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An improved technique of finding the coefficient of rolling friction by inclined plane method

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Abstract. During the design process the mechanical engineer faces a challenge: the choice of the tribological parameters. Even most influential technical references offer ranges for the frictional characteristics. An emblematic example is the coefficient of rolling friction (CRF). The handiest solution in establishing the correct value of the CRF for a certain contact is to find it experimentally. One of the methods is the inclined plane manner. The weak point of the method consists in the particularly small values of the slope required to evidence the effect of rolling friction. The present work shows a technique for finding the CRF. The principle of the method consists in considering both the slope of the inclined plane and the torque of rolling friction as unknowns. The two parameters can be established by experimental finding of the acceleration of an axi-symmetric body, both for ascending and descending motions.

1. Introduction

The rolling friction occurs in all nonconforming mechanical contacts [1]. The classical examples to take into account are the cam mechanisms and the rolling bearings. In Hertzian point contacts, besides the rolling friction the spinning friction appears, concretized by a spinning torque directed along the normal in the theoretical point of contact [2]. When the motion from the higher pair where friction occurs is a spatial one (the relative velocity has components both on the normal and contained in the tangent plane) the two types of friction are simultaneously present and when the experimental estimation of these frictions is aimed, difficult situations arise.

Though there are works concerning the rolling friction [3-6], in the recent technical literature there are met numerous papers in which new hypothesis and experimental techniques are presented. Perhaps the most significant aspect to be mentioned is the transition from the model that considers the proportionality between the rolling friction and the normal force [7], model created based on the analogy to the dry sliding friction, to the model based on hypothesis of the theory of elasticity which considers nonlinear dependency [8].

From experimental point of view, two main classes of methods are applied for the estimation of the rolling friction: methods where an oscillatory motion exists in contact and different types of pendula are used, and methods where the velocity from the contact point is monotonous and here the inclined plane technique can be mentioned. In the first case the coefficient of rolling friction is found as reveal by the damping of the oscillation of the pendulum while for the second case, the coefficient of rolling friction is estimated based on the acceleration of an axi-symmetric body. Both methods present advantages and drawbacks - the option for one of the other of the experimental techniques should take into account the particular conditions. For instance, if the two contacting bodies are made of materials with an important spatial gradient of the mechanical properties, then the method of oscillations is recommended since the dimensions of the analyzed region are reduced.

2. The principle of the method

The method proposed in the present work aims to establish the coefficient of rolling friction by analyzing the motion of an axi-symmetric body on an inclined plane. The technique was used in [9] where the evaluation of the coefficient of rolling friction is based on the measurements upon the acceleration of a revolute body moving downwards on an inclined plane, for diverse tilting angles. The main disadvantage of the method is the fact that the torque of rolling friction is noticed for small values of the tilting angle of the plane. Thus, the rolling friction torque influences the motion of the body when the angle of inclination of the plane is at least:

$$\alpha \cong s_r / R \tag{1}$$

where s_r is the coefficient of rolling friction and R is the radius of the contact point with respect to the axis of the rolling body. Considering that for the metallic bodies in contact, $s_r = (10^{-5} \div 10^{-4})m$ [10], it results that for a body with the radius of the contact point $R = 0.01m$ the tilting angle of the plane must take values within the range:

$$\alpha \cong (10^{-3} \div 10^{-2}) = 0.057^o \div 0.57^o \tag{2}$$

These are extremely small values, inferior to the precision of the measurement instruments. To avoid this aspect, the present paper proposes a method for estimation of the coefficient of rolling friction that also accepts as unknown the value of the angle of inclination of the plane. The rolling body used in the present study is a body made of two identical bearing balls of radius R, connected by a cylindrical part of radius r as seen in figure 1.

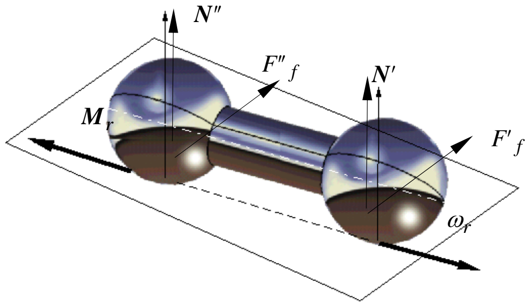


Figure 1. The rolling body, the relative angular velocity and the moment of rolling friction.

An inextensible wire is winded on the cylindrical part and passed over a pulley of negligible mass. A body is suspended by the end of the wire and moves on vertical direction as in figure 2. For reasons to be explained afterward, in figure 2 there were considered both the case when the body moves upward ($''u''$ index) and the case when the body moves downwards ($''d''$ index) along the line of the steepest slope of the plane. The motion of the body is a plane-parallel one. The unknowns of the problem are the reactions from the contact point (the normal component N, the friction force F_f, the moment of rolling friction M_r), the axial tension in the wire T and the characteristics of the motion (the angular acceleration ε and the acceleration of the center of mass a).

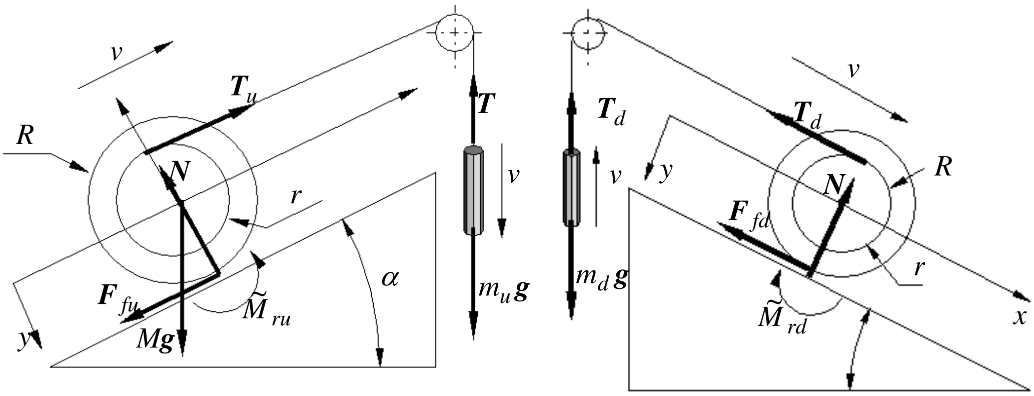
The hypothesis of pure rolling is expressed using the relation:

$$a_{u,d} = R\varepsilon_{u,d} \tag{3}$$

Three scalar equations can be written for each of the situations presented in figure 2: two equations for the motion of the center of mass and an equation corresponding to the moment of momentum theorem. Additionally, for the motion of the suspended body a scalar equation can be written. As conclusion, for any of the cases from figure 2, five scalar equations are available and the number of the unknowns is six, namely: $T, a, \varepsilon, N, F_f, M_r$. Therefore, a supplementary equation is required to ensure the compatibility of the problem. This equation is a constitutive equation describing the dependency of the moment of rolling friction on the normal force. Subsequently, the model that considers the rolling friction torque proportional to the normal force is proposed [7]:

$$M_r = s_r N \tag{4}$$

To be remarked that for both cases specified in figure 2, the normal force N and the moment of rolling friction M_r, have the same values, respectively, and so the indexes are abandoned.



a)The upward motion of the body b)The downward motion of the body

Figure 2. The cases considered for the motion of the rolling body on the inclined plane

3. The equations of motion. Obtaining and discussions

For the ascending motion on the inclined plane, the equations of motion have the form:

$$\begin{aligned} &Ma_u = T_u - F_{fu} - Mg\sin\alpha \\ &0 = N - Mg\cos\alpha \\ &J_z a_u / R = T_u r + F_{fu} R - M_{ru} \end{aligned} \tag{5}$$

and the equation of motion for the suspended body is:

$$m_u a_u = m_u g - T_u \tag{6}$$

When the body moves downhill, the equations of motion have the following form:

$$\begin{aligned} &Ma_d = Mg\sin\alpha - T_d - F_{fd} \\ &0 = N - Mg\cos\alpha \\ &J_z \frac{a_d}{R} = -T_d r + F_c R - M_{rd} \end{aligned} \tag{7}$$

and the equation of motion for the suspended body takes the form:

$$m_d a_d = T_d - m_d g \tag{8}$$

The acceleration of the body in uphill motion is obtained from the equations (5) and (6):

$$a_u = \frac{\frac{m_u}{M}\left(1+\frac{r}{R}\right) - \sin\alpha}{1+\frac{J_z}{MR^2}+\left(1+\frac{r}{R}\right)\frac{m_u}{M}} g - \frac{\cos\alpha}{1+\frac{J_z}{MR^2}+\left(1+\frac{r}{R}\right)\frac{m_u}{M}} \frac{s_r}{R} g \tag{9}$$

and using the equations (7) and (8), the acceleration for the downward motion results as:

$$a_d = \frac{\sin\alpha - \frac{m_d}{M}\left(1+\frac{r}{R}\right)}{1+\frac{J_z}{MR^2}+\left(1+\frac{r}{R}\right)\frac{m_d}{M}} g - \frac{\cos\alpha}{1+\frac{J_z}{MR^2}+\left(1+\frac{r}{R}\right)\frac{m_d}{M}} \frac{s_r}{R} g \tag{10}$$

The equations (9) and (10) may be written under the following from:

$$\begin{cases} \sin\alpha + \frac{s_r}{R}\cos\alpha - \left(1+\frac{r}{R}\right)\frac{m_u}{M} + \frac{a_u}{g}\left[1+\frac{J_z}{MR^2}+\left(1+\frac{r}{R}\right)\frac{m_u}{M}\right] = 0 \\ \sin\alpha - \frac{s_r}{R}\cos\alpha - \left(1+\frac{r}{R}\right)\frac{m_d}{M} - \frac{a_d}{g}\left[1+\frac{J_z}{MR^2}+\left(1+\frac{r}{R}\right)\frac{m_d}{M}\right] = 0 \end{cases} \tag{11}$$

Assuming that the accelerations a_u and a_d are known, the system of equations (11) allows for finding the unknowns α and s_r. At this moment, the demarcation of the two situations, for upward and downward motion, can be explained. The problem requires finding the two parameters, α and s_r. For two parameters are necessary two tests performed under different conditions. To exemplify, it is considered that these would be finding the accelerations a_{u1} a_{u2}, for the uphill motions for two values $m_{u1,2}$. It results that the system (11) has two equations of the form of the first relation in the system (11). In this case, the parameters s_r and α could not be found since in both equations of the obtained system there is no change in the left member where the unknowns are contained, ($\sin\alpha + s_r \cos\alpha / R$). As consequence, two test must be performed, one for the upwards moving body and the other for the downwards moving body and so, the moment of rolling friction changes its sign.

From the system of equations (11) the angle α is first obtained:

$$\alpha = \operatorname{asin} \frac{\frac{m_u + m_d}{M}\left(1+\frac{r}{R}\right) + \frac{a_d - a_u}{g}\left(1+\frac{J_z}{MR^2}\right) + \frac{m_d a_d + m_u a_u}{Mg}\left(1+\frac{r}{R}\right)}{2} \tag{12}$$

and afterwards, using the found expression, the relation of the coefficient of rolling friction is deduced:

$$s_r = \frac{\frac{m_u - m_d}{M}\left(1+\frac{r}{R}\right) - \frac{a_d + a_u}{g}\left(1+\frac{J_z}{MR^2}\right) - \frac{m_d a_d + m_u a_u}{Mg}\left(1+\frac{r}{R}\right)}{2\cos\alpha} \tag{13}$$

4. Conclusions

The paper proposes a method for evaluation of the coefficient of rolling friction based on the inclined plane technique. A revolution body is considered, consisting in two identical bearing balls and a

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cylinder, with the radius smaller than the radius of the balls, attached between the balls and thus shaping an axi-symmetric body. A wire is coiled over the cylinder and a mass it added to its end. The body is placed on an inclined plane with the revolution axis normal to the steepest slope of the plane. The wire passes over a pulley positioned at the highest point of the plane. The attached mass is chosen so that the body, starting from the base, moves uphill. Assuming dry friction conditions, the motion of the center of mass is uniformly accelerated with constant acceleration. The test is repeated with another mass chosen for the downward motion, with constant acceleration. The accelerations for the two cases are found by means of the time required to pass a stipulated distance.

The theoretical expressions for the uphill and downhill acceleration are obtained and the system of two equations has as unknowns the coefficient of rolling friction and the tilting angle of the plane. To be mentioned that, in the case of finding the coefficient of rolling friction, the task to evaluate the angle of the inclined plane is not simple.

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