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# Study on transverse vibrations of a lathe

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**Abstract.** This paper is focused on the importance of applying integral transforms in the determining of the dynamic response of a lathe because of vibrations during work.

## 1. Introduction

Because the machine-tool is a complex system with a large number of degrees of freedom, in the dynamic study it will be taken only those modes of vibrations, which depending on the required accuracy and the intended purpose, have a predominant role in the development of the researched vibratory phenomenon.

For example, in the paper [1] is presented a mathematical model that allows the simulation of the three-dimensional dynamic behavior of planetary gears/epicycle to the helical gears. By simultaneously combining a time integration scheme and a contact algorithm for network are solved the appropriate motion equations.

The paper [2] presents the same mechanical and mathematical model (for a tool and a support unit of a milling device). Specifically, it is determined a mechanical model with three degrees of freedom for which was applied the Lagrange formalism resulting a system of three second-degree differential equations. In order to solve the mathematical model, is used the integral Laplace transformation obtaining a polynomial equation that could easily be solved.

Using unilateral Laplace transform in relation to time, in the paper [3] were determined the transverse movements of some kinematic elements being in the vibratory motion.

In this paper is also used a similar process.

In paper [4], was applied an iterative method to determine the fields of longitudinal and transverse movements of the linear elastic connecting rod of a mechanism R {RRT}. In addition, the field of accelerations of the rod of a crank mechanism was determined. The same was realized in [5], but for a linear-elastic connecting rod, part of a parallelogram mechanism.

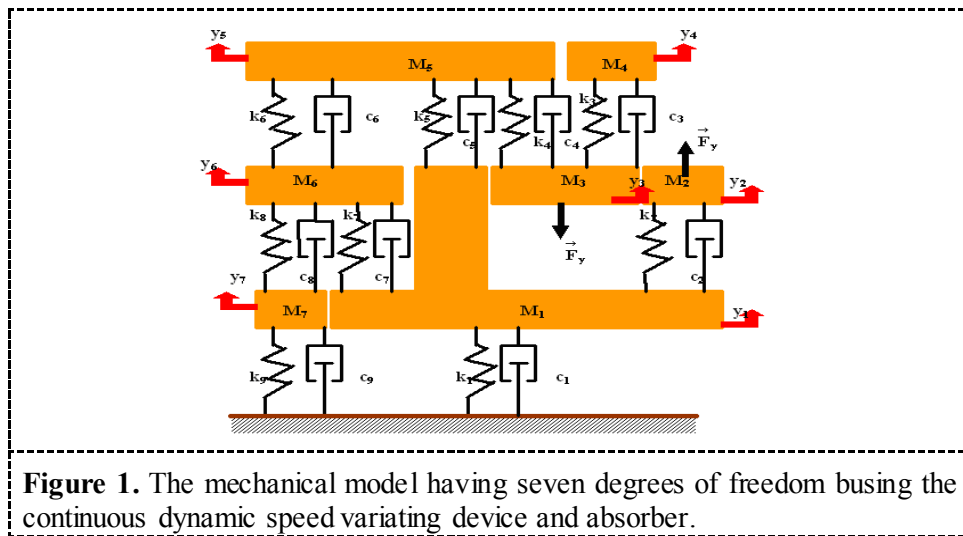
The fundamental problems of the discrete structural systems analysis subject to time analyses are resolved in the work [8]. An approach to solve these problems is the idealization of these systems by an assembly of discrete structural elements, obtaining sets of equations conveniently treated by matrix algebraic operations.



## 2. Mechanical model. mathematical model.

One of the qualities on that a machine tool must have is the dimensional precision of the shape and quality of the surface of the processed piece. The relative displacements appear in the cutting area between the tool and the piece and mainly are a direct consequence of a vibratory phenomenon. The relative displacements determine the dimensional precision. One way to decrease these displacements is to use a dynamic absorber in the processing zone.

Figure 1 presented the mechanical model with seven degrees of freedom of a lathe with continuous dynamic speed varying device and absorber, in case all the constituent elements of the lathe execute the vertical oscillatory translations.



**Figure 1.** The mechanical model having seven degrees of freedom basing the continuous dynamic speed varying device and absorber.

The mathematical model of motion we obtained in Lagrange's formalism (we obtained this model in a similar way and in [6]).

Kinetic energy is the date of the relationship:

$$T = \sum_{j=1}^7 M_j \dot{y}_j^2 = \sum_{j=1}^7 M_j \dot{y}_j^2, \quad (1)$$

where:

$M_1$  – the base mass;  $M_2$  – the mass of the sleigh supporting tool and the tool;  $M_3$  – the mass of the work piece;  $M_4$  – the mass of the dynamic absorber;  $M_5$  – the main shaft mass;  $M_6$  – the mass of dimmer of speed;  $M_7$  – the mass of the drive engine;  $q_j = y_j$ ,  $j = \overline{1,7}$  – Lagrange coordinates.

The following relations give the generalized forces:

$$Q_j = Q_{j,c} + Q_{j,a} + Q_{j,e}, \quad j = \overline{1,7}, \quad (2)$$

where:

$Q_{j,c}$  are the conservative generalized forces given by the relations  $Q_{j,c} = -\frac{\partial V}{\partial q_j}$ ,  $j = \overline{1,7}$ ;

$Q_{j,a}$  are the generalized forces dissipative given by the relations  $Q_{j,a} = -\frac{\partial D}{\partial \dot{q}_j}$ ,  $j = \overline{1,7}$ ;

$Q_{j,e}$  are the generalized forces due to the disturbance which appear during processing and which, in a matrix written, are given by the relationship:

$$\begin{aligned} [Q_e] &= [0 \quad F_y \quad -F_y \quad 0 \quad 0 \quad 0 \quad 0]^T; \\ V &= \frac{1}{2}k_1 y_1^2 + \frac{1}{2}k_2 (y_2 - y_1)^2 + \frac{1}{2}k_3 (y_4 - y_3)^2 + \frac{1}{2}k_4 (y_5 - y_3)^2 + \frac{1}{2}k_5 (y_5 - y_1)^2 + \\ &+ \frac{1}{2}k_6 (y_5 - y_6)^2 + \frac{1}{2}k_7 (y_6 - y_1)^2 + \frac{1}{2}k_8 (y_6 - y_7)^2 + \frac{1}{2}k_9 y_7^2; \\ D &= \frac{1}{2}c_1 \dot{y}_1^2 + \frac{1}{2}c_2 (\dot{y}_2 - \dot{y}_1)^2 + \frac{1}{2}c_3 (\dot{y}_4 - \dot{y}_3)^2 + \frac{1}{2}c_4 (\dot{y}_5 - \dot{y}_3)^2 + \frac{1}{2}c_5 (\dot{y}_5 - \dot{y}_1)^2 + \\ &+ \frac{1}{2}c_6 (\dot{y}_5 - \dot{y}_6)^2 + \frac{1}{2}c_7 (\dot{y}_6 - \dot{y}_1)^2 + \frac{1}{2}c_8 (\dot{y}_6 - \dot{y}_7)^2 + \frac{1}{2}c_9 \dot{y}_7^2. \end{aligned}$$

Because the constraints system is totally holonomic, Lagrange's equations have the form:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} = Q_j, \quad j = \overline{1,7}. \quad (3)$$

Replacing relations (1) and (2) in the Lagrange's equations (3) results, in matrix writing, the mathematical model of the movement in the form of:

$$[M] \left\{ \ddot{y} \right\} + [C] \left\{ \dot{y} \right\} + [K] \{y\} = [Q_e], \quad (4)$$

where

$$\begin{aligned} \left\{ \ddot{y} \right\} &= \left\{ \ddot{y}_1, \ddot{y}_2, \ddot{y}_3, \ddot{y}_4, \ddot{y}_5, \ddot{y}_6, \ddot{y}_7 \right\}^T; \quad \left\{ \dot{y} \right\} = \left\{ \dot{y}_1, \dot{y}_2, \dot{y}_3, \dot{y}_4, \dot{y}_5, \dot{y}_6, \dot{y}_7 \right\}^T; \\ \{y\} &= \{y_1, y_2, y_3, y_4, y_5, y_6, y_7\}^T; \\ [M] &= \begin{bmatrix} M_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_7 \end{bmatrix}; \end{aligned}$$

$$\begin{aligned}
[C] &= \begin{bmatrix} c_1 + c_2 + c_5 + c_7 & -c_2 & 0 & 0 & -c_5 & -c_7 & 0 \\ -c_2 & c_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_3 + c_4 & -c_3 & -c_4 & 0 & 0 \\ 0 & 0 & -c_3 & c_3 & 0 & 0 & 0 \\ -c_5 & 0 & -c_4 & 0 & c_4 + c_5 + c_6 & -c_6 & 0 \\ -c_7 & 0 & 0 & 0 & -c_6 & c_6 + c_7 + c_8 & -c_8 \\ 0 & 0 & 0 & 0 & 0 & -c_8 & c_8 + c_9 \end{bmatrix}; \\
[K] &= \begin{bmatrix} k_1 + k_2 + k_5 + k_7 & -k_2 & 0 & 0 & -k_5 & -k_7 & 0 \\ -k_2 & k_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_3 + k_4 & -k_3 & -k_4 & 0 & 0 \\ 0 & 0 & -k_3 & k_3 & 0 & 0 & 0 \\ -k_5 & 0 & -k_4 & 0 & k_4 + k_5 + k_6 & -k_6 & 0 \\ -k_7 & 0 & 0 & 0 & -k_6 & k_6 + k_7 + k_8 & -k_8 \\ 0 & 0 & 0 & 0 & 0 & -k_8 & k_8 + k_9 \end{bmatrix}.
\end{aligned}$$

### 3. Dynamic response

For finding the dynamic response is applied the matrix equation (4) unilaterally Laplace transform in relation to time, in homogeneous initial conditions.

The disruptive force that appears during the cutting process is:

$$F_y = F_0 \mu \frac{a}{a_0} \sin(\omega_e t),$$

where  $F_0$  - nominal cutting force;  $\mu$  -the overlap factor between the preceding and the present passing s of the knife;  $\frac{a}{a_0}$  -the ratio between actual and nominal depth;  $\omega_e$  -the pulse of the disruptive

force, which's the value is given by  $\omega_e = \frac{2\pi v}{l}$ , with  $l$  - the wavelength of the undulations of the work piece and  $v$  - cutting speed.

These conditions result in algebraic system (2):

$$[[M]s^2 + [C]s + [K]]\{\tilde{y}(s)\} = \frac{\omega_e}{s^2 + \omega_e^2} \left\{ 0, F_0 \mu \frac{a}{a_0}, -F_0 \mu \frac{a}{a_0}, 0, 0, 0, 0 \right\}^T, \quad (5)$$

that has as unknowns the Laplace images of the displacements:

$$\{\tilde{y}(s)\} = \{\tilde{y}_1(s), \tilde{y}_2(s), \tilde{y}_3(s), \tilde{y}_4(s), \tilde{y}_5(s), \tilde{y}_6(s), \tilde{y}_7(s)\}.$$

In extenso, we can write the matrix equation (5) in the form of the following algebraic system:

$$\left\{ \begin{array}{l} \left[ Ms^2 + (c_1 + c_2 + c_5 + c_7)s + (k_1 + k_2 + k_5 + k_7) \right] \tilde{y}_1(s) - \\ - (c_2s + k_2) \tilde{y}_2(s) - (c_5s + k_5) \tilde{y}_5(s) - (c_7s + k_7) \tilde{y}_6(s) = 0 \\ - (c_2s + k_2) \tilde{y}_1(s) + (Ms^2 + c_2s + k_2) \tilde{y}_2(s) = F_0 \mu \frac{a}{a_0} \frac{\omega_e}{s^2 + \omega_e^2} \\ \left[ Ms^2 + (c_3 + c_4)s + (k_3 + k_4) \right] \tilde{y}_3(s) - (c_3s + k_3) \tilde{y}_4(s) = -F_0 \mu \frac{a}{a_0} \frac{\omega_e}{s^2 + \omega_e^2} \\ - (c_3s + k_3) \tilde{y}_3(s) + (Ms^2 + c_3s + k_3) \tilde{y}_4(s) = 0 \\ - (c_5s + k_5) \tilde{y}_1(s) - (c_4s + k_4) \tilde{y}_3(s) + \\ + \left[ Ms^2 + (c_4 + c_5 + c_6)s + (k_4 + k_5 + k_6) \right] \tilde{y}_4(s) = 0 \\ - (c_7s + k_7) \tilde{y}_1(s) - (c_6s + k_6) \tilde{y}_5(s) + \\ + \left[ Ms^2 + (c_6 + c_7 + c_8)s + (k_6 + k_7 + k_8) \right] \tilde{y}_6(s) - (c_8s + k_8) \tilde{y}_7(s) = 0 \\ (Ms^2 - c_8s - k_8) \tilde{y}_6(s) + \left[ Ms^2 + (c_8 + c_9)s + (k_8 + k_9) \right] \tilde{y}_7(s) = 0 \end{array} \right.$$

Elementary solving the algebraic system (5) results the solution of the differential equation system (4), in Laplace images, as how follows:

$$\begin{aligned} \tilde{y}_1(s) &= \frac{(c_3s + k_2)(c_4s^3 - Fs^2 - Gs - K)}{(c_5s + k_5)(s^2 + \omega_e^2)(M^2s^4 + Bs^3 + Cs^2 + Ds + E)}; \\ \tilde{y}_2(s) &= \frac{a_0(c_2s + k_2)(c_3s + k_2)(c_4s^3 - Fs^2 - Gs - K)}{(Ms^2 + c_2s + k_2)(M^2s^4 + Bs^3 + Cs^2 + Ds + E)(s^2 + \omega_e^2)^2 a_0} + \\ &+ \frac{F_0 \mu a \omega_e (M^2s^4 + Bs^3 + Cs^2 + Ds + E)}{(Ms^2 + c_2s + k_2)(M^2s^4 + Bs^3 + Cs^2 + Ds + E)(s^2 + \omega_e^2)^2}; \\ \tilde{y}_3(s) &= \frac{A(Ms^2 + c_3s + k_3)}{(s^2 + \omega_e^2)(M^2s^4 + Bs^3 + Cs^2 + Ds + E)}; \\ \tilde{y}_4(s) &= \frac{1}{(s^2 + \omega_e^2)(M^2s^4 + Bs^3 + Cs^2 + Ds + E)}; \\ \tilde{y}_5(s) &= \frac{-EID^2FA - B - (c_7s + k_7)}{HDF - CEF - G[D(A - B) + E(c_7s + k_7)]DF}; \\ \tilde{y}_6(s) &= \frac{-EID^2F}{D\{HDF - CEF - G[D(A - B) + E(c_7s + k_7)]\}}; \\ \tilde{y}_7(s) &= \frac{ID^2F}{HDF - CEF - G[D(A - B) + E(c_7s + k_7)]}, \end{aligned} \quad (6)$$

where:

$$A = \frac{F_0 a \mu \omega_e}{a_0}; B = M(2c_3 + c_4); C = M(2k_3 + k_4) + c_3 + c_4;$$

$$D = c_3k_4 + c_4k_3; E = k_3k_4; F = c_4c_3 + k_4M - M;$$

$$G = -c_4k_3 + c_3k_4 - (c_4 + c_5 + c_6); H = k_3k_4 - (k_4 + k_5 + k_6);$$

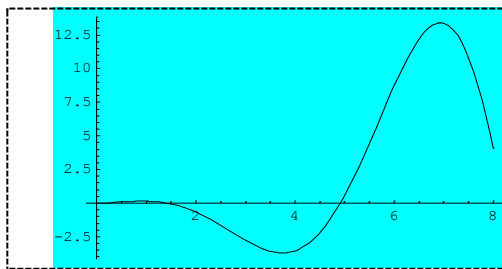
$$\alpha = \left[ Ms^2 + (c_1 + c_2 + c_5 + c_7)s + (k_1 + k_2 + k_5 + k_7) \right] \cdot \frac{(c_3s + k_2)(c_4s^3 - Fs^2 - Gs - K)}{(c_5s + k_5)(s^2 + \omega_e^2)(M^2s^4 + Bs^3 + Cs^2 + Ds + E)}$$

$$\begin{aligned}
\gamma &= [Ms^2 + (c_6 + c_7 + c_8)s + (k_6 + k_7 + k_8)]; \delta = (Ms^2 - c_8s - k_8); \\
\varphi &= (c_5s + k_5); \eta = [Ms^2 + (c_8 + c_9)s + (k_8 + k_9)]; \\
\lambda &= (c_6s + k_6); \mu = (c_8s + k_8); \nu = (c_7s + k_7); \\
\beta &= -(c_2s + k_2) \left\{ \frac{a_0(c_2s + k_2)(c_3s + k_2)(c_4s^3 - Fs^2 - Gs - K)}{(Ms^2 + c_2s + k_2)(M^2s^4 + Bs^3 + Cs^2 + Ds + E)(s^2 + \omega_e^2)^2 a_0} + \right. \\
&\quad \left. + \frac{F_0 \mu a \omega_e (M^2s^4 + Bs^3 + Cs^2 + Ds + E)}{(Ms^2 + c_2s + k_2)(M^2s^4 + Bs^3 + Cs^2 + Ds + E)(s^2 + \omega_e^2)^2 a_0} \right\}.
\end{aligned}$$

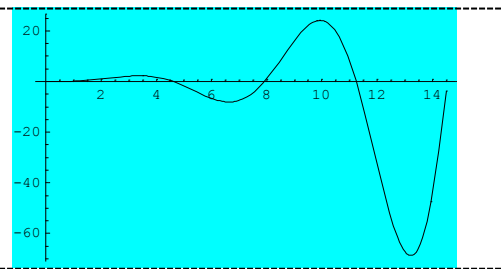
We considered only those movements whose behavior is relevant to obtain a better dimensional accuracy, namely the movement of base, the port-tool sledge and the tool and the movement of the work piece. Applying the inverse of the Laplace transform in (6) and using the numerical data specific of such tool machine (see [7]), the solution of the differential equation system (4) is apparent in the form of time functions:

$$\begin{aligned}
y_1(t) &= -2\cos t - \frac{1}{10}e^{-\frac{1}{4}(1+\sqrt{5})t} \left\{ 2(-5 + 2\sqrt{5})\cos\left(\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}t\right) - 2(5 + 2\sqrt{5})e^{\frac{\sqrt{5}t}{2}}\cos\left(\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{2}}t\right) + \right. \\
&\quad \left. + \frac{1}{\sqrt{10}} \left[ (5 + \sqrt{5})^{\frac{3}{2}} \sin\left(\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}t\right) + (5 - \sqrt{5})^{\frac{3}{2}} e^{\frac{\sqrt{5}t}{2}} \sin\left(\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{2}}t\right) \right] \right\}; \\
y_3(t) &= \cos t + \frac{1}{20}e^{-\frac{1}{4}(1+\sqrt{5})t} \left\{ 2(-5 + 3\sqrt{5})\cos\left(\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}t\right) - 2(5 + 3\sqrt{5})e^{\frac{\sqrt{5}t}{2}}\cos\left(\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{2}}t\right) + \right. \\
&\quad \left. + \sqrt{2} \left[ (-1 + \sqrt{5})\sqrt{5 + \sqrt{5}} \sin\left(\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}t\right) + \sqrt{5 - \sqrt{5}}(1 + \sqrt{5})e^{\frac{\sqrt{5}t}{2}} \sin\left(\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{2}}t\right) \right] \right\}; \\
y_4(t) &= \sin t + \frac{1}{20}e^{-\frac{1}{4}(1+\sqrt{5})t} \left\{ 4\sqrt{5}\cos\left(\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}t\right) - 4\sqrt{5}e^{\frac{\sqrt{5}t}{2}}\cos\left(\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{2}}t\right) - \right. \\
&\quad \left. - \sqrt{2} \left[ (-3 + \sqrt{5})\sqrt{5 + \sqrt{5}} \sin\left(\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}t\right) + \sqrt{5 - \sqrt{5}}(3 + \sqrt{5})e^{\frac{\sqrt{5}t}{2}} \sin\left(\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{2}}t\right) \right] \right\}.
\end{aligned}$$

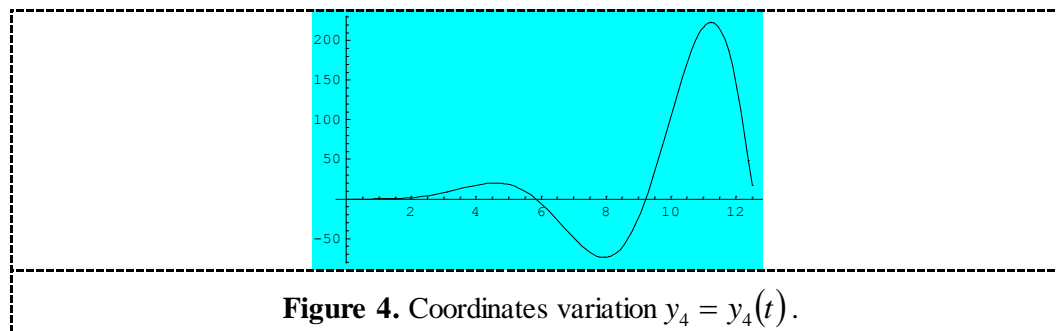
The figures 2, 3 and 4 present the graphical representation of these functions.



**Figure 2.** Coordinates variation  $y_1 = y_1(t)$ .



**Figure 3.** Coordinates variation  $y_3 = y_3(t)$ .



**Figure 4.** Coordinates variation  $y_4 = y_4(t)$ .

#### 4. Conclusions

An important role is the fine estimation of the errors related to modeling and the influence of disturbances when are put in the application some more effective strategies to control the vibratory behavior of complex mechanical structures.

As a result, the mastery of various vibratory phenomena in complex mechanical systems raises fundamental and not trivial research problems. There are phenomenological knowledge of various sources of vibration, thereby reducing efforts in the search for innovative methodologies not only in terms of modelling, but also from that of a pertinent thinking in relation to the objectives of various applications in which constraints appear in the pursuit of ever higher performance when are used these systems.

One of the research activities for this purpose relates to the modeling and study of vibratory phenomena in manufacturing systems, aimed at integrating the dynamic behavior of these systems into the process of developing the manufacturing lines (machining, process planning).

It must note that there is little likely to be serious scientific projects that do not have tangency with mathematics and which can be completed without the use of computers. A high level of safety of calculations can ensure the use of several systems of computational mathematics.

In this context, this attempt to demonstrate the validity of the method used here in researching the vibrations of such machine tools is also included.

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