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# Theoretical and CFD investigations on a 200 Hz thermoacoustic heat engine using pin array stack for operations in a pulse tube cryocooler

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**Abstract.** Thermoacoustic heat engines (TAHEs) convert heat energy into acoustic energy without using any moving part. The acoustic energy can lead to a high frequency linear oscillating motion in a gas which in turn can be used to drive a pulse tube cryocooler (PTC). Thus, the PTC becomes contamination free and offers an extended maintenance-free life with high reliability. In any TAHE, the stack is the heart of the TAHE. Out of various stack designs, e.g. parallel plates, circular pores, rectangular pores and pin array stack etc., reported studies indicate pin array stacks offer superior performance over others. In this paper, investigations are carried out on the pin array stack for a 200 Hz TAHE. The 100 mm long stack is designed using many numbers of 0.3 mm diameter SS wires inside a 24.5 mm diameter resonance tube. Theoretical analyses are carried out for finding the optimum stack centre position. The theoretical results show that maximum acoustic efficiency of 24.5% occurs when the stack centre position is at a distance of  $\lambda/32$  from the closed end. Pressure and velocity oscillations near the stack are also studied using Ansys<sup>®</sup> Fluent 17.1 software.

## 1. Introduction

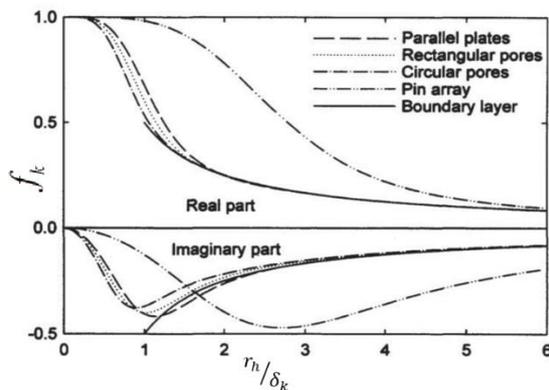
Thermoacoustics is a simple technology which converts heat energy into acoustic energy without having any moving parts. Gas particles undergo spontaneous pressure and velocity oscillation when they are closed to a solid material (stack) which possesses a temperature gradient [1]. Acoustic power can be used to drive PTC [2] or liner alternator [3]. PTC driven by thermoacoustic has an advantage of no moving components and thus a permissible long lifetime. The components of TAHE are stack, resonator, working fluid and hot and cold heat exchanger. Out of these components, stack is the critical element of a TAHE. The heat energy is converted into acoustic energy inside the stack region [4]. There are various types of stack geometries like parallel plate stack, pin array stack, tube array stack and honeycomb stack etc. Stack material should have low thermal conductivity as heat conduction through stack has a negative impact on the performance of thermoacoustics devices. The performance of thermoacoustic devices depends upon the spacing between stacks and thickness of stack. The spacing between stacks should be in the order of few (2 to 4 times) thermal penetration depth ( $\delta_k$ ) through which acoustic oscillations occur [5]. The time-averaged acoustic power gradient  $\left(\frac{d\dot{E}}{dx}\right)$  produced in stack [4] is given as equation (1).

$$\frac{d\dot{E}}{dx} = \frac{1}{2} |P_1|^2 \frac{(\gamma-1)}{\gamma} \frac{(\omega A \text{Im}[-f_k])}{P_m} \left( \frac{dT_m}{dx} \sqrt{T_{(crit)}} - 1 \right) \quad (1)$$

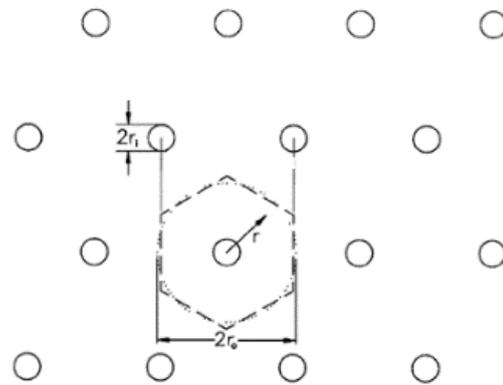


Here  $P_1, \gamma, \omega, A, Im[f_k], p_m, T_m$  and  $\nabla T_{crit}$  are pressure amplitude, heat capacity ratio, frequency, area, imaginary part of Rott's function, mean pressure, mean temperature and critical temperature respectively.

Equation 1, shows that acoustic power is directly proportional to the magnitude of the imaginary part of Rott's function ( $Im[-f_k]$ ). Therefore, it is desirable to have a large magnitude of the imaginary part of Rott's function. Figure 1 shows the variations of  $f_k$  for various stack geometries versus the ratio of the pore's hydraulic radius,  $r_h$ , to the thermal penetration depth,  $\delta_k$ . The hydraulic radius is defined as the area of a pore divided by its perimeter. Pin arrays and parallel plates stacks have larger  $Im[-f_k]$  as compared to the circular, hexagonal or square pore of honeycomb structure [4]. Swift et al. [6] had done analytical analysis on different geometries of stacks and found out that pin array stack is more efficient than parallel plate stack as viscous dissipation for pin array is less than parallel plate stack. They had done analytical analysis on a TAHE operating at 35 Hz using 3.5 MPa helium gas and found out that by using pin stack, there is an increase in 15 % efficiency relative to their best parallel plate stack design.



**Figure 1.** Rott's function " $f_k$ " as a function of the ratio of hydraulic radius and thermal penetration depth [6].



**Figure 2.** Design pattern for pin array stack [6]

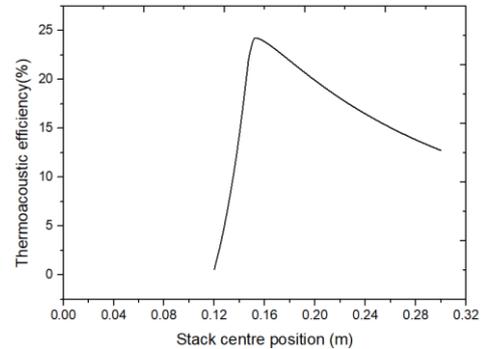
Earlier works on numerical analysis of TAHE have been done to demonstrate thermoacoustic phenomena [7-8] using CFD software. Zink et al. [7] simulated thermoacoustic cooling by supplying thermal energy to TAHE's stack as input and got refrigeration effect across the cooling stack. G. Yu et al. [8] applied the commercial CFD software FLUENT 6.1 to demonstrate the working of a 300 Hz standing wave thermoacoustic heat engine. However, in their work parallel plate stack has been configured into co-axially stacked tubes, so that, a 2D axis-symmetric simulation could be performed. As from figure 1, we can see that stack geometry has a significant impact on the performance of TAHEs. In 2D modelling, only co-axial stack tube be modelled with the help of axis-symmetric boundary condition. Hence, for stack geometries like parallel plate, pin array stack and honeycomb structure stack, a 3D modelling is required to identify the effects of three dimensionalities.

## 2. Design Methodology

The aim of current work is to develop a 200 Hz quarter wavelength ( $\lambda/4$ ) type standing wave TAHE using pin array stack. DeltaEC software [9] is used for designing the TAHE. DeltaEC is a computer program that can calculate details of how thermoacoustic equipment performs and to design the thermoacoustics devices. A 0.3 mm SS wire of 100 mm length is selected for designing pin array stack. Figure 1 shows that the imaginary part of Rott's function will be maximum when  $r_h/\delta_k$  is around 2.85. So, for designing purpose  $r_h/\delta_k$  is taken as 2.85. Pins of radius  $r_i = 0.15$  mm, are arranged in a hexagonal pattern and the distance between two pins is  $2r_o$  as shown in figure 2. A preliminary choice of 4 bar helium at 300 K is considered for design. At this condition,  $\delta_k = 0.265$  mm ( $\delta_k = \sqrt{2k/\omega\rho c_p}$ ). Using  $r_h/\delta_k = 2.85$ , we get  $r_h = 0.755$  mm, now  $r_h = r_o^2 - r_i^2 / 2r_i$  so,  $r_o = 0.5$  mm. The total length of  $\lambda/4$  standing wave TAHE is 1.2 m. Hot exchanger and cold heat exchanger's temperature is fixed to 700 K and 300 K respectively. Stack centre position is varied from 0.12 m to 0.3 m from the closed end. DeltaEC results show that for these conditions,

pin array stack type TAHE gives 24.5% thermoacoustic efficiency when the stack centre position is 0.15 m ( $\lambda/32$ ) as shown in figure 3. Dimensions of the designed TAHE is presented in table 1.

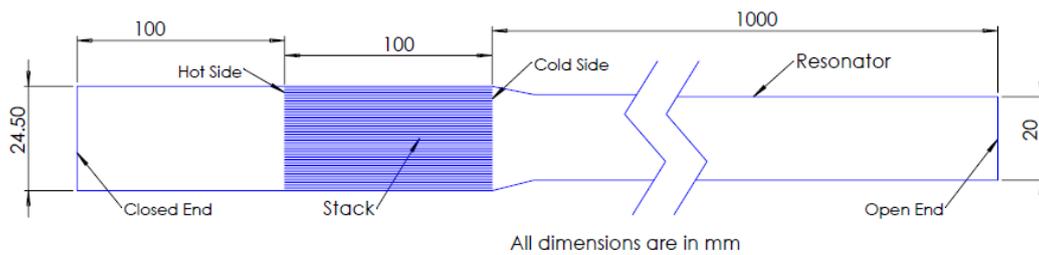
Pin Radius ( $r_i$ )	0.15 mm
Gap between two pins ( $r_o$ )	0.5 mm
Stack Length	100 mm
Stack centre position	150 mm
Hot buffer diameter	24.5 mm
Resonator length	1000 mm
Resonator diameter	20 mm



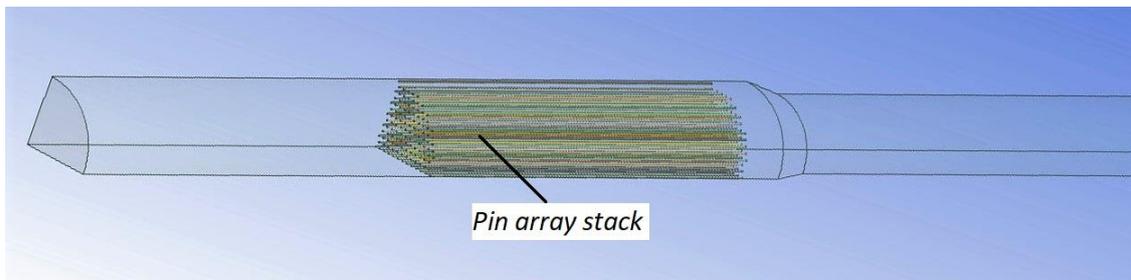
**Figure 3.** Stack centre position vs thermoacoustics efficiency.

### 3. CFD Modelling

Numerical simulations of TAHE were done by solving governing equations for conservation of mass, momentum, and energy in Ansys® Fluent 17.1 software package [10]. Figure 4(a) shows the schematic of a TAHE and figure 4(b) shows the computational domain of a pin array TAHE used for numerical simulations.



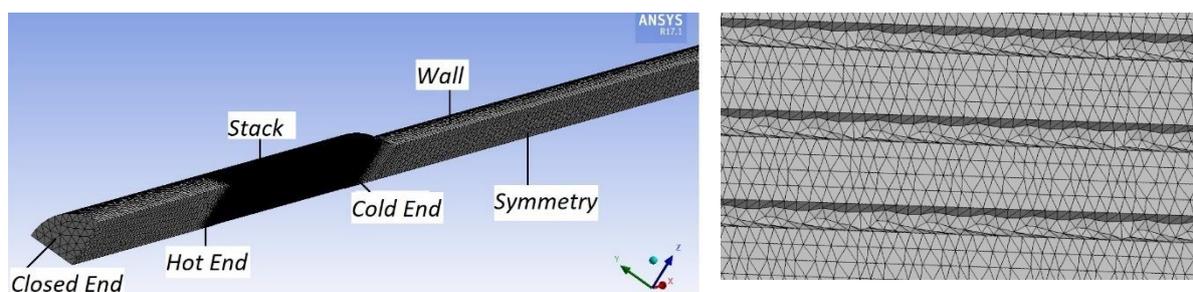
(a)



(b)

**Figure 4.** (a) Schematic of a pin array TAHE (b) Computational domain of a pin array TAHE

#### 3.1 Meshing



(a)

(b)

**Figure 5.** (a) Generated mesh (b) Mesh at stack region

Anslys meshing software is used to do the meshing of the computational domain of TAHE. Final mesh contains 22952487 mesh elements and 4112608 number of nodes. Mesh element size around the stack is kept at 0.16 mm, so that mesh elements around the stack should always be less than thermal penetration depth ( $\delta_k$ ). Y+ value varies from 0.09 to 35.14. Figure 5(a) shows the generated mesh, and figure 5(b) shows the enlarged view of mesh at the stack region.

**Table 2.** Mesh data

	Mesh 1	Mesh 2	Mesh 3
Mesh Elements	16699305	17776911	22952487
Nodes	282532	3201002	4112608
Gauge pressure (Pa) at closed end	18032	19342	20919

**Table 3.** Initial setups

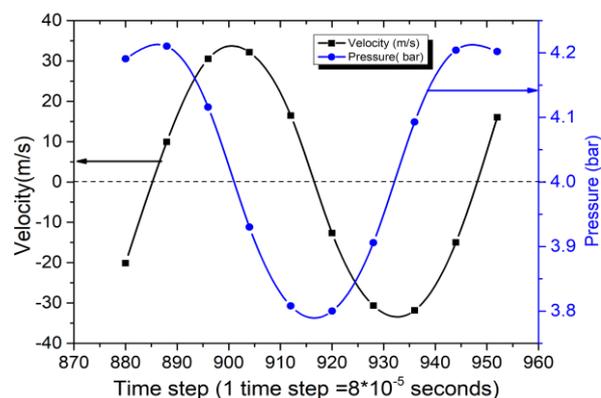
Parameters	choice
Operating Pressure	Helium gas 4 bar
Equation of state	Ideal gas
Viscosity ( $\mu$ )	$(0.04T + 8.23) \mu\text{Pa}\cdot\text{s}$
Turbulence model	k- $\epsilon$
Transient time step	$8 \times 10^{-5}$ s

### 3.2 Boundary conditions and initial conditions

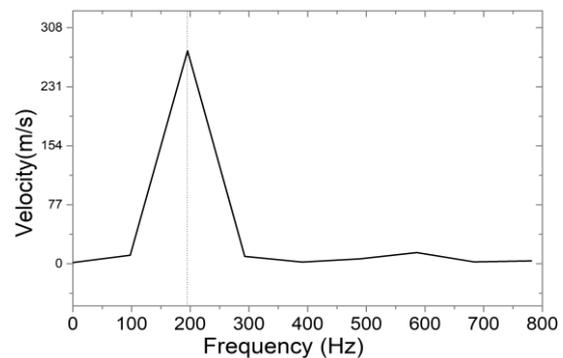
The computational domain includes an open end (a 0 Pa gauge pressure “pressure outlet”) and a closed end wall. In the present study, the simulation uses adiabatic walls with no-slip boundary condition for all walls (closed end, resonator wall and hot buffer wall). Two planes of symmetry are used in the computational domain to reduce the computational time. A linear temperature gradient from 700K to 300 K is imposed over the stack wall using a “user define function” file. Details of the initial setups for the simulations are given in table 3.

## 4. Results and Discussion

Self-exciting oscillations of pressure and velocity wave are generated due to the application of temperature gradient across the stack. Figure 6 shows the pressure and velocity oscillations after reaching their peak pressure and velocity oscillatory amplitude respectively. The amplitude of pressure wave reaches to 21013 Pa gauge at the closed end of TAHE. Thus, drive ratio (ratio of maximum pressure amplitude to the mean pressure) of the designed engine is 5.25%.

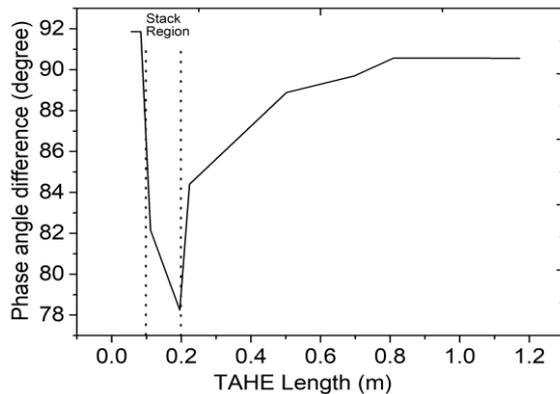


**Figure 6.** Pressure variation at closed end with respect to simulation time steps

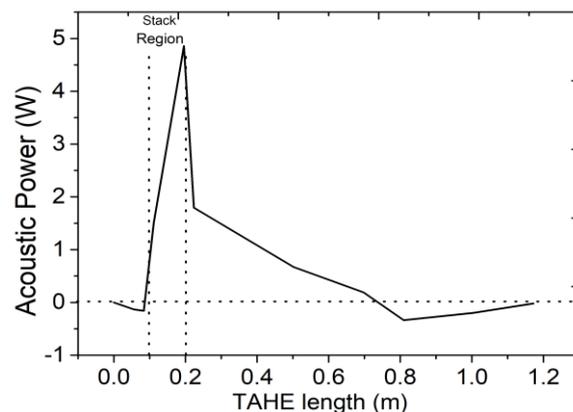


**Figure 7.** FFT analysis on velocity wave

The frequency of TAHE is 195.31 Hz as obtained by FFT analysis (Figure 7) on velocity waveform and frequency obtain from DeltaEC analysis is 198.95 Hz which is comparable with CFD result. For calculating acoustic power 10 points across the length of the engine is taken to calculate phase angle and acoustic power. Pressure waveform is found to be leading the velocity waveform. Phase angle difference between pressure and velocity waveform is plotted in figure 8. The presence of stack shifts the phase angle from 91.8 degrees to 78.2 degrees. Acoustic power across TAHE length is calculated using equation  $\dot{E} = \frac{1}{2} |P_1| |U_1| \cos(\phi)$  [4], and it is plotted in figure 9. Maximum acoustic power is generated near the cold side of the stack, and there is negative acoustic power near the hot end which signifies the acoustic power dissipated at the hot buffer side.



**Figure 8.** Phase angle difference between pressure and velocity wave



**Figure 9.** Acoustic work across TAHE Length

## 5. Conclusion

A  $\lambda/4$  standing wave pin array stack type of TAHE is designed by taking  $n_h/\delta_k = 2.85$  and pin diameter of 0.3 mm for operating pressure of 4 bar. Wavelength ( $\lambda$ ) of the developed engine is 4.8 m. DeltaEC software is used to find the optimum stack centre position which is found to be 0.15 m ( $\lambda/32$ ) from the closed end. 3D modelling of a pin array stack type TAHE is simulated in Ansys® Fluent 17.1 after getting all the dimensions from the theoretical analysis. Self-exciting oscillations of pressure and velocity waves are observed in this model. The drive ratio of the designed TAHE is 5.25 %. The shift in phase angle difference from 91.8 degrees to 78.2 degrees is observed at the end of the cold side of the stack which is responsible for delivering acoustic power. Data obtained from this analysis will be used to develop an experimental setup to validate this numerical model and establishment of a design methodology for TAHE.

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