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A survey on gear meshing quality based on tooth contact analysis

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Abstract. Gears are common mechanical elements in power transmission systems. In order to avoid failure of transmission systems, it is necessary to inspect the gear meshing condition with respect to kinematic and dynamic parameters. Tooth contact analysis (TCA) method represents a useful tool for designing and analysing gear performance. It represents an important method to examine gear tooth contact points, contact ratios and kinematic errors. The primary objective of TCA is to establish the contact trail on gear tooth surfaces, and also to determine the transmission error functions generated by gear misalignments. TCA has several branches such as the loaded tooth contact analysis, unloaded tooth contact analysis and tooth contact analysis with errors. The determination of the initial contact point is necessary in order to achieve an accurate solution. The present survey allows a new insight into the improvement of the gear meshing quality, based on tooth contact analysis and transmission error.

1. Introduction

Gears are common mechanical elements in power transmission systems. Numerous industries benefit from their use. Their main advantages are robustness, small dimension, constant transmission ratio, high performance etc. In order to avoid failure of transmission systems, it is necessary to inspect the gear meshing condition in terms of kinematic and dynamic parameters. Gears also demand high accuracy of machining and assembly process which may influence gear meshing system by spreading the vibration and noise. The transmission error is considered the main cause for noise occurrence. Gearing noise is one of the main concerns for mechanical engineers.

Walker [1] was the first to study noise and vibration by analyzing the modifications of spur gear tooth profile. Gear noise has three important elements: gear whine, gear clatter and gear rattle. Gear whine is a result of time-varying mesh stiffness and is regarded as a clean tonal noise [2]. Whining noise represent the noise produced by the meshing gears under load [3]. Its existence is an effect of the static transmission error resulted from the meshing process [4,5]. Gear rattling occurs as a consequence of gear backlash and may be present when the meshing gears are under a slight load [6]. Gear clatter is generated by torsional oscillations when the gear mesh is under load.

Tooth contact analysis (TCA) algorithm is a useful tool for designing and analysing gears performance. It represents an important method to examine gear tooth contact points, contact ratios and kinematic errors. Involute tooth surfaces are usually used to achieve linear contact and lower transmission error functions. Usually, in order to reduce noise and vibration in transmission systems, high contact ratio gears are used. The primary objective of TCA is to establish the contact path on gear tooth surfaces and also to determine the transmission error functions generated by gear misalignments. Gear meshing quality can be assessed through a number of elements, like the transmission error and contact pattern.

This paper is a state-of-the-art on the theory of TCA algorithm that determines the gear meshing.



2. Condition of continuous tangency

Three stiff independent coordinate systems (S_1, S_2, S_f) interacting with both gears and the framework are considered. In order to reproduce the misalignment, an extra settled coordinate system is connected (S_q). In S_1 and S_2 coordinate systems, two tooth surfaces (Σ_1, Σ_2) are examined. The first gear with the corresponding tooth surface (Σ_1) spin about a settled axis situated in S_f . Therefore, the coordinate system S_f contains a group of gear tooth surfaces. The second gear with the corresponding tooth surface (Σ_2) rotates about another settled axis situated in S_q . The position and direction of the coordinate system S_q , considering S_f , reproduces the misalignment of the pinion [7]. When the location vectors and the normal fit at any moment, it is considered that contacting surfaces are in continuous tangency (Fig. 1).

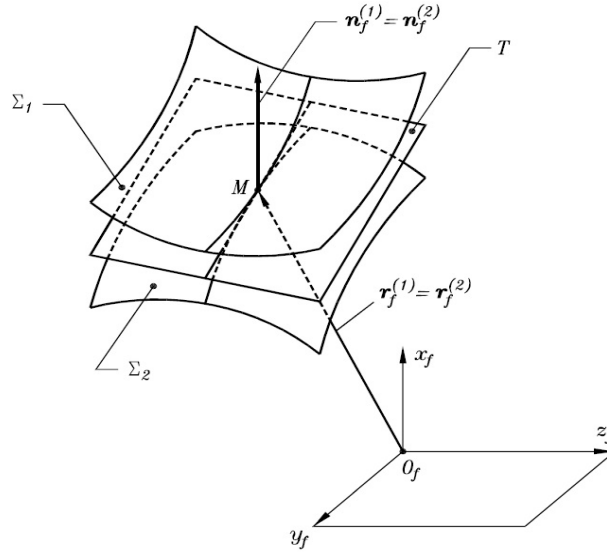


Figure 1. Tangency of surfaces in ideal gear train [7]

The equation of continuous tangency of pinion and gear surfaces is reached when establishing the vector conditions:

$$r_f^{(1)}(u_1, \theta_1, \phi_1) - r_f^{(2)}(u_2, \theta_2, \phi_2) = 0; \quad n_f^{(1)}(u_1, \theta_1, \phi_1) - n_f^{(2)}(u_2, \theta_2, \phi_2) = 0 \quad (1)$$

where r_f represent the position normal vector, n_f is the unit normal vector, u_i and θ_i are the parameters of the gear surface and ϕ_i represent the angle of gear revolution.

3. Mathematical pattern of surface contact analysis

The involute curve represents the most common gear profile and also the ideal shape of the unmodified tooth flank. When the modified tooth flank case is considered, a section of the pressure line is kept unchanged during the gear meshing, in order to maintain the involute curve [8].

$$L_{Ca} = 0.5 \times (g_\gamma - p_e) \quad (2)$$

where L_{Ca} is the length of tip and root relief, g_γ represent the length of the pressure line and p_e is the base pitch length.

Tooth flank is set by covering the curvature family of the cutting route. Tooth flank geometry can be described using a universal generation model which is developed as a widespread machine setting. Usually, eight fundamental generic machine tools settings are well-known. Using conversion matrices for symbolizing the kinematic chain, tooth flank generic mathematical model was achieved [9]:

$$\begin{cases} r_b(\mu, \theta, \phi) = M_{bc}(R_a, S_r, E_M, X_D, X_B, \gamma_m, \sigma, \zeta; \phi) \times r_c(\mu, \theta) \\ n_b(\mu, \theta, \phi) = M_{bc}(R_a, S_r, E_M, X_D, X_B, \gamma_m, \sigma, \zeta; \phi) \times n_c(\mu, \theta) \\ n_b(\mu, \theta, \phi) \times v(\mu, \theta, \phi) = 0 \end{cases} \quad (3)$$

where $n_b \times v = 0$ form the gearing theory, μ and θ are the cutter blade parameters, also known as Gaussian parameters and ϕ is a motion parameter. M_{bc} represents the transformation matrix. $R_a, S_r, E_M, X_D, X_B, \gamma_m, \sigma, \zeta$ are the fundamental machine tool settings used to change the polynomial terms through the manufacturing process.

The cutter blade usually has three types of blade shape: straight, Top-Rem and parabolic profile.

3.1. Determining the points of contact

Cao et al. [10] presented in their article the mathematical pattern for estimating the edge contact, tooth contact area and depicted the tooth contact analysis ease-off numerical procedure.

The common algorithm presents the following contact conditions: the unit normal vectors are aligned at the point of contact and the surface position vectors are coincidences. The equations can be presented as [10]:

$$\begin{cases} r_h^1(u_1, v_1, \varphi_1) = r_h^2(u_2, v_2, \varphi_2) \\ n_h^1(u_1, v_1, \varphi_1) = n_h^2(u_2, v_2, \varphi_2) \end{cases} \quad (4)$$

where \mathbf{r}_i are the position vectors of pinion and gear, \mathbf{u}_i and \mathbf{v}_i are the face generating characteristics and φ_i represent the angular position.

When the \mathbf{r}_1 and \mathbf{r}_2 functions are considered, a string of instantaneous contact points are created considering each angular position of the gear drive. The equivalent transmission error function can be expressed as [10]:

$$\Delta\varphi_2(\varphi_1) = \varphi_2 - (z_1/z_2) \times \varphi_1 \quad (5)$$

where the Transmission Error is considered to be the gap between the current position of the rotation angle of the output gear and the theoretical one if the meshing gears are ideally conjugate [11,12].

The common method for the estimation of the instantaneous contact point ensures a good precision as a result of the necessity for an answer to solve the nonlinear equations.

3.2. Transmission Error and the separation of the point of contact

A function $H(j)$ is considered to be the difference between the contact curve (CC) and the changed curve at the considered mating point taking into account the gear drive angular position [10]. The instantaneous unloaded TE represents the minimum difference $\Delta(i)$ between the instantaneous conjugate CC and the changed curve. The unloaded TE can be written as [10]:

$$TE(i) = [\Delta(i) / (-z_1^* n_{y1}^* + y_1^* n_{z1}^*)] (z_1/z_2) \quad (6)$$

where \mathbf{n}_1^* and \mathbf{r}_1^* represent the unit normal and position vectors of the point p_1^* , and the length deviation $\Delta(i)$ had to be transformed into angular deviation.

4. Contact characteristics

Considering shaft misalignment and tip corner contact, it is necessary to inspect the impact of the flank changes on the contact features [8]. Lately, numerous researches have been conducted to investigate the contact characteristics. Pedrero determines the surface resistance and tooth bending strength by analysing a load distribution sample developed on the minimum resilient potential principle. Munro [13] proposed an analytical method for estimating the transmission error of spur gears heaving high contact ratio and corner contact.

Zhuo et al. [14] used spiral and hypoid bevel gears to obtain a method for the optimization of the transmission error and tooth contact pattern. Their method includes optimization objectives, a genetic algorithm and control parameters in order to found the pinion tooth face. They determined the correlation between the curvature elements and the pitch cone and achieved a new list of basic equations for the hypoid gear pitch cone elements. A perfect geometry for the tooth contact pattern cannot be established using standard machine adjustments and cutter specifications for the fabrication of hypoid and spiral bevel gear controls. Tooth contact pattern may present errors related to shape, location or size. Many types of control parameters exist, such as pressure and spiral angle, pitch angle and pitch cone distance, root angle or the coefficient of contact zone length. Still, all these parameters

are not appropriate to use for the design of meshing features. Changes in machine-tool settings lead to different types of transmission errors. The shape of the tooth surface is influenced by the machine-tool parameters and also by the type of the cutter blade. Curvature characteristics are determined by the cutter blade section while the adjustment parameters have a strong correlation with the pitch cone elements.

The need to reduce the gear noise and sensitivity to misalignment represents the main issue for the aerospace and automotive industries. The original contact point had to be identified at the beginning of the process in order to acquire an exact response. The initial contact point affects the dimension of the contact pattern, the contact pressure and the amplitude of TE [15]. Huang et al. article [16] present the modification of the contact pressure produced between the meshing teeth. The contact stress changes are investigated by the FEM method.

Jabbour and Asmar [17] determined a method for the estimation of contact stress throughout each contact line of spur and helical gears. In other paper [18] they created a pattern for the simulation of the conduct of a plastic helical gear.

Liang et al. [19] present the estimation strategy for the deviation of tooth areas of the conjugate-curve pinion. Zhou [20] described the micro-contact characteristics of gear tooth shape using an elastic-plastic roughness contact pattern considering or not the friction case. The micro-contact characteristics of different rough surfaces are very important to friction, fatigue, wear or to the transmission performances.

Fernandez del Rincon et al. [21] proposed a connection between a regional and a global term to establish the deformation of each contact point. The local term was established using the Hertzian contact method, and the FEM analysis was used to determine the global term. Winter et al. [22] study the load distribution when profile changes are considered. Sanchez et al. [23] also had approached the problem of a load distribution model for inward gear transmission who used the minimum elastic potential energy. Their model established the boundary load regime and the level of the contact and bending stress. The elastic potential energy can be described as the sum of the compressive part with the shear component and the bending component. The contact stress can be determined using the Hertz's equation. Contact stress was also investigated by Chen et al. [24] who studied the case of concave conical involute meshing gears having a non-parallel axis.

In the signal-vibration study for the improvement of gearbox diagnostic models, it is necessary to determine the impact of gear faults in order to estimate the errors and boundary operation conditions [25]. Tharmakulasingam [2] used numerical methods and FEA to highlight the result of tooth profile changes on the TE in spur gears. Sheveleva et al. [26] suggested a way for tracking the contact path where the relative location among tooth surface pairs was important and the revolution of a gear wheel was established when the contact was acquired.

Kawasaki et al. [27] used Klingelnberg cyclo-paloid model to present the design process, the determination of the contact path and TE and the effect of assembly errors on gear meshing properties.

Digital image correlation represents an efficient instrument for the measurement of in-plane deformation considering a planar object area. It specifically gives full-field displacements with a very good precision and strains by looking at the computerized pictures of a test subject area obtained before and after the deformation [28].

4.1. Basic contact conditions for meshing teeth

The engaging teeth having the specific displacements (δ_1, δ_2) come into contact at the point Q_y . The contact situation of two different points on the mating teeth, having a partition distance h , is described by the following equations [29]:

$$h + w_{k1} + w_{k2} = \delta, \text{ when the two points are in contact} \quad (7)$$

$$h + w_{k1} + w_{k2} > \delta, \text{ when the two points are out of contact} \quad (8)$$

where w_{k1} and w_{k2} represent the specific deformation.

4.2. Discretization of the contact zone of the meshing tooth

The contact surface of the mating teeth is usually discretized into a large number of units on the mutual tangential plane. The influence coefficient method is used to determine the deformations

having the assigned stresses on the separated elements. As shown in fig.2 the deformation w_{kj} of the medium point P_k of a single element, which is generated by a stress p_j that influence another spot P_j , can be interpreted using the influence coefficient $f_{k,i}$ [29].

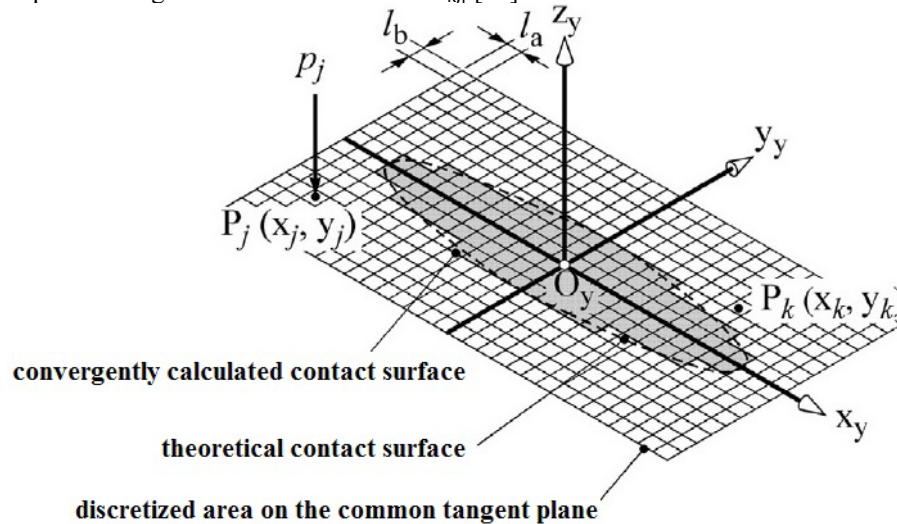


Figure 2. Gear meshing on the mutual tangent plane [29]

4.3. Corner contact

In their work Yu and Mechefske [30] developed an analytical procedure to indicate the impact of corner contact on the static transmission error and TVMS of loaded spur gears. They used FEM to calculate the dynamic response of meshing gears and the static transmission error (STE).

Corner contact represents the contact which is external to the standard contact length and it takes place as a result of the resilient deviation of the loaded teeth. This situation is considered to be the cause for early meshing. The contact point is influenced by the implemented torque. The elastic deformation grows with the increase of the transmitted load. Gear teeth partition distance represents the length between mating teeth without connection when there is no resilient deformation. The contact situation is established by comparing the distance along the pressure line with the LSTE. Lin et al. [31] presented the link between transmission error and corner contact without taking into consideration gear tooth profile faults.

For spur gears having a standard contact ratio, corner contact appears when the approached tooth couple arrive at the presumed starting point of the meshing and the separating tooth couple is leaving the finish point of the meshing considered. Yu et al. [30] made the following assumptions: corner contact concerns only the gear mesh at the beginning and the end of a singular tooth couple part in meshing and have no effect on the double engagement.

4.4. Tip relief optimization

In Fuentes et al. paper [32] a parabolic tip relief is used by changing the cross-section geometry of the generating tool.

Spur and helical gear use involute tooth faces which ensure linear contact and a reduced function of TE considering normal meshing situation. Spur gears can be considered a special case of the helical gears when the helix angle is nil. Tip relief is generally necessary in order to achieve better contact conditions throughout gear meshing. Tip relief represents the permanent elimination of the material from the top edge of the tooth face. It can be implemented using linear and parabolic functions of deflection of the tooth profile. In the absence of tip relief, different surfaces of high contact stress arise on the top edge of the tooth face. The use of an ideal level of tip relief allows for improvement of contact solicitation throughout the entire meshing period and bypass the occurrence of the different surfaces with intense contact solicitation.

As a result of the tip relief changes, bending stress rises slightly influenced by the decrease of the actual contact ratio.

5. Tooth contact analysis

TCA software programs predict meshing and contact of gear tooth areas taking into account the localized bearing contact, which provides the surface contact point. Using the TCA algorithm the following objectives are set [7]:

- a) the transmission errors generated by the misalignment between gear axes;
- b) the contact route on pinion tooth surfaces;
- c) bearing contact as the arrangement of momentary contact ellipses.

Considering misalignment, the place and direction of gear axes are known and gear tooth surfaces are perfectly determined.

TCA technique represents an important instrument for achieving spiral and hypoid gears and also determines gear tooth contact performances [9]. TCA outcome is strongly influenced by the original contact point. TCA equations are established when the considered original values that affect the original contact point are acceptable, otherwise the computational procedure is stopped or the TCA outcome is incorrect.

There are currently several ramifications of the tooth contact analysis technology, such as the loaded tooth contact analysis (LTCA), the unloaded tooth contact analysis (UTCA) or the tooth contact analysis with errors (ETCA).

Vogel et al. [33] used a so-called “ease-off topography” to estimate the tooth contact analysis of hypoid bevel gears. In their research, the transmission error, the curvature properties and the path of contact parameters were obtained. A formulation of the ease-off was given by Stadtfeld [34] who also determined the instantaneous contact between two areas.

In Su Jinzhan et al. paper [35] it was achieved a seventh-order polynomial function of transmission error for spiral bevel gears. They used both TCA and LTCA algorithms to compare the seventh-order polynomial function and the parabolic function of transmission error. The Finite Element Method, mathematical programming and flexibility matrixes are part of the numerical simulation LTCA technique. A reverse TCA procedure is set for the derivation factor of the changed roll.

Jiang Jinke and Fang Zongde [36] presented a solution to diminish noise and vibration of cylindrical spur gears by changing the tooth face using a verifiable higher sequence polynomial function of TE (H-TE) and also described the process of achieving the mathematical pattern. They optimized the loaded transmission error minimal amplitude (ALTE) using TCA and LTCA of pinions and used particle swarm optimization technique in order to achieve the polynomial coefficients of H-TE. The TCA algorithm represents a solution to analyse the profile of transmission errors and the changes of contact pattern caused by manufacturing and assembly errors. Under lightly load tooth meshing features and TE can be established using TCA technics. A derivation of gear drive tooth surface and additional rotation factors were created using a reverse TCA pattern.

TCA algorithm and FEA software are the methods by which computer simulation of contact pattern and gear meshing is performed. Bearing contact among different surface pairs do not influence the TCA procedure. The estimation of TCA does not require solving nonlinear equations, but it has to take into account the impact of the neighbouring tooth couple on the contact pattern [32].

Vilmos Simon [37] described a procedure for computer-aided TCA in inconsistent spiral bevel gears. In his work he presented the contact path and also the possible contact lines. His procedure considers the minimization of a particular function that causes the detachment of the tooth surfaces throughout the face width. The possible contact lines are represented by the points having this minimum detachment. The method was created using a software program. The use of this software enables the determination of the tooth faces that separate throughout the contact lines, the contact path, the angular displacement and the changing of the angular speed ratio over a meshing period.

Chao et al. [38] developed a TCA algorithm for spherical gear sets. TCA outcome delivers important information about the path of contact, kinematic errors and the contact ellipses. From the geometrical point of view, two types of spherical gear tooth shape are known: concave and convex. Unlike helical or spur gears, spherical gears enable the changeable shaft angle and the existence of misalignment without having gear interference throughout the meshing cycle.

The gear tooth presents a number of errors among which we can list: pressure angle errors, lead angle errors, tooth profile errors etc. The most common assembly errors are the horizontal and vertical axial misalignment and the centre distance errors. Location and unit normal vectors are considered

inside a coordinate system when the TCA algorithm is applied. Thus, the unit normal vectors are collinear. Tooth surface contact point is located inside an elliptical area, due to the fact that gear tooth surfaces exhibits elasticity. TCA outcomes allow the estimation of the immediate contact point of the gear mesh. The consideration of continuous tangency of the tooth faces gear drive represent the hypothesis on which the TCA procedure is based. Tooth surface contact pattern is obtained from the succession of the contact ellipses which are generated by the motion of the gear drives. The methods used for the investigation of the contact pattern are divided into two main categories: rigid body and the elastic body method. The best-known elastic body method is the FEM which is used to examine the contact surface with respect to the resilient deformation of tooth areas. The curvature research process belongs to the contact pattern investigation which represents a rigid body method [38].

Hsu and Su [39] analyze TCA and TE of a gear meshing pair for the case of a crowned pinion and a common involute gear.

Daniele Vecchiato [40] creates a TCA algorithm for misaligned planetary gear train. He establishes that changes for the contacting teeth area are necessary in order to bypass edge contact and also gives the parabolic function of TE.

Guan et al. [41] study the manufacturing procedures for crown gear coupling having misalignment. They located the contact points and examine the crown gear coupling having misalignment using a precise solution based on the TCA algorithm.

5.1. Loaded tooth contact analysis

The influence coefficient model is used to determine the LTCA algorithm with respect to the interconnection between the deformation and the loading on a contact region. According to Yu et al. [8] research contact stress had a powerful correlation with the separation distances. In their LTCA algorithm, the influence coefficients used for deformations are subject to tooth bending, shear loading and Hertzian contact of the engaged teeth. Real models are also affected by the twisted shafts due to the existing torque on the contact pressure.

The LTCA algorithm used by Jinke and Zonde was developed by Zhang and Fang [42,43]. During the meshing cycle, a couple of mating tooth (indicated as I and II) are in contact at some point, as presented in fig. 3 [36].

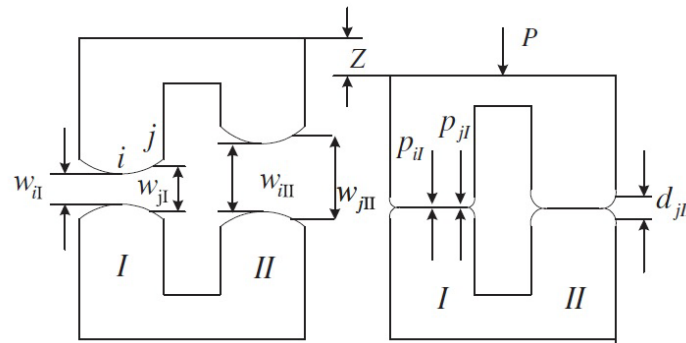


Fig. 3 A model for LTCA [36]

The contact load among two reaching tooth surfaces is considered to be initiated throughout the relative main direction. The intersection between the normal plane throughout the relative main direction and the tooth surface represents the tooth curvature. The clearance among a pair of mating tooth surfaces, when tooth deformation is not taken into account, can be described by the following set of equations [36]:

$$\begin{cases} F_k p_k + w_k = Z + d_k, \dots k = I, II \\ \sum_{j=1}^n p_{jl} + \sum_{j=1}^n p_{jii} = P \\ d_{jk} = 0, \dots (p_{jk} > 0) \\ d_{jk} > 0, \dots (p_{jk} = 0) \end{cases} \quad (9)$$

where ik represent the contact point, jk indicates a point throughout the relative main direction, p_k represent the column matrix, F_k signify the assembled adaptability matrix of contact place conditioned by the FEM analysis, w_k represent a column matrix that includes the underlying tooth difference from TCA, and Z indicates the tooth approximation throughout the pressure line.

According to Li [44], LTCA of spur gears had been determined by implementing FEM and mathematical programming method.

Considering element deformation and flank topology, LTCA pattern can reproduce the loaded contact particularities [29]. Among the methods used for establishing LTCA, the FEM is the most useful for analysing the contact between gear teeth. High precision and the deformations of the elements being analysed are the main FEM advantages. Influence coefficient method was used by Wu and Tsai [45] to create a computerized LTCA.

Ye and Tsai [8] created an effective computerized instrument using the effect of misalignment and tip corner contact to establish the LTCA.

In Sanchez-Marin et al. work [46] a new geometric approximation was suggested for the TCA.

5.2. Unloaded tooth contact analysis

In Astoul et al. [47] article a numerical algorithm for UTCA is presented. His suggested algorithm is considered steady and offered a decent estimation of the contact design area. In order to obtain a higher robustness, a lower complexity level was chosen. Mesh density influences the precision of the contact point place and also the computing time.

In Fong's research [48] was developed a model to make less demanding the improvement of a general hypoid gear generator. An adjustment of Fong's model [49] was realized by Shih. He was able to reproduce different face-hobbing processes.

5.3. Tooth contact analysis with errors

ETCA technology had been used to reproduce the gear meshing operation [50]. Using ETCA and considering the benefit of numerical techniques, the contact points of the gear meshing process can be established. The most important element of the calculation process represents the precise and efficient determination of the primary contact point. This first contact point immediately influences the dimension and course throughout the contact length of the tooth flank. The original contact point is assessed by various numerical models, such as the improved Powell hybrid algorithm [51], the Newton-Raphson algorithm [52] or FEM. In order to determine the initial contact point, it is necessary to introduce a string of possible primary values [50]. In other TCA researches, the original contact point value is established as an estimated value that leads to human errors. The examination of the original point value having minimum distance requires many iterations and some coefficients must be randomly considered. Litvin et al. [51] presented an iterative procedure to obtain the estimated values, taking into consideration the minimum deviation of the normal vectors. However, this solution is not very efficient as a large number of operations are required to compute a set of complex nonlinear equations. Considering ETCA, errors mainly arise from the assembly errors of the gear meshing and the geometric and kinematic errors of the machine tool.

Han et al. [50] presented a number of noteworthy improvements, like the subdivision procedure, the counting of matching tooth face points or the coordinating methodology. In their work, the actual tooth flank with errors is represented using a generic term of the ease-off. Manufacturing errors that are specific to the active flank are generated by the effects of different faults. The ease-off topography can be described by the following equations [50]:

$$\varepsilon_i^j = [r_i^{*j}(q_i, \theta_i) - r_i^j(q_i, \theta_i)] \times n_i^j(q_i, \theta_i) \quad (i = 1, 2; j \in [1, n \times m]) \quad (10)$$

where r_i represents the tooth flank vector, n_i is the tooth flank normal vector, i is considered the drive or driven gear and q and θ are the basic design parameters of the tooth flank.

Using the Coordinate Measuring Machine (CMM), it was possible to obtain the proper coordinate parameters of the calculated locations on the tooth flank. The nonlinear ETCA equations are determined by the tooth flank, cutter blade and the tooth contact conditions. The contact pattern is made from all existing contact ellipses. The contact ellipses can be determined for each contact point

using Euler and Bertrand formula. The occurrence of the transmission error is caused by the fact that the transmission ratio is different in each contact point [50].

6. Time-varying mesh stiffness and meshing simulation

The vibration study of the gear meshing can be determined by analysing the time-varying mesh stiffness (TVMS), which represents an internal excitation of the gears [53]. The TVMS calculation can be realized using several methods, among which we list: analytical, FEM and experimental methods. The most effective way to determine TVMS represents the use of the FEM method, which takes into account geometric and assembly errors. However, the FEM method presents an important drawback as the processing time may be long.

In Astoul et al. article [47] the algorithm used for the meshing simulation considers the relative displacement of the two sections. Relative displacement presents four separate elements: the master part axial displacement, the slave part axial displacement, the shaft angle and the offset displacement. Tooth flank shape affects the relative displacement and the kinematic and geometric conduct of the contact pattern. The Gleason principle is used to estimate the value and sign, as well as the displacement direction. The tooth flank creation procedure is the only parameter that influences the stability of the algorithm. On the master flank, an ordinary mesh is created and its nodes are projected on the slave flank. The driven gear may have the master part functionality. The slave flank is considered to be stationary, while the master flank spins. In order to detect if the contact point is inside the contact pattern, it was necessary to compare the lowest calculated angle value at any place in the gear mesh with the value calculated at the previous step. If the first value was smaller, then the contact point was present in the meshing area, otherwise it does not exist in the specified area.

In Chen et al. research [54] a refined analytical TVMS pattern is presented for gear tooth root break having deep spread irregular along tooth width.

Analytical computation of an individual tooth pair can be separated into a number of elements, such as [30]:

- a) the narrow deformation of the tooth determined by the Hertzian contact;
- b) the tooth beam deviation having the route of the load, considering gear tooth as an irregular cantilever when the tooth basis is entirely stiff;
- c) the tooth deviation having the route of the load provoked by the elasticity of the basis when tooth is considered to be entirely stiff.

Mesh stiffness influences the load distribution of a tooth pair, which varies at each contact point [23]. Load measured per every unit of length also differs at each point of the contact line and contact length.

Wu et al. [55] described a FEA method for the dynamic contact, taking into account the loaded contact deformation, sliding friction and changes of the tooth meshing couple. Dynamic Transmission Error (DTE) is determined when the investigation is conducted at low speed. Dynamic meshing features are influenced by the sliding friction which generates a pitch point effect. As a result of this pitch point effect, the dynamic contact force decreases immediately. When the investigation takes place at high speed the dynamic meshing features with an existing TVMS is analysed by the described method.

7. Contact fatigue

Shen et al. [56] determined a solution to estimate the contact fatigue life by studying the properties of a gear pump. Their method merges the Finite Element Method with the nominal stress method pursuing the Miner cumulative damage hypothesis. A gear pump was used as it represents the fundamental energy source of a hydraulic device. Tooth face wear and fatigue resulted from the high contact strain that gear tooth face had to support.

According to Miner cumulative damage assumption, the damage produced by cyclic stress at every stage is self-sufficiency and permanence!when is under a periodic load and fatigue failure of a component take place when an accumulated damage threshold level is reached. When a nominal stress method is observed, it is necessary to take into account many material elements such as surface processing coefficient, size factor or the actual stress concentration coefficient. It had been noticed that gears present a lower noise level when the pressure angle is greater than 20 degrees and the contact

ratio is greater than two. Gear noise and vibration and tooth fatigue are influenced by dynamic gear loads. To increase gear efficiency, lower the noise levels, avoid tooth breaking and eliminate the pitting effect, it is necessary to reduce the dynamic gear loads. The change of the pressure angle affects the meshing properties [57].

Gear mesh stiffness changes during the meshing process. It represents the main source of excitation for noise and vibration. When a crack or other fault is present in the gear tooth then gear mesh stiffness is also altered [58].

Many kinds of research were made for the investigation of the crack propagation path. Researches outcomes are applied to prevent unexpected failure, to monitor the health status by calculating the dynamic vibration response or to diagnose gear faults [59]. Considering the case of bevel gears, the maximum bending stress takes place on the highest point of singular tooth contact and maximum contact fatigue solicitation is in the proximity of the pitch line [60].

8. Conclusion

The article reviews the basic types of tooth contact analysis and also the mathematical pattern for the surface contact analysis as well as gear mesh contact features.

Tooth contact analysis algorithm represents an important instrument for designing and examining the gear meshing quality. One of the most important purposes of TCA is to determine the contact path on gear tooth faces, and also to establish the transmission error functions produced by gear misalignments.

The profile of transmission error function is affected by the TCA algorithm.

A precise geometry of the tooth contact pattern cannot be determined using common machine adjustments and cutter specifications for the fabrication of hypoid and spiral bevel gear controls. Tooth contact model features errors associated with size, location or shape.

The identification of the initial contact point is very important for achieving an accurate response. Initial contact point affects contact stress, transmission error amplitude and the size of the contact shape. The investigation of the contact stress changes is accomplished using the Finite Element Method. Some authors suggested a union between a global term and a local term in order to determine the deformation of each contact point. The contact point is also affected by the applying torque.

The contact conditions are established by comparing the distance considered along the pressure line with the loaded static transmission error. Tip relief is usually required to achieve better contact conditions throughout the meshing.

TCA software programs predict meshing and contact of gear tooth areas taking into account the localized bearing contact, which provides the surface contact point. TCA equations can be determined if the considered original values that affect the initial contact point are admissible, if not, the computational process will be terminated or the TCA result will be wrong. Tooth contact analysis method represents an answer for the investigation of the transmission error shape and for the modification of the contact model induced by manufacturing and assembling errors. TCA result provides significant information about the contact path, contact ellipses, kinematic errors and the evaluation of the instantaneous contact point of the meshing gear. The LTCA method is determined using the influence coefficient considering the interconnection between the deformation and the loading on a contact region. In the ETCA algorithm, the initial contact point can be assessed using the improved Powell hybrid algorithm, the Newton-Raphson algorithm and the FEM.

This paper represents the introductive part for an in-deep research. A number of analytical, numerical and optimization procedures will be developed in order to achieve the transmission error functions as well as the proper TCA algorithm. All these methods will be implemented in non-circular gears.

9. References

- [1] Walker H 1938 Gear Tooth Deflection and Profile Modification *Engineer* **166** pp 410-435
- [2] Tharmakulasingam R 2009 *Transmission Error in Spur Gears: Static and Dynamic Finite-Element Modeling and Design Optimization* (London: School of Engineering and Design Brunel University – Doctoral Thesis) pp 6-50

- [3] Fietkau P, Bertsche B 2013 Influence of tribological and geometrical parameters on lubrication conditions and noise of gear transmissions *Mechanism and Machine Theory* **69** pp 303–320
- [4] Gregory R W, Harris S L, Munro R G 1963 Dynamic behaviour of spur gears *Proceedings of the Institution of Mechanical Engineers* **178** pp 207–218
- [5] Garambois P, Perret-Liaudet J, Rigaud E 2017 NVH robust optimization of gear macro and microgeometries using an efficient tooth contact model *Mechanism and Machine Theory* **117** pp 78–95
- [6] Ottewill J R, Neild S A, Wilson R E 2010 An investigation into the effect of tooth profile errors on gear rattle *Journal of Sound and Vibration* **329** pp 3495–3506
- [7] Litvin F L, Fuentes A 2009 *Gear Geometry an Applied Theory – Second Edition* (New York: Cambridge University Press) Chapter 9.4
- [8] Ye S-Y, Tsai S-J 2016 A computerized method for loaded tooth contact analysis of high-contact-ratio spur gears with or without flank modification considering tip corner contact and shaft misalignment *Mechanism and Machine Theory* **97** pp 190–214
- [9] He D, Ding H 2018 A new analytical identification approach to the tooth contact points considering misalignments for spiral bevel or hypoid gears *Mechanism and Machine Theory* **121** pp 785–803
- [10] Cao X, Deng X, Wei B 2018 A novel method for gear tooth contact analysis and experimental validation *Mechanism and Machine Theory* **126** pp 1–13
- [11] Welbourn D B 1979 Fundamental Knowledge of Gear Noise *Proceedings of the Institution of Mechanical Engineers* pp 9–14
- [12] Gregory R W, Harris S L, Munro R G 1963 A Method of Measuring Transmission Error in Spur Gears of 1:1 Ratio, *Journal of Scientific Instruments* **40** pp 5–9
- [13] Munro R G, Morrish L, Palmer D 1999 Gear transmission error outside the normal path of contact due to corner and top contact *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science* **213** pp 389–400
- [14] Zhuo Y-B, Xiang X-Y, Zhou X-J, LV H-L, Teng G-Y 2017 A method for the global optimization of the tooth contact pattern and transmission error of spiral bevel and hypoid gears *Journal of Zhejiang University-Science A (Applied Physics & Engineering)* **18**(5) pp 377–392
- [15] Mermoz E, Astoul J, Sartor M, Linares J M, Bernard A 2013 A new methodology to optimize spiral bevel gear topography *CIRP Annals – Manufacturing Technology* **62** pp 119–122
- [16] Hwang S-C, Lee J-H, Lee D-H, Han S-H, Lee K-H 2013 Contact stress analysis for a pair of mating gears *Mathematical and Computer Modelling* **57** pp 40–49
- [17] Jabbour T, Asmar G 2015 Tooth stress calculation of metal spur and helical gears *Mechanism and Machine Theory* **92** pp 375–390
- [18] Jabbour T, Asmar G 2009 Stress calculation for plastic helical gears under a real transverse contact ratio *Mechanism and Machine Theory* **44** pp 2236–2247
- [19] Liang D, Chen B, Peng S, Hua C, Liao R 2017 Deviation Calculation and Analysis of Tooth Surface of Conjugate-Curve Gear Drive *International Journal of Precision Engineering and Manufacturing* **18** (5) pp 689–696
- [20] Zhou C, Huang F, Han X, Gu Y 2017 An elastic–plastic asperity contact model and its application for micro-contact analysis of gear tooth profiles *Int J Mech Mater Des* **13** pp 335–345
- [21] Fernandez del Rincon A, Viadero F, Iglesias M, Garcia P, de-Juan A, Sancibrian R 2013 A model for the study of meshing stiffness in spur gear transmissions *Mechanism and Machine Theory* **61** pp 30–58
- [22] Winter H, Placzek T 1991 Load distribution and topological flank modification of helical and double helical gears *European Journal of Mechanical Engineering* **36**
- [23] Sanchez M B, Pleguezuelos M, Pedrero J I 2016 Calculation of tooth bending strength and surface durability of internal spur gear drives *Mechanism and Machine Theory* **95** pp 102–113
- [24] Chen Y-C, Liu C-C 2011 Contact stress analysis of concave conical involute gear pairs with non-parallel axes *Finite Elements in Analysis and Design* **47** pp 443–452
- [25] Lee D-H, Moon K-H, Lee W-Y 2016 Characteristics of transmission error and vibration of

- broken tooth contact *Journal of Mechanical Science and Technology* **30**(12) pp 5547-5553
- [26] Sheveleva G I, Volkov A E, Medvedev V I 2007 Algorithms for analysis of meshing and contact of spiral bevel gears *Mechanism and Machine Theory* **42** pp 198-215
 - [27] Kawasaki K, Tamura H, Iwamoto Y 1999 Klingelnberg spiral bevel gears with small spiral angles *Proceedings 4th World Congress on Gearing and Power Transmissions* pp 697-703
 - [28] Raghuwanshi N K, Parey A 2017 Experimental measurement of spur gear mesh stiffness using digital image correlation technique *Measurement* **111** pp 93-104
 - [29] Tsai S-J, Ye S-Y 2018 A computerized approach for loaded tooth contact analysis of planetary gear drives considering relevant deformations *Mechanism and Machine Theory* **122** pp 252-278
 - [30] Yu W, Mechefske C K 2016 Analytical modeling of spur gear corner contact effects *Mechanism and Machine Theory* **96** pp 146-164
 - [31] Lin H H, Wang J, Oswald F B, Coy J J 1994 Effect of extended tooth contact on the modeling of spur gear transmissions *Gear Technology* **July/August** pp 18-25
 - [32] Fuentes-Aznar A, Ruiz-Orzaez R, Gonzalez-Perez I 2017 Comparison of spur, helical and curvilinear gear drives by means of stress and tooth contact analyses *Meccanica* **52** pp 1721-1738
 - [33] Vogel O, Griewank A, Bar G 2002 Direct gear tooth contact analysis for hypoid bevel gears *Computer Methods in Applied Mechanics and Engineering* **191** pp 3965-3982
 - [34] Stadtfeld H. J. 1993 Handbook of Bevel and Hypoid Gears (Rochester Institute of Technology)
 - [35] Jinzhan S, Zongde F, Xiangwei C 2013 Design and analysis of spiral bevel gears with seventh-order function of transmission error *Chinese Journal of Aeronautics* **26** 1310-1316
 - [36] Jinke J, Zongde F 2015 Design and analysis of modified cylindrical gears with a higher-order transmission error *Mechanism and Machine Theory* **88** pp 141-152
 - [37] Vilmos S 2007 Computer simulation of tooth contact analysis of mismatched spiral bevel gears *Mechanism and Machine Theory* **42** pp 365-381
 - [38] Chao L C, Tsay C B 2008 Contact characteristics of spherical gears *Mechanism and Machine Theory* **43** pp 1317-1331
 - [39] Hsu R-H, SU H-H 2014 Tooth contact analysis for helical gear pairs generated by a modified hob with variable tooth thickness *Mechanism and Machine Theory* **71** pp 40-51
 - [40] Vecchiato D 2006 Tooth contact analysis of a misaligned isostatic planetary gear train *Mechanism and Machine Theory* **41** pp 617-631
 - [41] Guan Y, Fang Z, Yang X, Chen G 2018 Tooth contact analysis of crowned gear coupling with misalignment *Mechanism and Machine Theory* **126** pp 295-311
 - [42] Zhang Y, Fang Z 1997 Analysis of transmission errors under load of helical gears with modified tooth surfaces *J. Mech. Des.* **119** pp 120-126
 - [43] Zhang Y, Fang Z 1999 Analysis of tooth contact and load distribution of helical gears with crossed axes *Mechanism and Machine Theory* **34** pp 41-57
 - [44] Li S 2008 Effect of addendum on contact strength, bending strength and basic performance parameters of a pair of spur gears *Mechanism and Machine Theory* **43** pp 1557-1584
 - [45] Wu S-H, Tsai S-J 2009 Contact stress analysis of skew conical involute gear drives in approximate line contact *Mechanism and Machine Theory* **44** pp 1658-1676
 - [46] Sanchez-Marin F, Iserte J L, Roda-Casanova V 2016 Numerical tooth contact analysis of gear transmissions through the discretization and adaptive refinement of the contact surfaces **101** pp 75-94
 - [47] Astoul J, Geneix J, Mermoz E, Sartor M 2013 A simple and robust method for spiral bevel gear generation and tooth contact analysis *Int J Interact Des Manuf* **7** pp 37-49
 - [48] Fong Z-H 2000 Mathematical model of universal hypoid generator with supplemental kinematic flank correction motions *J. Mech. Des.* **122** pp 136-142
 - [49] Shih Y-P, Fong Z-H, Lin G C 2006 Mathematical model for a universal face hobbing hypoid gear generator *J. Mech. Des.* **129** pp 38-47
 - [50] Ding H, Zhou Y, Tang J, Zhong J, Zhou Z, Wan G 2017 A novel operation approach to determine initial contact point for tooth contact analysis with errors of spiral bevel and hypoid gear *Mechanism and Machine Theory* **109** pp 155-170

- [51] Litvin F L, Vecchiato D, Fuentes A, Gonzalez-Perez I 2004 Automatic determination of guess values for simulation of meshing of gear drives *Computer Methods in Applied Mechanics and Engineering* **193** pp 3745-3758
- [52] Litvin F L, Sheveleva G I, Vecchiato D, Gonzalez-Perez I, Fuentes A 2005 Modified approach for tooth contact analysis of gear drives and automatic determination of guess values *Computer Methods in Applied Mechanics and Engineering* **194** pp 2927-2946
- [53] Ma H, Zeng J, Feng R, Pang X, Wen B 2016 An improved analytical method for mesh stiffness calculation of spur gears with tip relief *Mechanism and Machine Theory* **98** pp 64-80
- [54] Chen Z, Zhai W, Shao Y, Wang K, Sun G 2016 Analytical model for mesh stiffness calculation of spur gear pair with non-uniformly distributed tooth root crack *Engineering Failure Analysis* **66** pp 502-514
- [55] Wu Y-J, Wang J-J, Han Q-K 2012 Contact finite element method for dynamic meshing characteristics analysis of continuous engaged gear drives *Journal of Mechanical Science and Technology* **26** pp 1671-1685
- [56] Shen H, Li Z, Qi L, Qiao L 2018 A method for gear fatigue life prediction considering the internal flow field of the gear pump *Mechanical Systems and Signal Processing* **99** pp 921-929
- [57] Gupta K, Chatterjee S 2018 Analysis of Design and Material Selection of a Spur gear pair for Solar Tracking Application *Material Today: Proceedings* **5** pp 789-795
- [58] Raghuwanshi N K, Parey A 2016 Experimental measurement of gear mesh stiffness of cracked spur gear by strain gauge technique *Measurement* **86** pp 266-275
- [59] Pandya Y, Parey A 2013 Crack behavior in a high contact ratio spur gear tooth and its effects on mesh stiffness *Engineering Failure Analysis* **34** pp 69-78
- [60] Song D, Lin H, Xing-hui H, Song H 2013 Finite element analysis of contact fatigue and bending fatigue of a theoretical assembling straight bevel gear pair *J. Cent. South Univ* **20** pp 279-292

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