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To cite this article: Rui Zha *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **502** 012038

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# Investigations on the three-stage gas-coupled Stirling-type pulse tube cryocooler

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**Abstract.** This paper establishes a theoretical model of the three-stage gas-coupled SPTC based on the electrical circuit analogy (ECA) model. A further improved ECA model for the three-stage arrangement is proposed to achieve the dynamic pressure and volume flow rate at any position of each stage, in which the effects of both the real gas and the dynamic temperature are considered to improve the accuracy. The temperature profiles in regenerators are studied by considering the temperature-induced enthalpy flow rate. Based on the above theoretical analyses, a gas-coupled three-stage SPTC driven by a moving-coil linear compressor is developed and tested. A no-load temperature of 4.8 K and cooling capacity of 40 mW/ 6 K are achieved experimentally with a total input power of 320 W, which is a considerable improvement of performance compared with the thermally-coupled three-stage SPTC counterpart developed by the same authors.

## 1. Introduction

The pulse tube cryocooler (PTC) eliminates the moving component at the cold end, thereby gaining several significant advantages over the conventional regenerative cryocoolers such as Stirling or GM in both the absence of wear-out of the cold finger and the substantial reduction of vibration, electromagnetic interference level and structural complexity at the cold end [1]. The Stirling-type PTC (SPTC), driven by the linear compressor based on clearance seal and flexure springs, also achieves high reliability and long operation time of the driver at the warm end [2-3]. As a result, the SPTC has a strong appeal to many special fields such as in space.

In recent years, the development of the SPTCs working in the temperature range of 4–10 K are gaining interest due to their potential applications in deep space exploration, security defense, and low- $T_c$  superconducting application, etc. In practice, for a SPTC, to achieve the effective cooling capacity at 10 K or below usually needs a three-stage arrangement, for which two typical coupling methods, namely, thermally-coupled and gas-coupled, are often employed. The thermally-coupled arrangement is simple to design the configuration and easy to control the internal flow as well. However, the indispensable thermal links often introduce considerable irreversible losses and also lead to a complicated system. By contrast, the gas-coupled arrangement eliminates all the external thermal



links between stages and thus avoids the potential irreversible losses, which might result in a higher cooling efficiency.

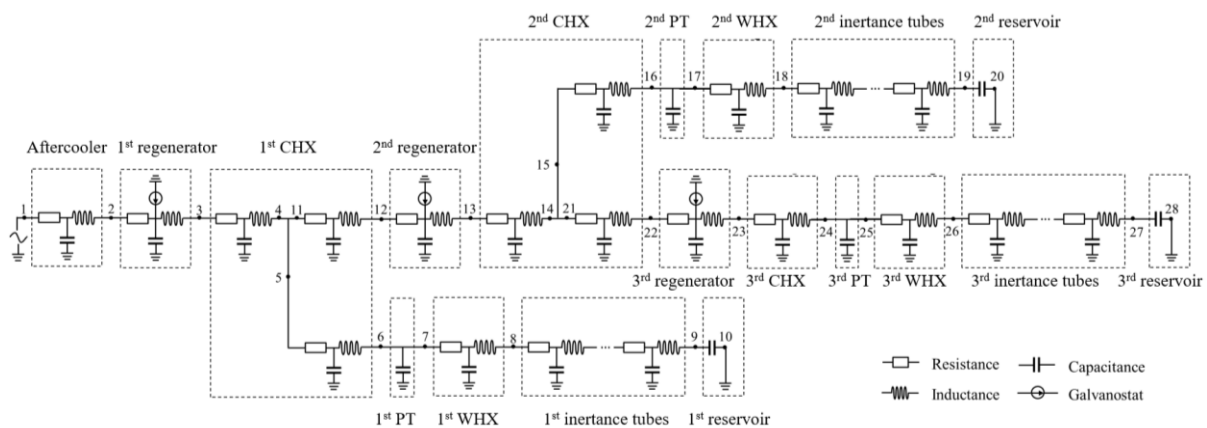
However, in practice, despite the above definite advantages, the application of the gas-coupled multi-stage SPTC is rare, partly due to the great challenge to the design posed by the considerable complexity of the internal flow in a gas-coupled multi-stage SPTC system. In some cases, the mixed coupling approach [4] is often used instead, in which the first two stages are gas-coupled while the third stage is thermally coupled to the first two ones. Gao and Dang [5] developed a three-stage thermally-coupled SPTC and investigated the losses in regenerators and pulse tubes. This paper aims to set up a theoretical model of the three-stage gas-coupled SPTC based on the improved electrical circuit analogy (ECA) model, and experimental verification will be given to provide detailed explanations and quantitative analyses about the effects of several important operating parameters on the system efficiency.

## 2. Electrical circuit analogy model

### 2.1 Establishment of the model

The ECA model is an effective approach of systematically analyzing a regenerative cryocooler. With the aid of the analogy, Swift [6] systematically defined the equivalent elements of the key components such as regenerator, pulse tube, inertance tube and reservoir. Tan and Dang [7] proposed an ECA model for the whole system of a single-stage SPTC, and then Dang et al. [8] further improved the model by considering the effects of heat exchangers and the compression space and by starting the calculation from the phase shifter as well. For the discussed three-stage gas-coupled SPTC model with the coldest stage working at or even below 10 K, the real gas effect should be considered. In addition, the temperature at any specific position in the SPTC, especially in the regenerator, cannot be regarded as fixed, but should be taken as dynamic instead, provided that a considerably high accuracy is pursued. Therefore, a still further improved ECA model considering both of the effects of the real gas and the dynamic temperature is proposed as follows.

The three-stage cold fingers are driven by a single linear compressor in order to ensure the compactness of the system, and the gross input acoustic power is  $W_1$ . All of the inertance tubes and reservoirs are put in the room temperature environment. A schematic of the further improved ECA model for three-stage gas-coupled SPTC is shown in figure 1, in which the positive direction is defined to be from the aftercooler to reservoirs. For different components of the system, the relations between  $p_d$  and  $\dot{U}$  can be obtained from equations (1)-(2). For example, for the first regenerator, the pressure decreases due to the flow resistance and the inertance, while the volume flow rate is affected by the compliance [6]:



**Figure 1.** ECA model for the three-stage gas-coupled SPTC.

$$p_{dx} = p_{d2} - \int_0^x \left( \frac{i\omega\rho}{\varphi A} + r_g \right) \dot{U}_x \cdot dx \quad (1)$$

$$\dot{U}_x = \dot{U}_2 - \int_0^x \left[ \frac{i\omega\varphi A}{\gamma_m p_m} p_{dx} - \frac{i\omega\varphi A \cdot \ln p_m}{\gamma_m^2} \left( \frac{\partial\gamma}{\partial p} p_{dx} + \frac{\partial\gamma}{\partial T} T_{dx} \right) + g \dot{U}_x \right] \cdot dx \quad (2)$$

where  $\varphi$ ,  $\rho$ ,  $A$ ,  $x$ , are porosity, density, area and relative position in the cold finger,  $p_d$ ,  $\dot{U}$ ,  $T_d$  are dynamic pressure, volume flow rate and dynamic temperature,  $\gamma$  is the specific heat ratio,  $g$  is the complex gain factor for volume flow rate [6], and  $r_g$  represents the flow resistance in regenerators per unit length [5,9]. The subscript numbers are referring to positions in figure 1 and the subscript m stands for mean values.

In the first pulse tube, both the resistance and the inertance are much smaller than the compliance. Therefore, the dynamic pressure is considered as constant, while the volume flow rate varies with the position. When the process is assumed to be adiabatic, we have:

$$p_{dx} = p_{d6} = p_{d7} \quad (3)$$

$$\dot{U}_x = \dot{U}_6 - \int_0^x \left[ \frac{i\omega\varphi A}{\gamma_m p_m} p_{dx} - \frac{i\omega\varphi A \cdot \ln p_m}{\gamma_m^2} \left( \frac{\partial\gamma}{\partial p} p_{dx} + \frac{\partial\gamma}{\partial T} T_{dx} \right) \right] \cdot dx \quad (4)$$

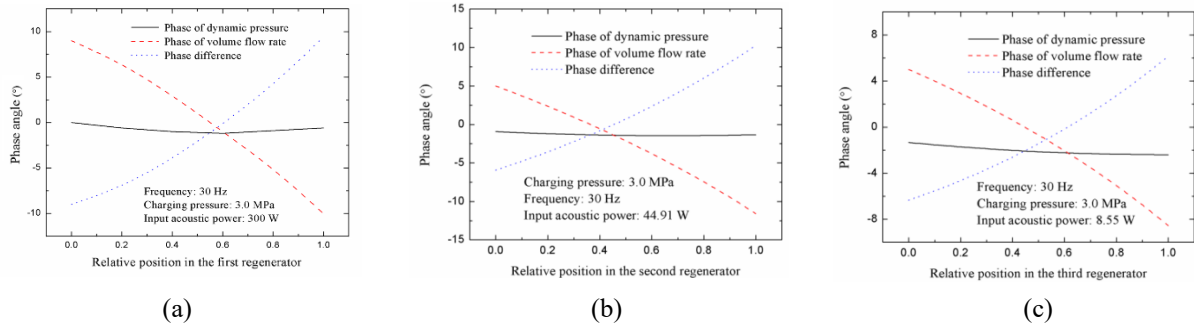
Similar analyses can be made on the last two stages based on equations (1)-(4). In addition, as shown in figure 2, the gas is divided into two parts at point 4, one flows in the first stage toward point 5, while the other flows into the second stage toward point 11. They keep at the same dynamic pressure, but their volume flow rates are different, and then we have the following relationships:

$$p_{d4} = p_{d5} = p_{d11} \quad (5)$$

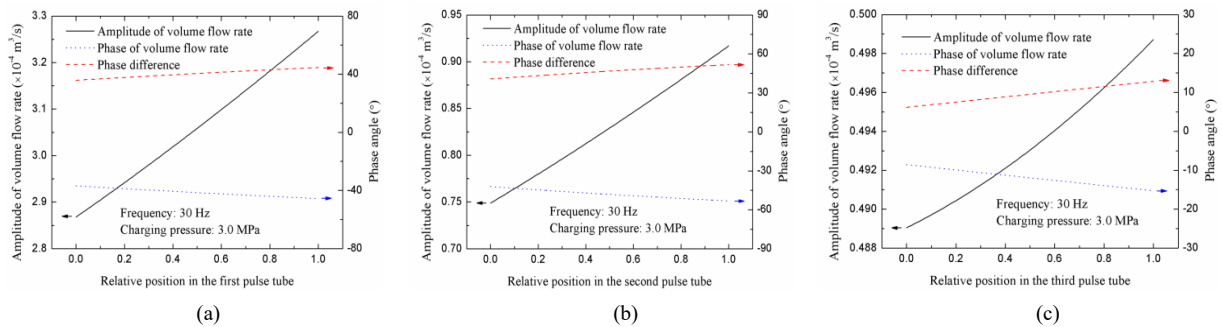
$$\dot{U}_4 = \dot{U}_5 + \dot{U}_{11} \quad (6)$$

## 2.2 Simulations and discussions based on the ECA model

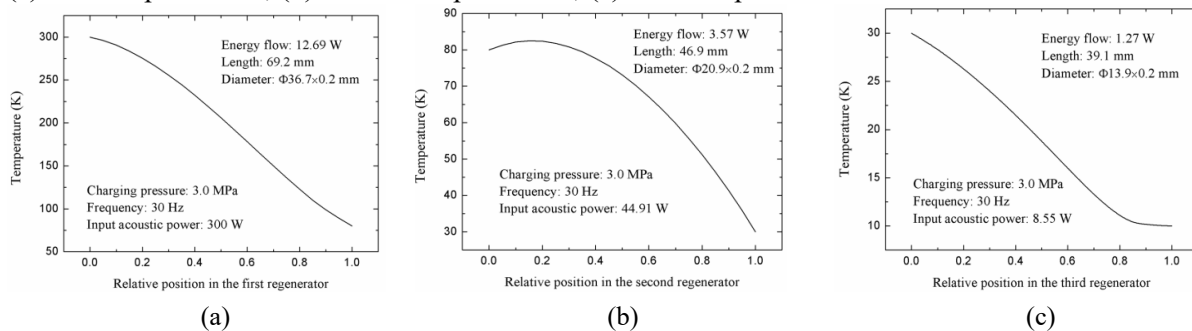
Base on the above analyses, the dynamic pressure and the volume flow rate at any position of the three-stage gas-coupled SPTC can be calculated. Figure 2 (a) shows phase angles for the dynamic pressure and the volume flow rate in the first regenerator. The variation in the phase of dynamic pressure is small, decreasing at first and then increasing, but the phase of volume flow rate decreases monotonically. Hence, the phase difference varies from  $-9.0^\circ$  to  $9.6^\circ$ . Figure 2 (b)-(c) show the variations of the corresponding parameters with the relative position in the last two regenerators, respectively, which keep the similar variation trends to those in the first regenerator.



**Figure 2.** Distributions of phase angles of dynamic pressure and volume flow rate in the first (a), second (b) and third (c) regenerator.



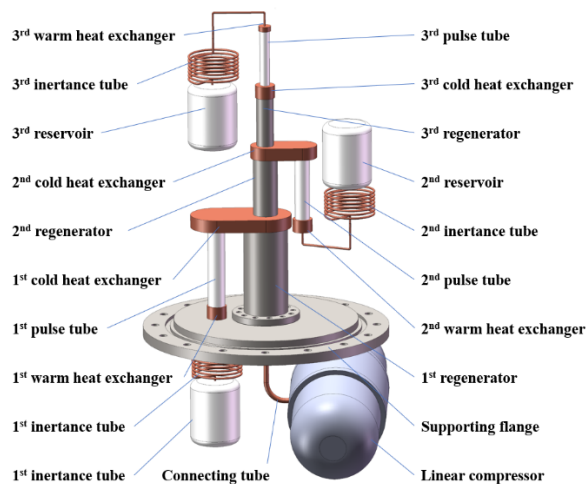
**Figure 3.** Distributions of the volume flow rate and the phase difference in the three stage pulse tubes: (a) the first pulse tube; (b) the second pulse tube; (c) the third pulse tube.



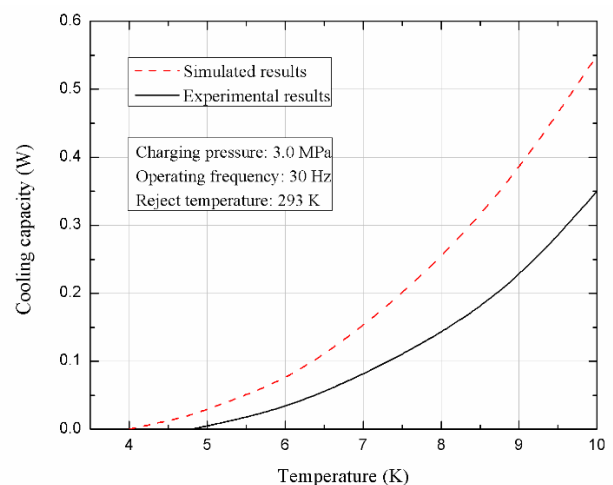
**Figure 4.** Temperature profiles in the first (a), second (b) and third (c) regenerator.

In pulse tubes, the flow resistance and inertance are negligible. Figure 3 (a) shows the variations in  $|\dot{U}|$  and the phase of  $\dot{U}$  in the first pulse tube against the relative position. The variations in phase difference are given for reference as well. The results indicate that,  $|\dot{U}|$  increases monotonically and sharply while the phase of  $\dot{U}$  decreases slightly. By contrast, the phase difference increases slightly. Similar variation tendencies can also be observed for the last two stages, as shown in figure 3 (b)-(c).

Figure 4 shows temperature profiles in regenerators. Figure 4 (a) indicates that the temperature profile in the first regenerator is almost linear. In the second regenerator, the temperature increases at first and then shows a decrease tendency as shown in figure 4 (b). In the third regenerator, the temperature profile almost keeps linear but tends to level off near the cold end of the regenerator, as shown in figure 4 (c).



**Figure 5.** Configuration of the SPTC.



**Figure 6.** Simulated and experimental results.

### 3. Experimental verifications

To verify the validity of the above results, a three-stage gas-coupled SPTC is developed and tested. The configuration of the SPTC is shown in figure 6. It is driven by one linear compressor. The simulated and experimental results of cooling performance are shown in figure 7. The SPTC can reach a no-load temperature of 4.8 K experimentally with total input power of 320 W, while the simulated result of the no-load temperature is 4.0 K.

### 4. Conclusions

This paper establishes a theoretical model of the three-stage gas-coupled SPTC based on the electrical circuit analogy (ECA) model. Some significant conclusions are arrived at as follows:

- (1) About the ECA model, some important improvements are made to make it suitable for the three-stage gas-coupled arrangement, in which the effects of both the real gas and the dynamic temperature are considered. The non-ideal properties of the working fluid are taken into account especially for the second and the third stages, and the influences of the dynamic temperatures are also considered especially in the regenerators. Based on the model, the dynamic pressure and the volume flow rate at any position of each stage could be achieved quantitatively with improved accuracy.
- (2) A three-stage gas-coupled SPTC is developed to verify the above simulations. With a total input acoustic power of 320 W, the three-stage gas-coupled SPTC can reach a no-load temperature of 4.8 K and achieve a cooling capacity of 40 mW/ 6 K.
- (3) In view of the obvious advantages such as a single driver, compactness, absence of thermal links between stages and high cooling efficiency, the gas-coupled SPTC will have a strong appeal to the relevant practical applications.

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### Acknowledgements

This work is financially supported by the Aeronautical Science Foundation of China (No. 20162490005) and the Science and Technology Commission of Shanghai Municipality (Nos. 18511110100, 18511110101, 18511110102).