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Effect of correlation between different ply failures on reliability of laminate composites

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Abstract. The present study focuses on the correlation between different ply failures in laminate composites and its effect on the laminate reliability. First order reliability method (FORM) is employed to compute the failure probability of each ply, and the the correlation between different ply failures is derived from the linearized limit state function of each ply. Significant correlation between different ply failures is observed, and the magnitude of the correlation depends on the uncertainty level of stochastic variables. The correlation is demonstrated laying heavy influence on the laminate reliability. It is clarified that the max algorithm (adopted by previous studies) may provide a heavy underestimation on the laminate failure probability, depending on the magnitude of the correlation coefficient between different ply failures. On the contrary, the sum algorithm (adopted by previous studies) is found providing huge overestimation of the laminate failure probability due to its complete neglect of the correlation between different ply failures. To well account for the correlation between different ply failures in composite reliability evaluation, the integration of multidimensional normal distribution (IMND) is recommended to be employed.

1. Introduction

Due to superior performances provided by laminate composites, they are widely used as important components in aircraft, marine and civil industries, etc. The superior performances of laminate composites are achieved from the specific constituent, micro-configuration and lamination. However, corresponding complex fabrication processes often introduce high variability on composite performances [1, 2]. As a result, deterministic approaches on composite structure design often incorporate high safety factors, which may result unnecessary conservatism and hence scarify competitive capabilities of composites.

To achieve a more reasonable composite design and utilization, a range of probabilistic techniques have been employed in order to represent the reliability (or failure probability) of laminate composites [3]. Among these probabilistic techniques, the first order reliability method (FORM) is possibly mostly adopted due to its good accuracy and meanwhile low computation effort. For example, Lekou and Philippidis [4] used FORM to investigate the failure probability of $[\pm 45/0_2]_S$ laminate plate subjected to in-plane pressure loads, where uncertainty of both mechanical and thermal properties are accounted. Kam and Chang [5] used FORM to calculate the failure probability of several different symmetric laminate plates, focusing on the effect of failure criteria and statistical distribution type of random variables; Gomes et al [6] proposed a reliability based design optimization (RBDO) approach based on genetic algorithms and artificial neural networks, and in the optimization process the reliability is calculated by FORM. In these studies, the common place is that the failure probability of each ply is derived using FORM and then the laminate reliability is derived by a certain algorithm. The max algorithm (represent the laminate failure probability as the maximum ply failure probability) and the sum algorithm (represent the laminate failure probability as the sum of failure probabilities of all plies) have been adopted to derive the laminate failure probability [4, 6-8], but the accuracy of these algorithms were not clarified.

It needs to be kept in mind that the laminate is not a typical series of plies because all plies of a certain laminate are normally cured with very similar qualities, and thereby in composite reliability evaluation it is well accepted that the mechanical properties of different plies in one laminate are considered to be identical. The stochastic variability is mainly represented in the ply mechanical properties of different laminate batches or different in-plane positions from the same laminate batch. Thereby, significant correlation may exist between different ply failures and this correlation would heavily affect the laminate reliability. However, to the authors' knowledge, the correlation between different ply failures in laminate composites has not been properly addressed in related previous studies.

The objective of the present study is to achieve a comprehensive understanding on the correlation between different ply failures in laminate composites and its effect on the laminate reliability. The failure probability of each ply is calculated by FORM, and the correlation coefficient between different ply failures is computed from the linearized limit state function (LSF) of each ply. Significant correlation is observed between different ply failures and the correlation magnitude is found highly dependent on the uncertainty of the load and the ply thickness. To appropriately account for the correlation, integration of multivariate normal distributions (IMND [9, 10]) is adopted to compute the laminate reliability, and the results are compared to the max algorithm and the sum algorithm which have been used in [4, 6-8]. Monte Carlo simulation (MCS) is also adopted to calculate the ply and laminate failure probabilities, as a reference of the exact solution. Finally, the selection of an appropriate algorithm to derive laminate reliability is discussed regarding the correlation between different ply failures.

2. Laminate composite failure criteria

Regarding the lamina (or ply) failure, a range of failure criteria have been developed, such as the maximum stress, maximum strain, Hoffman, Tsai-Wu and Tsai-Hill failure criteria, etc. Generally, the Tsai-Wu criterion yields rather accurate prediction of lamina failure, and it is employed in the present study. By the Tsai-Wu criterion, the survival condition of a ply (or lamina) while considering only in-plane stresses is expressed as:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_{12}\sigma_1\sigma_2 - 1 < 0 \tag{1}$$

and

$$F_1 = \frac{1}{X_t} - \frac{1}{X_c} \quad F_2 = \frac{1}{Y_t} - \frac{1}{Y_c} \quad F_{11} = \frac{1}{X_t X_c} \quad F_{22} = \frac{1}{Y_t Y_c} \quad F_{66} = \frac{1}{S^2}$$

Here, X_t is the tensile strength in fiber direction, X_c is the compressive strength in the fiber direction, Y_t is the tensile strength in transverse direction, Y_c is the compressive strength in the transverse direction, and S is the in-plane shear strength. It has been shown that a good approximation of F_{12} is $-0.5\sqrt{F_{11}F_{22}}$ [11], and this value is used in the present work. It is important to notice that F_{12} may vary for different composite type, and an accurate value of F_{12} can be obtained by multi-axial loading experiments.

Regarding the laminate failure, first ply failure criterion (FPF) is adopted in the present study. The FPF criterion assumes that the laminate has failed if any ply fails. The FPF criterion may give a

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conservative estimation on the laminate composite reliability, as the laminate maybe still capable of bearing certain load after a ply has failed. However, due to the quick progress of composite failure from one ply to all plies, the FPF criterion of composite is still widely employed in industries for safety issues.

3. Reliability analysis

The failure probability of a structure can be written as:

$$P_f = \int_{\mathbf{x}|g(\mathbf{x})\leq 0} f_X(\mathbf{x})d\mathbf{x} \tag{2}$$

where $\mathbf{x}=[x_1, x_2,\ldots x_n]^T$ is a n-dimension vector of random variables, $f_x(\mathbf{x})$ is the joint probability density function (PDF) of $\mathbf{x}$, and $g(\mathbf{x})$ is widely named as the limit state function (LSF) in which $g(\mathbf{x})\leq 0$ represents a subset of the variable space where structure failure occurs. If the ply failure probability is calculated following the Tsai-Wu failure criterion, $g(\mathbf{x})$ is expressed as $1-(F_1\sigma_1+F_2\sigma_2+F_{11}\sigma_1^2+F_{22}\sigma_2^2+F_{66}\tau_{12}^2+F_{12}\sigma_1\sigma_2)$.

It is known that direct integration of Eq. (2) is extremely time-consuming and sometimes impractical when complicated non-linear LSF is encountered. Alternatively, the first order reliability method (FORM) provides an approximation of the failure probability by linearizing the LSF in the vicinity of a most possible failure point (MPP) or named as design point. Hasofer and Lind [12] has proposed an efficient procedure to calculate the reliability using FORM in which the random variables ($\mathbf{x}$) is transformed to uncorrelated standard normal variables ($\mathbf{u}$), and then in the $\mathbf{u}$ space the reliability index β is determined as the shortest distance from the coordinate origin to the failure boundary, as depicted in Figure 1. The derivation of β is then essentially transformed to solve an optimization problem as shown in the formula:

$$\beta=\min\sqrt{\mathbf{u}^T\mathbf{u}}, \text{ Subject to } G(\mathbf{u})=0 \tag{3}$$

where $G(\mathbf{u})$ is the mapping of $g(\mathbf{x})$ from the $\mathbf{x}$ space to the $\mathbf{u}$ space. The reliability index β and the corresponding MPP, $\mathbf{u}^*=[u_1^*, u_2^*,\ldots, u_n^*]^T$, can be obtained via an iteration procedure described in [12].

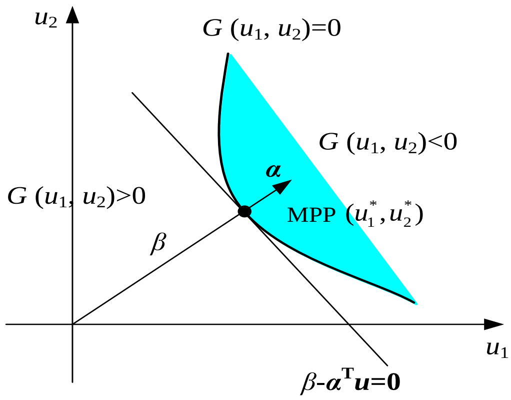

Figure 1. Sketch of FORM method with one design point.

The failure probability Pf is then computed as:

$$P_f = 1-\Phi(\beta) \tag{4}$$

The above algorithm is suitable to compute the failure probability regarding one failure mode with the LSF having only one design point, such as the failure probability of a ply. However, the laminate may fail at different plies, and thereby the LSF of the laminate is a combination of the LSFs of all plies such as illustrated in Figure 2. In Figure 2, multiple design points need to be accounted. If the reliability index corresponding to the i-th design point is β_i, the LSF corresponding to i-th design point can be approximately expressed as:

$$G_i(\mathbf{u}) \approx \beta_i - \boldsymbol{\alpha}_i^T \mathbf{u} \tag{5}$$

where $\boldsymbol{\alpha}_i$ is the unit vector shown in Figure.
2. Then, the linear correlation coefficient between i-th and j-th LSFs can be approximately obtained as:

$$\rho_{ij} = \frac{Cov\left(G_i(\mathbf{u}), G_j(\mathbf{u})\right)}{\sqrt{D[G_i(\mathbf{u})]}\sqrt{D[G_j(\mathbf{u})]}} = \boldsymbol{\alpha}_i^T \boldsymbol{\alpha}_j \tag{6}$$

which indicates that the correlation coefficient ρ_{ij} is actually the cosine of the angle between vectors $\boldsymbol{\alpha}_i$ and $\boldsymbol{\alpha}_j$, as depicted in Figure 2, where $\cos(\theta_{23}) = \rho_{23}$. Using Eq.(6), the correlation coefficient between different ply failures in laminate composites can be obtained.

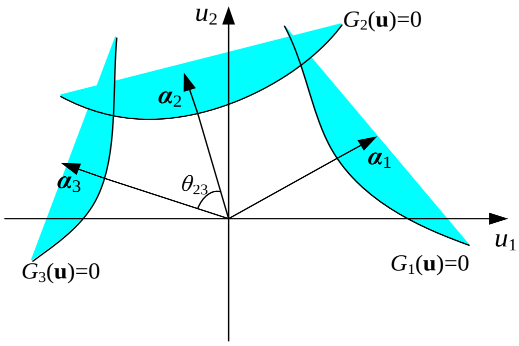

Figure 2. Sketch of FORM with multiple design points.

Following the FPF criterion, the failure probability of the laminate composite can be written as:

$$P_f = P\left\{\bigcup_{i=1}^{N}\left(G_i(\mathbf{u}) \le 0\right)\right\} \tag{7}$$

where P_f is the laminate failure probability, $G_i(\mathbf{u})$ is the LSF of the i-th ply in the uncorrelated standard normal space, and N is the total ply number of the laminate.

By a combination with Eq. (5), Eq.(7) can be expressed as:

$$P_f = 1 - P\left\{\bigcap_{i=1}^{N}\left(\beta_i - \boldsymbol{\alpha}_i^T \mathbf{u} > 0\right)\right\} = 1 - \Phi_N\left(\boldsymbol{\beta}; \boldsymbol{\rho}\right) \tag{8}$$

where $\boldsymbol{\beta}=[\beta_1, \beta_2, \ldots, \beta_3]^T$ is the reliability index vector, $\boldsymbol{\rho}=[\rho_{ij}]_{N\times N}$ is the linear correlation coefficient matrix. Eq.(8) shows that the calculation of the laminate probability is essentially transformed to an integration of multivariate normal distributions (IMND). Direct numerical integration of $\Phi_N(\boldsymbol{\beta}; \boldsymbol{\rho})$ with N > 5 is known to be impractical owing to prohibitively high computer running time and accumulation of numerical errors. The recently developed improved product of conditional marginal (I-PCM) method is proved capable of providing fairly accurate approximation of IMND and as well maintains the simplicity [9]. In the present work, the I-PCM is employed to compute laminate failure probability, and the results are compared with that computed by the max algorithm and sum algorithm.

4. Reliability analysis of laminate plates - examples

In this section, reliability analysis of laminate composite plates is performed in different situations. Classic lamination theory (CLT) is adopted to derive the laminate stress/strain components. The CLT assumes: (a) straight lines perpendicular to the mid-plane before deformation remain straight after deformation; (b) the cross-section plane are inextensible; (c) the cross-section plane rotate such that they remain perpendicular to the mid-plane after deformation. The failure probability of each ply is calculated by both FORM and Monte-carlo simulation (MCS). The correlation between different ply failures is obtained using Eq.(6). Laminate reliability is calculated by the max algorithm, the sum algorithm, the IMND and the MCS. In the MCS, if the failure probability is smaller than 10^{-3}, a sample size of 10^7 is used to obtain the failure probability; otherwise, a sample size of 10^6 is employed. The simulation number provides the failure probability with a relative error smaller than 6%.

A typical 4-layer orthotropic laminate composite plate with $[0/90]_s$ ply orientations is studied in this example. The laminate plate is subjected to bi-axial in-plane load per unit length N_x and N_y as shown in Figure 3. The ply mechanical properties are listed in Table 1 [2]. In the present study, the ply strength parameters (X_t, X_c, Y_t, Y_c, S) are considered as random variables, while the ply elastic parameters (E_1, E_2, G_{12}, υ_{12}) are considered as constants. This is mainly because the stiffness parameters of composites were observed with small variation but the strength parameters were found to be highly scattered [1]. One reasonable explanation of this phenomenon is that composite strength is very sensitive to manufacture defects such as premature infusion voids or debond, but composite stiffness is not. In addition to ply strength parameters, the uncertainty of the load and ply thickness is also considered. Overall, the reliability of the laminate plate is investigated in 3 cases: (1) only the strength parameters are considered as random variables; (2) both of the strength parameters and the load are considered as random variables; (3) both of the strength parameters and the ply thickness are considered as random variables, but the thicknesses of symmetric layers are assumed to be identical.

Table 1. lamina mechanical properties of carbon-epoxy [2].

Symbol	Unit	Mean value	COV
E_1	GPa	133.92	-
E_2	GPa	8.84	-
G_{12}	GPa	4.45	-
υ_{12}	-	0.336	-
X_t	GPa	1.79	0.143
X_c	GPa	1.19	0.135
Y_t	MPa	58.36	0.111
Y_c	MPa	249.96	0.074
S	MPa	92.60	0.041

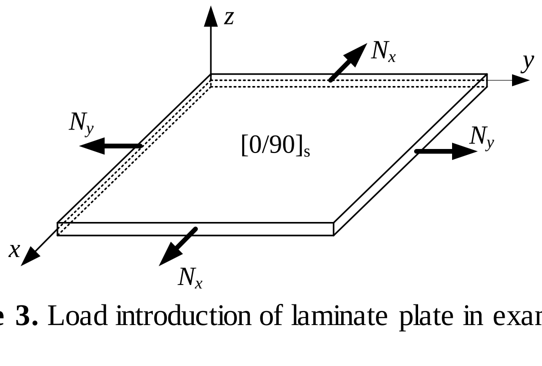

Figure 3. Load introduction of laminate plate in example 1.

Table 2 shows the failure probability of the 0° and 90° plies at the three different cases and the correlation between the 0° and 90° ply failures. In case (1), the values of N_x and N_y are both 550 kN/m and the thickness of each layer is 0.5 mm. In case (2), the mean values of N_x and N_y are both 550 kN/m and their coefficient of variance (COV) of are given as 0.1; the thickness of each layer is 0.5 mm. In case (3), the mean values of each ply thickness is 0.5 mm and its COV is give as 0.1; the N_x and N_y are both 550 kN/m. From Table 2, it is seen that the failure probabilities of the 0° ply and 90° ply are identical in each case, but the ply failure probabilities at difference cases are different. The ply failure probability derived using the FORM is about 10% different from the MCS. This error is most possibly due to the error in the linearization of the complicated non-linear LSF which adopts the Tsai-Wu failure expression. The correlation coefficient between the 0° and 90° ply failures are different in the 3 cases, which is 1 for case (1), 0.935 for case (2) and 0.922 for case (3). The correlation coefficient equaling to 1 in case (1) is as expected, because if only the strength parameters are considered as random variables the Tsai-Wu numbers of different plies are identical. It is not normally noticed that if the load or ply thickness are also considered as random variables (in case (2) and (3)), the correlation coefficient between 0° and 90° ply failures is no longer 1, but a positive value between 0 and 1. This indicates that in these scenarios the failures of the 0° and 90° plies are neither independent nor completed correlated.

Table 2. The failure probability of 0°/90° layers and their correlation coefficient ρ.

	P_f (0° layer)		P_f (90° layer)		ρ
	FORM	MCS	FORM	MCS	
Case (1)	1.42×10^{-3}	1.90×10^{-3}	1.42×10^{-3}	1.90×10^{-3}	1.000
Case (2)	3.15×10^{-3}	3.90×10^{-3}	3.15×10^{-3}	3.90×10^{-3}	0.935
Case (3)	3.50×10^{-3}	4.80×10^{-3}	3.50×10^{-3}	4.80×10^{-3}	0.922

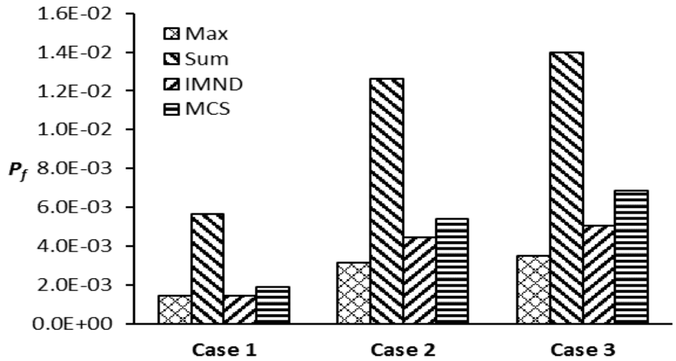

Figure 4. Laminate reliability in case (1), case (2) and case (3) calculated by different algorithms.

From a viewpoint of the stochastic mathematics as shown in Eq.(8), the correlation between different ply failures may lay a significant effect on the laminate reliability. Figure 4 shows the laminate reliability calculated by the max algorithm, the sum algorithm, the IMND and the MCS. It can be seen that the max algorithm, the IMND and the MCS provide similar results on the laminate reliability in case (1). This indicates that if the all ply failures are completely correlated, the max algorithm is proper to derive the laminate reliability. However, in case (2) and (3), it is clearly shown that the max algorithm provides a large underestimation (by about 42%) on the laminate reliability, and this underestimation should be explained as the laminate may not fail at the ply with the maximum failure probability. In all cases, the sum algorithm provides the laminate reliability with a heavy overestimation (by about 77%), while the IMND provides the closest reliability evaluation to the MCS. Thereby, it is clearly shown that correlation magnitude between different ply failures lays an important effect on the laminate reliability, and the selection of a proper approach to derive laminate reliability must consider this correlation.

5. Conclusions

The present work highlights that significant correlation may exist between different ply failures in laminate composites, and the correlation lays important effect in the laminate reliability. Case studies are conducted on symmetric laminate plates subjected to in-plane loads, where the ply strength parameters, ply thicknesses and load are considered as random variables. 2. If not only the strength parameters but also the load or ply thickness possess stochastic variability, the correlation coefficient between different ply failures is a value between 0 and 1. In this scenario, the max algorithm introduces underestimation on the laminate failure probability. The sum algorithm completely neglects the significant correlation between different ply failures, and hence it generates huge overestimation on the laminate failure probability. The IMND is demonstrated capable of providing good laminate reliability prediction despite of the magnitude of correlation between different ply failures, though it is more complicated to implement than the max algorithm. This study highlights a very important factor in the composite reliability estimation, the correlation between different ply failures, which have not been appropriated considered or even neglected in many previous studies. Accounting for the correlation is vital for composite structure design where weight is critical and laminate reliability needs to be accurately estimated. Still, it is important to notice that the present study is based on the FPF failure criterion of composites. The FPF failure criterion is sometimes too conservative as the composite maybe still capable of carrying load if one ply has failed. Future studies will focus on correlation effect between different ply failures considering a consequent ply failure process of laminate composites.

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