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An integrated production and inventory model considering reworks and two types of demand simultaneously in a two level supply chain network

Jonrinaldi¹, T Rahman², Henmaidi¹, Eri Wirdianto¹, Asmuliardi Muluk¹

¹ Department of Industrial Engineering, Universitas Andalas, Padang, 25163, Indonesia

² Institut Teknologi Batam, Batam, Kepulauan Riau, 29424, Indonesia

jonrinaldi@eng.unand.ac.id

Abstract. This study proposed an integrated production and inventory model for multiple items or products considering two types of demand (continuous and discrete demand) and rework process of defect products and disposal in a two level supply chain network consisting of a vendor and a buyer. The multiple items or products are processed and produced in a single facility of the vendor. This model developed is applied for the system consisting of a vendor and a buyer where the vendor produces multiple items and delivers apart of them to the buyer as the discrete demand and others as continuous demand to end customers. During the production time there will be some defect products where apart of them will be reworked to be a good ones and some others will be disposed of. A mathematical model is built to describe the problem studied referred to the basic inventory models. The continuous demand coming from the other customers are served at anytime while the discrete demand from the buyer are served by the vendor using a number of deliveries shipments during the cycle time. The aim of the model developed is to minimize the total operational cost of the supply chain network. The model then is solved by algorithm and optimization methods as solution procedures. A numerical example is given to demonstrate the practical usage of the model and results are discussed. Sensitivity analysis to show the changes of optimal solutions which are affected by the changes of input parameters is also performed. Last, conclusions and potential future research are noted and explained.

Keywords: inventory, production, discrete demand, continuous demand, simultaneously, rework

1. Introduction

Inventory problem has take much attention over the years and become an important competitive factors in industries for many years [1,2]. Inventory models were used by the industries as one of tools to control their items in inventory [3]. Many extensions of inventory models were developed referred to the basic models those are EOQ and EPQ models [4]. The two basic inventory models are combined to determine an optimal production and inventory quantity with considering setup or ordering cost and inventory holding cost [6]. Some years after the first inventory policy was introduced, the basic optimal production lot size policy was developed referred to the basic inventory one (EOQ) [7]. Those limitations of both EOQ and EPQ models considered sometimes as unreal conditions, such as only continuous demand fulfilled and producing perfect quality products [8,9]. In fact, multiple delivery policies, named as discrete demand, are usual performed [9] and defect products during production cycle is inevitable [8,9,10]. Tai [9] proposed an extended production lot size model considering defect



products using rework activity. Al-Salamah [12] improving EPQ model with both defect products and inspection using both acceptance sampling techniques, destructive and non-destructive ones, in two-level system. Finally, integrated production and inventory model in supply chain had been discussed in [10] and [11].

This paper also considers imperfect items in production processing which parts both are serviceable and others are scraps. Therefore, this paper developed an integrated production and inventory model for multiple items considering reworks and two types of demand (continuous and discrete) simultaneously in the two level supply chain network. This paper extends Rahman *et al* [13] proposing the integrated production and inventory model considering two types of demand simultaneously but this paper excluded the condition in Rahman *et al* [13] that said there were not consumption of product during production process because the continuous demand was filled by the last production run. The aim of the paper is to determine the integrated production and inventory policy and number of deliveries with considering the two types of demand simultaneously and rework process of defect products and disposal for multiple products in the two level supply chain network. The rest of sections of this paper are; mathematical modelling is described in Section 2, numerical example and discussion in section 3. Finally, conclusion of this paper is given in section 4.

2. Mathematical modelling

2.1. System characteristics

The integrated production and inventory model is formulated from the real condition in a company having both continuous and discrete demand and considering defect products during production process. The system is applied for single machine line production and produce multiple items. The next type of item will be produce after one type item have been manufactured for both continuous and discrete demand and the machine has finished the setup process. During the production process, there are imperfect items which parts of it can be rework and others will be discard/disposal as scraps. The rework process will be begin as soon as the regular production process for item has been finished. For more detailed, the system characteristics can be seen in Figure 1.

Figure 1 describes, for an example, three types of products to be produce. The first type of item is produced with production rate (P_1) for several long time t_1 and will be produce imperfect item ($\alpha_1 z_1$). Then the imperfect products will rework for several long time r_1 and some of them will be dispose ($\alpha_1 \beta_1 z_1$). During the regular and rework production process (TP₁), some finished products is delivered to costumer to fulfil the order (continuous demand). The process will be same for the next types of items. When production of all products are completed, a number of discrete demand will be sent to customer with multiple delivery for every several time (w).

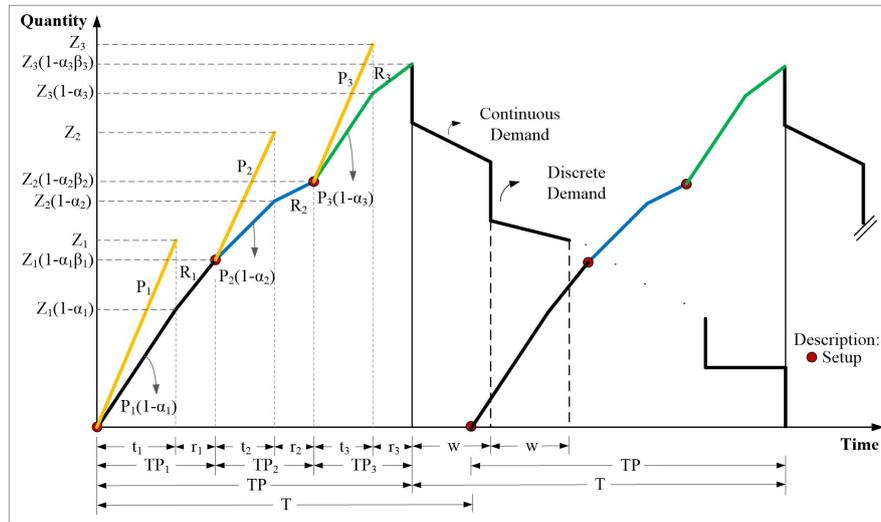


Figure 1. The illustration of system considering two types of demand and reworks

2.2. Notations and assumptions

The complete list of notations is in Appendix A. Assumption of the model developed are as follow:

- a. The planning horizon is infinite.
- a. Shortages are not allowed.
- b. There are single customer for discrete demand.
- c. The total of both demand is less than or equal to the rate of production.

$$(D_i + C_i) \leq P_i \quad \text{and} \quad \alpha_i \cdot (1 - \beta_i) (D_i + C_i) \leq R_i \tag{1}$$

- d. The ratio of demand and the rate of production for the products is less than or equal to 1.

$$\sum_{i=1}^n \left[\frac{(D_i + C_i)}{P_i} + \frac{\alpha_i \cdot (1 - \beta_i) (D_i + C_i)}{R_i} \right] \leq 1 \tag{2}$$

- e. The cycle time is more or equal to the total production time

$$\sum_{i=1}^n TP_i \leq T \approx TP \leq T \tag{3}$$

2.3. Model formulation

The aim of the model is to optimize the integrated production and inventory model and the number of deliveries for multiple products with considering the both types of demand and rework process of defect products and disposal in the two level supply chain network. The model developed consists of setup and holding costs, transportation cost, production cost, and rework and disposal costs.

A. *Production cost.* The cost for production is computed by considering demand and production cost per product for regular production and a number of rework product with rework cost per product. The formulation of total production cost (TPC) is as follow,

$$TPC = \sum_{i=1}^n (D_i + C_i)[A_i + B_i \cdot \alpha_i (1 - \beta_i)] \tag{4}$$

B. *Setup cost.* The formulation of the cost (TSC) for all products is as follow,

$$TSC = \frac{\sum_{i=1}^n S_i}{T} \tag{5}$$

C. *Holding costs.* The formulation of the total holding cost (THC) for the model consists of seven sections shown in Figure 2. The holding costs are; the cost during regular production cycle (I), rework process (II), the cost during production cycle of other product (III), the cost during the cycle time for the continuous demand (IV), the cost of scrap items (V), the cost during the cycle time for the discrete demand (VI) and the cost for buyer (VII).

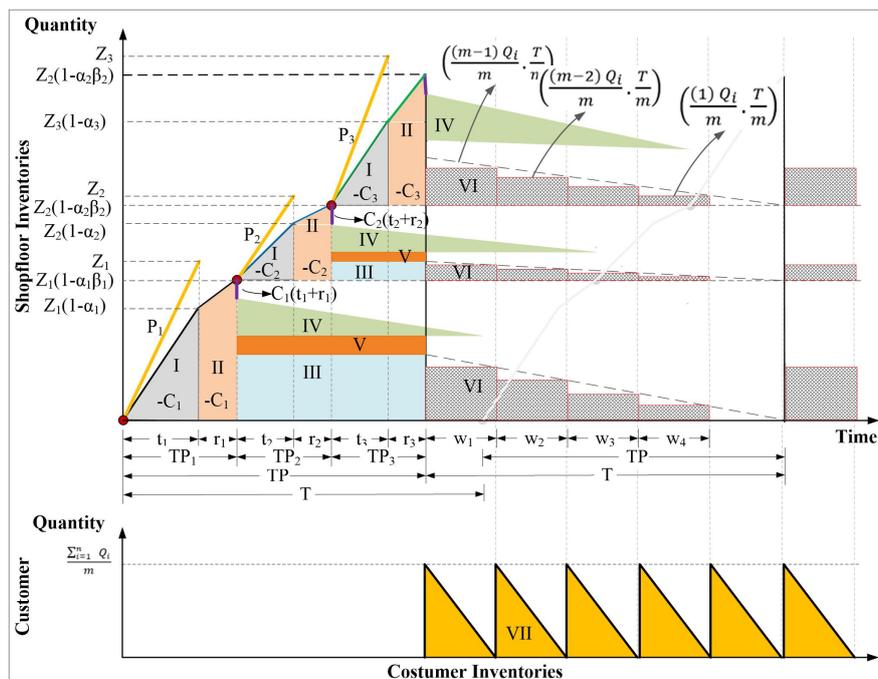


Figure 2. The illustration of the two level supply chain with two types of demand and reworks

a. The formulation of the holding cost of product *i* during production cycle is given as follow,

$$= \frac{1}{2} \cdot T \sum_{i=1}^n H_i \cdot (D_i + C_i)^2 \frac{(P_i - C_i)}{P_i^2} \tag{6}$$

b. The formulation of the cost for holding product *i* during rework cycle is given as follow,

$$= \frac{1}{2} T \sum_{i=1}^n \frac{(D_i + C_i)^2 \cdot \alpha_i (1 - \beta_i)}{R_i} \left\{ H_i \left[2 - \frac{2C_i}{P_i} - \frac{C_i \cdot \alpha_i (1 - \beta_i)}{R_i} \right] + \alpha_i \cdot \beta_i (2L_i - H_i) \right\} \tag{7}$$

- c. The formulation of the cost for holding product i in the production cycle of the other products for discrete demand is given as follow,

$$= T \sum_{i=1}^{n-1} H_i \cdot D_i (1 - \alpha_i \beta_i) \sum_{i=i+1}^n (D_i + C_i) \left(\frac{1}{P_i} + \frac{\alpha_i (1 - \beta_i)}{R_i} \right) \quad (8)$$

- d. The formulation of the cost for holding product i starting the production cycle of next other products until the next its production cycle for continuous demand is given as follow,

$$= \frac{1}{2} T \sum_{i=1}^n H_i \cdot C_i \left\{ 1 - \alpha_i \beta_i - (D_i + C_i) \left[\frac{1}{P_i} + \frac{\alpha_i (1 - \beta_i)}{R_i} \right] \right\} \cdot \left\{ 1 - (D_i + C_i) \left[\frac{1}{P_i} + \frac{\alpha_i (1 - \beta_i)}{R_i} \right] \right\} \quad (9)$$

- e. The formulation of the cost of disposal product i during in the production cycle of other products is given as follow,

$$= T \sum_{i=1}^{n-1} L_i \cdot \alpha_i \beta_i \cdot (D_i + C_i) \cdot \sum_{i=i+1}^n (D_i + C_i) \left[\frac{1}{P_i} + \frac{\alpha_i (1 - \beta_i)}{R_i} \right] \quad (10)$$

- f. The formulation of the cost for holding product i for discrete demand in delivery periods is given as follow,

$$= \frac{T \sum_{i=1}^n D_i \cdot H_i \cdot (1 - \alpha_i \beta_i)}{2} - \frac{T \sum_{i=1}^n D_i \cdot H_i \cdot (1 - \alpha_i \beta_i)}{2m} \quad (11)$$

- g. The formulation the holding cost for the buyer is given as follow,

$$= \frac{1}{2m} \cdot T \cdot \sum_{i=1}^n D_i \cdot (1 - \alpha_i \beta_i) \cdot K_i \quad (12)$$

D. Transportation cost. The cost consists of fixed and variable transportation costs. The variable cost is depended on number of product that sent to customer. The formulation of the total cost (TTC) for all products is as follow,

$$TTC = \frac{(m.F)}{T} + \sum_{i=1}^n (D_i + C_i) (1 - \alpha_i \beta_i) V_i \quad (13)$$

E. Disposal cost. The total disposal cost (TDC) is a cost for scrap items to be disposed of, that depended on number of scrap items and dispose cost per item.

$$TDC = \sum_{i=1}^n T (D_i + C_i) \alpha_i \beta_i E_i \quad (14)$$

F. Total operational cost. It is cumulative of the costs considered in this model. The formulation of the costs (TIPC) is as follow,

$$\begin{aligned} TIPC = & \sum_{i=1}^n (D_i + C_i) \{ A_i + V_i + \alpha_i [B_i + \beta_i (E_i - B_i - V_i)] \} + \frac{\sum_{i=1}^n S_i}{T} + \frac{1}{2} \cdot T \sum_{i=1}^n H_i \cdot (D_i + \\ & C_i)^2 \frac{(P_i - C_i)}{P_i^2} + \frac{1}{2} T \sum_{i=1}^n \frac{(D_i + C_i)^2 \cdot \alpha_i (1 - \beta_i)}{R_i} \left\{ H_i \left[2 - \frac{2C_i}{P_i} - \frac{C_i \cdot \alpha_i (1 - \beta_i)}{R_i} \right] + \alpha_i \beta_i (2L_i - \right. \\ & H_i) \} + \frac{1}{2} T \sum_{i=1}^n H_i \cdot C_i \left\{ 1 - \alpha_i \beta_i - (D_i + C_i) \left[\frac{1}{P_i} + \frac{\alpha_i (1 - \beta_i)}{R_i} \right] \right\} \cdot \left\{ 1 - (D_i + C_i) \left[\frac{1}{P_i} + \right. \right. \\ & \left. \left. \frac{\alpha_i (1 - \beta_i)}{R_i} \right] \right\} + T \sum_{i=1}^{n-1} [H_i \cdot D_i (1 - \alpha_i \beta_i) + L_i \cdot \alpha_i \beta_i \cdot (D_i + C_i)] \sum_{i=i+1}^n (D_i + C_i) \left[\frac{1}{P_i} + \right. \\ & \left. \frac{\alpha_i (1 - \beta_i)}{R_i} \right] + \frac{1}{2} T \sum_{i=1}^n D_i \cdot H_i (1 - \alpha_i \beta_i) + \frac{1}{2m} \cdot T \sum_{i=1}^n D_i \cdot (1 - \alpha_i \beta_i) (K_i - H_i) + \frac{(m.F)}{T} \end{aligned} \quad (15)$$

Doing second differential to equation (15), the optimal solution will be obtain if $(K_i - H_i > 0)$ and Equation (16) is fulfilled.

$$\left\{ \left(\frac{2 \sum_{i=1}^n S_i}{T^3} + \frac{2(m.F)}{T^3} \right) \left(\frac{1}{m^2} \cdot T \sum_{i=1}^n D_i \cdot (1 - \alpha_i \cdot \beta_i) (K_i - H_i) \right) \right\} - \left\{ - \left(\frac{1}{2m^2} \sum_{i=1}^n D_i \cdot (1 - \alpha_i \cdot \beta_i) (K_i - H_i) + \frac{F}{T^2} \right) \right\}^2 > 0 \quad (16)$$

2.4. Methods of solution

A. Algorithm method. Algorithm method is solved the model step by steps. There are:

Step 0: Satisfy the equation (1) and (2).

Step 1: Compute T^* (the optimal production cycle time) by using equation (17) and then go to step 2. For the details of differentiating equation (17), see in Appendix D.

$$T^* = \left[2 \sum_{i=1}^n S_i / \left(\sum_{i=1}^n H_i \cdot (D_i + C_i)^2 \frac{(P_i - C_i)}{P_i^2} + \sum_{i=1}^n \frac{(D_i + C_i)^2 \cdot \alpha_i (1 - \beta_i)}{R_i} \left\{ H_i \left[2 - \frac{2C_i}{P_i} - \frac{C_i \cdot \alpha_i (1 - \beta_i)}{R_i} \right] + \alpha_i \beta_i (2L_i - H_i) \right\} + \sum_{i=1}^n H_i \cdot C_i \left\{ 1 - \alpha_i \beta_i - (D_i + C_i) \left[\frac{1}{P_i} + \frac{\alpha_i (1 - \beta_i)}{R_i} \right] \right\} \cdot \left\{ 1 - (D_i + C_i) \left[\frac{1}{P_i} + \frac{\alpha_i (1 - \beta_i)}{R_i} \right] \right\} + 2 \sum_{i=1}^{n-1} [H_i \cdot D_i (1 - \alpha_i \beta_i) + L_i \cdot \alpha_i \beta_i \cdot (D_i + C_i)] \sum_{i=i+1}^n (D_i + C_i) \left[\frac{1}{P_i} + \frac{\alpha_i (1 - \beta_i)}{R_i} \right] + \sum_{i=1}^n H_i \cdot D_i (1 - \alpha_i \beta_i) \right]^{-1/2} \quad (17)$$

Step 2: Compute m^* (the optimal number of deliveries) by equation (18). If the result is integer number, then go the next step. If not, go to step 4.

$$m^* = \sqrt{\frac{\sum_{i=1}^n D_i \cdot (1 - \alpha_i \beta_i) (K_i - H_i)}{2F}} T^* \quad (18)$$

Step 3: Calculate the optimal total operational cost by using equation (15).

Step 4: Round up and down the values of m^* computed from step 2 and calculate the total operational cost by using equation (15) for both values of m^* . Then choose the optimal number of deliveries, m^* with minimum total operational cost.

B. Optimization Method. The mathematical model that have been formulated is in mixed integer nonlinear programming form. So that, the model can be solved by using software Lingo V14.0. the complete model for total cost can be state as follows:

$$\text{Min } TIPC = TPC + TSC + THC + TTC + TDC$$

$$S/t \quad (D_i + C_i) \leq P_i$$

$$\alpha_i \cdot (1 - \beta_i) (D_i + C_i) \leq R_i$$

$$\sum_{i=1}^n \left[\frac{(D_i + C_i)}{P_i} + \frac{\alpha_i \cdot (1 - \beta_i) (D_i + C_i)}{R_i} \right] \leq 1$$

$$TP \leq T, T, m > 0 \quad m \text{ is integer}$$

3. Numerical example

For the numerical example, it used data input adopted from Rahman *et al* [16] with additional data input such as percentage of imperfect/defect item, percentage of scrap/disposal item and rework production rate. The data is as shown in Table 1 and the analysis sensitivity is given in Table 2. The analysis sensitivity is used to investigate the effect of changing parameters to decision variables and total operational cost. The parameters is changed about $\pm 20\%$, $\pm 10\%$ and 5% , except for percentage of defect and scrap product which are changed about -80% , -40% , 50% , 100% and 150% .

Table 1. The data for the numerical example

Input	Product/Item (<i>i</i>)						
	<i>i</i> =1	<i>i</i> =2	<i>i</i> =3	<i>i</i> =4	<i>i</i> =5	<i>i</i> =6	
D_i	4,047,500	744,100	3,472,500	1,730,750	10,729,200	42,026,551	Units / Year
C_i	0	0	7,673,560	500,000	6,989,500	2,116,000	Units / Year
P_i	90,720,000	108,864,000	90,720,000	108,864,000	90,720,000	108,864,000	Units / Year
R_i	72,576,000	87,091,200	72,576,000	87,091,200	72,576,000	87,091,200	Units / Year
α_i	3%	3%	4%	3%	3%	5%	
β_i	10%	10%	10%	10%	10%	10%	
A_i Rp	3,000	2,300	3,000	2,300	3,000	2,300	/ Unit
B_i Rp	1,000	800	1,000	800	1,000	800	/ Unit
S_i Rp	20,000,000	20,000,000	20,000,000	20,000,000	20,000,000	20,000,000	/ Setup
H_i Rp	440	440	440	440	440	440	/ Unit.Year
K_i Rp	880	880	880	880	880	880	/ Unit.Year
L_i Rp	300	300	300	300	300	300	/ Unit.Year
E_i Rp	800	800	800	800	800	800	/ Unit
V_i Rp	100	100	100	100	100	100	/ Unit
F Rp			2,500,000				/ Shipment

operational cost (TIPC) is Rp 221,947,883,209 per year for solving the problem with algorithm method. This result is the same compared with using software Lingo V14.0 that give the optimal cycle time (T^*) is 0.067 year, m^* is 5 times shipments per cycle time and TIPC is Rp 221,947,900,000 per year. From Table 2, we can see that the changes of parameters will affect the decision variable in this model. The optimal numbers of shipments (m^*) is highly sensitive to manufacturing holding cost (H_i) and customer's holding cost (K_i) with give difference effects. The decision to decrease the numbers of shipments will be accept if the manufacturing holding cost is higher or the customer's holding cost is lower than the current cost. As for the optimal cycle time, it is highly sensitive to changes of number of discrete demand (D_i), manufacturing holding cost (H_i) and setup cost (S_i), but insensitive to both regular and rework production cost (A_i, B_i), disposal cost (E_i) and variable delivery cost (V_i).

4. Conclusions

This paper has proposed the mathematical model to optimize the integrated production and inventory model and number of deliveries considering both two types of demand simultaneously and rework process of defect products and disposal for multiple products in a two level supply chain network. This paper also given two alternative methods to solved the problem, algorithm and optimization methods with the help of software Lingo V14.0. Although this mathematical model can help the company with the same system to make a decision to determine the optimal lot production size and number of shipments, this model still have limitations, such as safety stock, back-order, and multiple buyers. Therefore, for the further research, the model can be extended by considering them in the model.

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Appendices

Appendix A

The list of notations of the model parameters

i	= index for product types ($i=1, 2, 3, \dots, n$)
α_i	= percentage of defect product i (%)
β_i	= percentage of disposal product i (%)
D_i	= discrete demand of product i (unit/year)
C_i	= continuous demand of product i (unit/year)
P_i	= regular production rate of product i (unit/year)
R_i	= rework production rate of product i (unit/year)
H_i	= holding cost of product i (Rp/unit.year)
K_i	= customer's holding cost of product i (Rp/unit.year)
L_i	= holding cost of product i for disposal product (Rp/unit.year)
A_i	= production cost of product i (Rp/unit)
B_i	= rework cost of product i (Rp)
E_i	= disposal cost of product i (Rp)
S_i	= setup cost of product i (Rp)
F	= fixed delivery cost (Rp)
V_i	= variable delivery cost of product i (Rp/unit)
w	= delivery time of discrete demand (year)

- t_i = regular production time of product i (year)
 r_i = rework process time of product i (unit)
 TP_i = production time for produce product i (year)
 TP = total of production time for all types products (year)
 TPC = total production cost (Rp/year)
 TSC = total setup cost (Rp/year)
 THC = total holding cost (Rp/year)
 TTC = total delivery cost (Rp/year)
 TDC = total disposal cost (Rp/year)
 $TIPC$ = total inventory-production cost (Rp/year)

Decision variables

- T = production cycle time (year)
 m = number of shipment (integer number)