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## Application of the Cabaret scheme in task of shock-wave loading

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# Application of the Cabaret scheme in task of shock-wave loading

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**Abstract.** A method for calculating problems of uniaxial shock-wave loading of material is proposed in this paper. The method is based on Cabaret finite difference scheme. The mathematical model of the impact loading is described in Lagrangian formulation in the one-dimensional case. The task is approximated by the scheme. A comparison is made with the analytical solution. Verification of the approach is carried out on experimental data.

## 1. Introduction

The problem of high-velocity interaction of solids is accompanied by various physical phenomena, such as heating, melting and evaporation, physical hardening of materials, destruction of materials, etc. The problems of cratering and penetration of materials are associated with shock-wave loading of bodies. There are many publications on this problem, including generalizing, for example, monograph [1]. However, the practical importance of the task makes this problem relevant and incomplete today. There are many approaches to the description of a deformable body, but all these approaches are divided into three types [2]: continual, microstructural and atomistic. It was considered only a first approach in present paper. The numerical implementation of the first approach is carried out by finite difference methods. Up-to-date way of realization of the problem is the family of Cabaret scheme. The initial version of the difference scheme was proposed independently in [3] and [4]. Today, the scheme includes a large number of different implementations [5]. Two-layer time variable scheme is widely used in hydrodynamic problem [6].

## 2. The system of equations for task of shock-wave loading of bodies. Lagrange formulation

The Maxwell model for describing a solid medium [7] in a one-dimensional formulation was considered. The system of conservation laws look like this:

$$\rho_t = -\rho \frac{\partial v}{\partial x}; v_t = \frac{1}{\rho} \frac{\partial \sigma}{\partial x}; (E + \frac{u^2}{2})_t = \frac{1}{\rho} \frac{\partial (u\sigma)}{\partial x} - \frac{1}{\rho} \frac{\partial W}{\partial x}; \frac{dx}{dt} = v. \quad (1)$$

Density  $\rho(x, t)$ , velocity  $v(x, t)$  and internal energy  $E(x, t)$  are desired values in the system. The parameter  $\sigma$  is a stressed state of the material. It is described by state of equation (2). The parameter  $W$  is the heat-flux density. It's defined by the Fourier law (6). The equation of state in the tensor form is as follows:

$$\sigma_{ij} = -P\delta_{ij} + S_{ij} \quad (2)$$

Here  $P$  is spherical and  $S_{ij}$  is deviator components of the stress tensor. Spherical tensor has following form equation of state:

$$P = P_x + P_H; E = E_x + E_H \quad (3)$$

Here  $P_x$  is the pressure of elastic compression or cold pressure component. This parameter depends only on the specific volume of the substance and is equal to the total pressure at absolute zero temperature.  $P_H$  is the hot component of pressure, depending on the intensity of the movement of molecules and thermal excitation by electrons. For the task of shock-wave loading were chosen the equation of state of the condensed material in the Theta form (4) and relationship between hot components pressure and internal energy through the specific heat (5)

$$P_x = A \left( \frac{\rho}{\rho_0} \right)^n - 1 \quad (4)$$

$$P_T = \gamma \rho E_T = \gamma \rho C_v (T - T_0) \quad (5)$$

Here  $A$  and  $n$  are parameters of material. They are determined by experiments.  $\rho_0$  is reference value of density,  $\gamma$  is the Gruneisen parameter,  $C_v$  is specific heat and  $T_0$  is the temperature value at the initial moment of time.

Fourier's law is as follows:

$$W = -\chi T_x \quad (6)$$

Here  $\chi$  is thermal conduction coefficient. It is a function of the thermodynamic state of the medium:  $\chi = \chi(\rho, T)$ . Here  $T$  is temperature of medium. The hyperbolicity of the system of equations is violated if the heat-flux density (6) is not equal to zero on the right in the energy conservation law (1). There are two methods to solve this problem:

- numerical solution of the problem using implicit difference schemes. However, the disadvantage of this approach is low accuracy compared with the Cabaret scheme [4];
- Application of the splitting method for this problem [9].

The temperature change in the process of shock-wave loading is negligible in this work. As a result, the mathematical model of the medium is as follows (7):

$$\rho_t = -\rho \frac{\partial v}{\partial x}; v_t = \frac{1}{\rho} \frac{\partial \sigma}{\partial x}; \sigma = -P_x = -A \left( \frac{\rho}{\rho_0} \right)^n - 1; \frac{dx}{dt} = v. \quad (7)$$

It is necessary to determine the characteristics and invariants of a system of differential equations in order to use the Cabaret scheme considered in [6]. The characteristics  $\lambda_1$  and  $\lambda_2$  and invariants  $w_1$  and  $w_2$  of the system are as follows:

$$\begin{aligned} \lambda_1 = -c & \quad w_1 = v - \frac{2}{n-1} c \\ \lambda_2 = c & \quad w_2 = v + \frac{2}{n-1} c \end{aligned} \quad (8)$$

Here  $c$  is critical velocity, which is determined by the formula (9) for the equation of state:

$$c = \sqrt{\frac{\partial P_x}{\partial \rho}} = \sqrt{An \frac{\rho^{n-1}}{\rho_0^n}} \quad (9)$$

### 3. Modified Cabaret scheme

Two calculation areas were defined. Construct a rectangular differential grid for the drummer and for the target:

$$\{x_m, t^n\}: x_i = ih; t^{n+1} = t^n + \tau^n \quad (10)$$

Here  $h$  is constant grid spacing and  $\tau^n$  is time step. It is determined from the condition of stability:

$$\tau^n = \frac{zh}{\max_{i,j} |(\lambda_j)_{i+1/2}^n|} \quad (11)$$

Here  $z = 0.5$  is Courant number. The characteristic values  $(\lambda_j)_{i+1/2}^n = \lambda_j(\vec{U}_{i+1/2}^n)$  are defined in the center of the cell on the  $n$ -th time layer. The minimum time step in both cases was determined for time synchronization in both computing areas. All variables are divided into streaming and conservative in the Cabaret scheme. The first variables are determined at the cell boundaries  $\vec{u}_i^n = u(x_i, t^n)$ , and the second ones are responsible for the propagation of disturbances in the medium and are located in the centers of the cells  $\vec{U}_{i+1/2}^n = U(x_{i+1/2}, t^n)$ . The numerical solution of problem (7) was determined in four stages. At the first stage, conservative variables were found on the semi-integral time layer using difference equations (12):

$$\begin{cases} \frac{P_{i+1/2}^{n+1/2} - P_{i+1/2}^n}{\tau^n / 2} + P_{i+1/2}^n \frac{v_{i+1}^n - v_i^n}{h} = 0 \\ \frac{V_{i+1/2}^{n+1/2} - V_{i+1/2}^n}{\tau^n / 2} - \frac{1}{P_{i+1/2}^n} \frac{\sigma_{i+1}^n - \sigma_i^n}{h} = 0 \end{cases} \quad (12)$$

In the second stage, the values of the flow variables  $v_i^{n+1/2}$  and  $\sigma_i^{n+1/2}$  were calculated on the half-integral time layer. At the third stage, the values of conservative variables at the new time layer are calculated according to the following difference equations (13):

$$\begin{cases} \frac{P_{i+1/2}^{n+1} - P_{i+1/2}^n}{\tau^n} + P_{i+1/2}^{n+1/2} \frac{v_{i+1}^{n+1/2} - v_i^{n+1/2}}{h} = 0 \\ \frac{V_{i+1/2}^{n+1} - V_{i+1/2}^n}{\tau^n} - \frac{1}{P_{i+1/2}^{n+1/2}} \frac{\sigma_{i+1}^{n+1/2} - \sigma_i^{n+1/2}}{h} = 0 \end{cases} \quad (13)$$

Values of the flow variables are calculated at the new time layer in the fourth stage. In conclusion, the new location of the impactor and target was determined by the following equations:

$$\begin{cases} x_{i+1}^{n+1} = x_i^n + dt \cdot v_i^{n+1} \\ X_{i+1/2}^{n+1} = X_{i+1/2}^n + dt \cdot V_{i+1/2}^{n+1} \end{cases} \quad (14)$$

A collision of impactor with a target is defined as an ideal mechanical contact [10]. The moment of collision is determined by the equality of the coordinates of the nodes of the extreme cells of the impactor and the target.

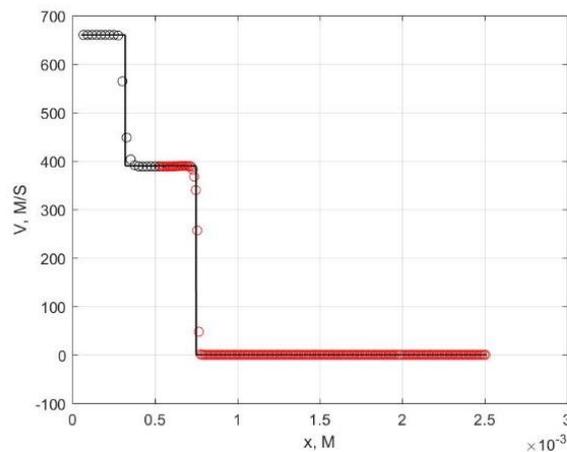
#### 4. Results and verification

The problem of shock-wave loading for two metallic homogeneous materials was considered to test the obtained calculation algorithm, which corresponds to uniaxial loading in the initial stage of the process.

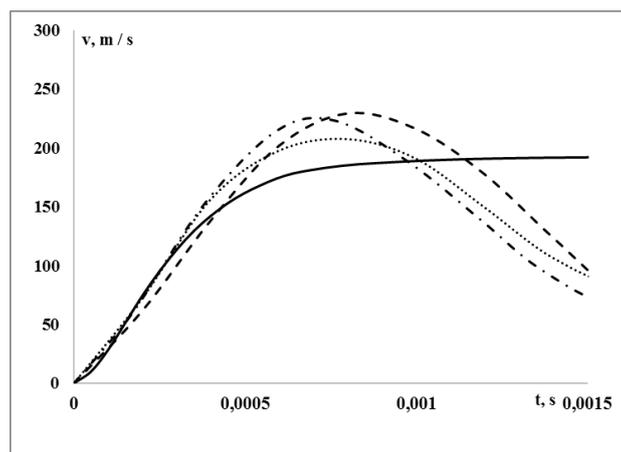
The first calculation was carried out for the case of a collision with a velocity of 660 m / s of two identical materials on the example of copper. The thickness of impactor is 0.5 mm, and the target – 2 mm. The number of grid nodes of the impactor is 20 and the target - 200. The convergence on a fine grid was confirmed: 40 nodes on the impactor and 400 nodes on the target. Black circles set the centers of the impactor cells. Red circles set the centers of the target cells. The comparison was carried out with the analytical solution and is shown in figure 1 using the velocity profile as an example. The graph shows an accurate coincidence of the calculation with the analytical solution. The scheme smoothes the

values in front of the shock front both in the impactor and in the target. The rest of the calculation accurately repeats the analytical solution.

The next step was the verification of the method with the software package. We used the software package LS-Dyna. The comparison was carried out on the dependence of the velocity of the contact surface of the target on time. The LS-Dyna software package does not include the equation of state in theta form; therefore, for comparison, some of the simplest hydrodynamic and elastic-plastic models of materials were considered. Impact took place for two identical materials. Impactor thickness 0.1 m, target thickness - 1 m. Impact velocity - 500 m / s. Comparison of calculations is shown in figure 2. At the initial moment, there is a qualitative convergence of the calculation by the Cabaret scheme with the calculations in LS-Dyna. However, further discrepancies are observed. It is due to the interference of shock waves. It's possible to solve only the 3D formulation of the problem in this software package. Therefore, the results after 5 ms can't be compared. More details on the models can be found in [11]. The data of the all numerical experiments are summarized in table 1.

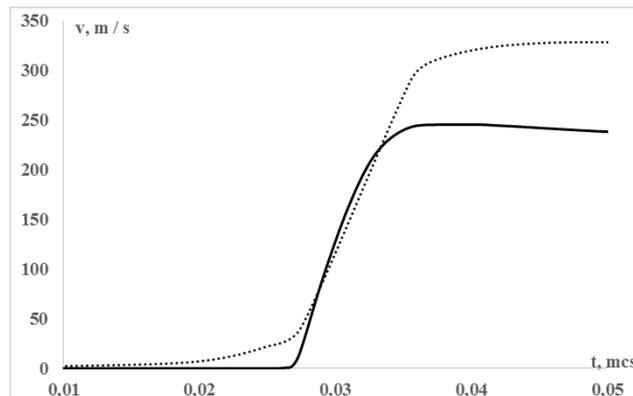


**Figure 1.** Velocity profile with uniaxial loading of a copper target with a copper plate. Comparison with analytical solution.



**Figure 2.** Velocity profile of the contact surface. Comparison of results with the LS-Dyna package. The solid line is a Cabaret solution. The point line is a Elastic fluid model in LS-Dyna. The stippled line is Plastic kinematic model in LS-Dyna. The dash-and-dot line is the Elastic-Plastic Hydro model in LS-Dyna

The final stage of verification was a comparative analysis of the Cabaret scheme with experimental data [12]. The following experiment was performed numerically. Aluminum plate 0.2 mm thick swoops at a velocity of 660 m / s on a target of homogeneous copper 0.7 mm thick. Comparison will be carried out along the velocity profile of the contact surface. A comparison was made on the velocity profile of the contact surface depending on time in figure 3. There is a qualitative coincidence of the result with experimental data in the first moments of time. However, later it is necessary to take into account the parameters of elasticity and plasticity of metals.



**Figure 3.** Velocity profile of the contact surface. Comparison of results with experiment. The solid line is a Cabaret solution. The point line is experimental data.

**Table 1.** Material parameters.

Parameters/ material	$\rho_0$ [kg/m <sup>3</sup> ]	A [GPa]	n	E [MPa]	$\nu$	K [MPa]	$Y_0$ [MPa]	$\gamma$	$C_p$ [J/(kg·K)]	G [MPa]
Copper	8930	37.75	4	$10^5$	0.32	$1.39 \cdot 10^5$	$2.8 \cdot 10^2$	1.4	3790	$4.6 \cdot 10^6$
Aluminium	2785	$1.97 \cdot 10^5$	4.2							

## 5. Conclusion

In the course of the work, it turned out that the algorithm implemented by the Cabaret scheme describes the algorithm well the initial stage of uniaxial shock-wave loading of materials. The calculation according to the Cabaret scheme gives an excellent match with the analytical solution. With the experimental data and the results of calculations in LS-Dyna, the calculation according to the Cabaret scheme is qualitatively well matched at the initial moments of loading. In the future, the scheme will be tested on heterogeneous materials, a mathematical model of which was proposed in [13].

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