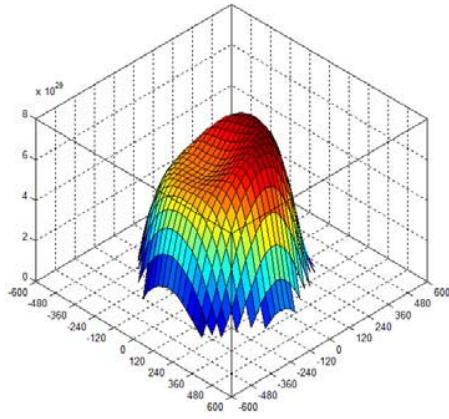


**Figure 10.** Function  $\det[B]$ ,  $z = -200\text{mm}$

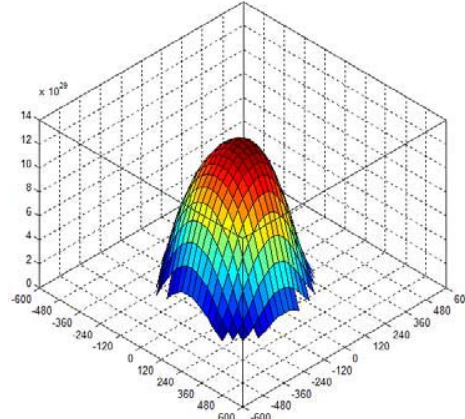


**Figure 11.** Function  $\det[B]$ ,  $z = -315\text{mm}$





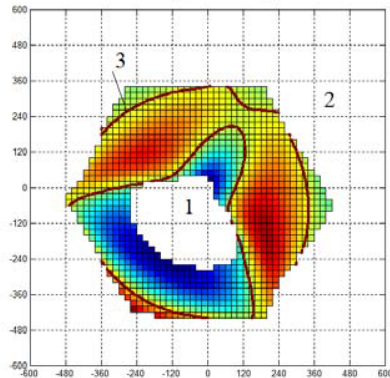
**Figure 12.** Function  $\det[B]$ ,  $z = -400\text{mm}$



**Figure 13.** Function  $\det[B]$ ,  $z = -495\text{mm}$

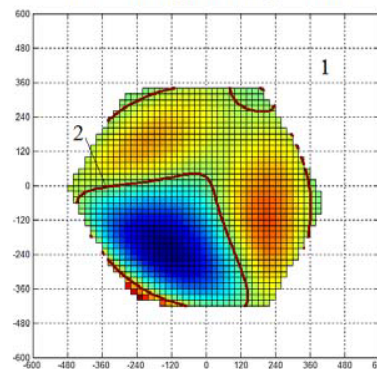
Based on these three-dimensional representations, sections with a plane corresponding to the zero value of the two functions  $\det[B]$  and  $\det[A]$  were done. These sections highlight those critical curves determined by the set of singular positions fulfilling one of the two conditions mentioned above (figures 14, 15). Also, with the help of the same program, it was possible to establish precisely the existence or the inexistence of the mechanism for a given position and orientation.

- 1,2 - Spaces in which the mechanism can not exist  
3 - Critical curves determined by the set of singular configurations



**Figure 14.** Section with a plane corresponding to the value  $\det[A] = 0$ , for  $z = -200\text{mm}$

- 1 - Spaces in which the mechanism can not exist  
2 - Critical curves determined by the set of singular configurations



**Figure 15.** Section with a plane corresponding to the value  $\det[A] = 0$ , for  $z = -315\text{mm}$

#### 4. Conclusions

This work contributes to an original approach to avoiding the single configurations of the new 6RSS parallel mechanism. The issue of singularities was highlighted based on the general kinematic model of the parallel manipulator determined by the kinematic screws. The mechanism presented in this paper is part of the manipulators with complex architectures that appear in the current robotics, characterized by new mechanisms of increasingly complex movements.

Based on the two Jacobian matrices, the two types of singularities encountered in parallel mechanisms were defined. A program was created to make three-dimensional representations of the determinate functions  $\det[B]$  and  $\det[A]$  of the direct and inverse Jacobian matrix. The developed method thus allows the determination of the singular configurations of the parallel mechanism for a particular position and orientation of the mobile platform. Viewing these sections with precise details on those trajectories that generate singular positions allows the motion capabilities of the manipulator to be determined. These diagrams can provide a map on which to identify those trajectories that do not cross singularity, in other words they are safe in terms of the robot's motion capabilities.

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