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Deflection of Composite Cantilever Beams with a Constant I-Cross Section

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Abstract. Laminated composite beams are widely used as structural components in aerospace and wind energy industries. For airplane wings and wind turbine blades, the beams, commonly called spars, provide principal stiffness against bending deformation of the structures. The present work is focused on composite cantilever beams with a constant I-cross section. A unidirectional (UD) Glass/Epoxy lamina was chosen for composite material. The overall dimension of the beam is 50 x 55.2 x 2,500 mm. Various stacking sequences were assigned to the flanges and the web of the beam. A uniform distributed load was applied to the upper flange. The deflection results from First-Order Shear Deformation Theory (FSDT) and Finite Element Analysis (FEA) are in good agreement. The effective longitudinal modulus (E_x) of the flanges has strong influence on the bending stiffness and the beam deflection. Understanding important roles of materials and beam construction can help improving the design of I-beams. The validated FEA procedure can be extended to the analysis of realistic spars, which is based on an I-cross section plus curve, taper, and twist along the length.

1. Introduction

Composite materials have excellent specific stiffness and strength. Lightweight structures such as airplanes and wind turbine blades use composite beams, commonly called spars, to provide principal stiffness against bending deformation. The relationship between load and deformation of composite laminates is complex due to the orthotropic nature of the materials. The behavior at ply level requires a stiffness matrix derived from the classical lamination theory (CLT). The behavior at laminate level can be simplified by using effective in-plane and flexural modulus [1].

Theories of laminated composite beams can be divided into two classes. One is the class of classical beam theories (CBT), which are suitable for thin beams. The other is the class of shear deformation beam theories (SDBT), which take into account the shear deformation and rotary inertia. The latter are more accurate for thick beams [2]. Fundamental equations using a first-order shear deformation theory (FSDT) for rectangular beams were developed and verified with 3D finite element analyses (FEA) [2].

Composite beams with I-cross section are regarded as thin-walled, open-section, orthotropic beams. The I-beam is made from flat laminates that can have different properties. Effective modulus of the upper flange, lower flange, and web can be different. Therefore, equivalent tensile and equivalent bending stiffness of the cross-section must be determined [3]. Moreover, thick solid beams and thin-walled beams are subjected to a transverse shear strain that adds a shear deformation to the total beam deformation [4]. The first-order shear deformation theory (FSDT) takes this shear deformation into account with the assumption that the cross section of a beam remains plane but not perpendicular to the beam's neutral axis.



The present work is focused on composite cantilever beams with a constant I-cross section. The calculation of deflection using CBT and FSDT [3-5] was investigated. Simplified I-beams representing a wing's spar or a wind turbine blade's spar with various laminate stacking sequences were analyzed and compared with the results from FEA.

2. Composite laminate behavior

The relationship between external loads and deformation of composite laminates can be described by the classical lamination theory [1]. The stiffness matrix is commonly called ABD matrix.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (1)$$

where [A]: Extensional stiffness matrix
[B]: Bending-extension coupling stiffness matrix
[D]: Bending stiffness matrix

Since the material properties are defined relative to fiber direction (coordinate 1-2), the effective modulus of a laminate relative to structural direction (coordinate x-y) have to be calculated from equation (2) to (4).

$$[a] = [A]^{-1}, [d] = [D]^{-1} \quad (2)$$

$$E_x = \frac{1}{Ta_{11}}, E_y = \frac{1}{Ta_{22}}, G_{xy} = \frac{1}{Ta_{66}}, \text{ and } \nu_{xy} = -\frac{a_{12}}{a_{11}} \quad (3)$$

$$E_x^f = \frac{12}{T^3 d_{11}}, E_y^f = \frac{12}{T^3 d_{22}}, G_{xy}^f = \frac{12}{T^3 d_{66}}, \text{ and } \nu_{xy}^f = -\frac{d_{12}}{d_{11}} \quad (4)$$

where T is the laminate thickness, equations (3) are effective in-plane properties and equations (4) are effective flexural properties.

3. Deflection of composite cantilever beams

A cantilever beam with the cross section symmetrical with respect to the x-z plane is shown in figure 1. A uniform transverse load p_z (N/m) is applied in the symmetry plane.

The deflection of the beam is due to both bending and shear deformation. The maximum deflection at the free end of the beam can be determined using equation (5) [4].

$$w = w_b + w_s = \frac{p_z L^4}{8EI} + \frac{p_z L^2}{2\bar{S}} \quad (5)$$

where L is the beam's length, \overline{EI} is the equivalent bending stiffness and \bar{S} is the shear stiffness.

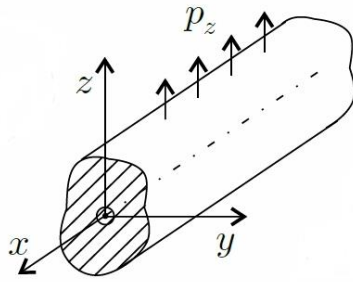


Figure 1. Orthotropic beam under transverse loading [4].

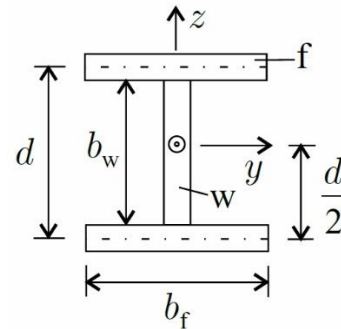


Figure 2. Cross section of composite I-beam [4].

The equivalent bending stiffness of a symmetrical thin-walled I-beam with each wall segment being a symmetric laminate can be determined using figure 2 and equation (6) [4].

$$\overline{EI}_{yy} = \frac{b_f}{(a_{11})_f} \frac{d^2}{2} + \frac{2b_f}{(d_{11})_f} + \frac{b_w^3}{12(a_{11})_w} \quad (6)$$

where the subscript f denotes the flange segments and the subscript w denotes the web segment.

The shear stiffness of the cross section is assumed to be from the web only. By the arrangement of the web segment, the transverse shear modulus G_{xz} in the beam coordinate is actually the in-plane shear modulus G_{xy} in the web laminate. The shear stiffness can be calculated by [6]:

$$\bar{S} = G_{xy} A_w \quad (7)$$

4. Composite I-beam analysis

The beam for deflection analysis has a constant I-cross section. One end is fixed and the other end is free, forming the cantilever beam configuration. The overall dimension of the beam is 50 x 55.2 x 2,500 mm as shown in figure 3. The flanges and the web have the cross section of 50 x 2.6 mm.

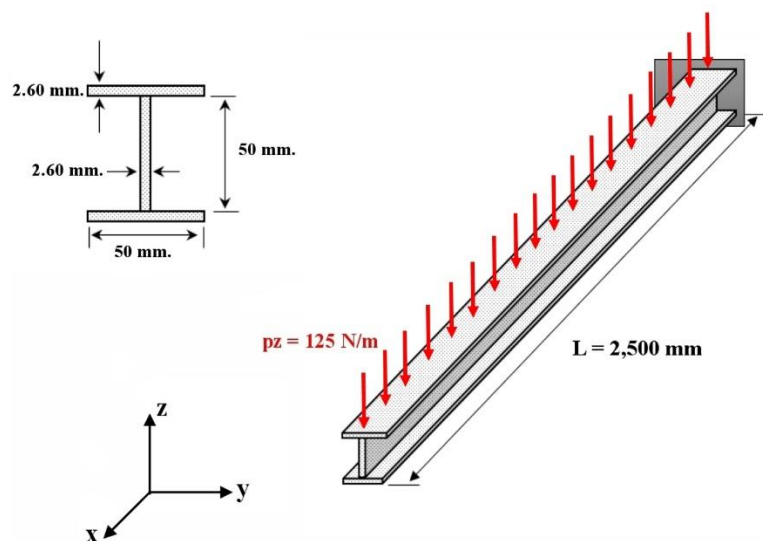


Figure 3. Cantilever beam under uniform transverse load.

A unidirectional (UD) Glass/Epoxy lamina was chosen for the material. The material properties are shown in table 1. Four stacking sequences were assigned to the flanges. They were selected to produce different level of deformation coupling [7]. Laminate F1 is for $[20/-70/-70/-70/20]_{2S}$, F2 for $[20_5]_{2S}$, F3 for $[0/90/0/90/0]_{2S}$, and F4 for $[90/45/-45/0/0]_{2S}$. The web has two stacking sequences: W1 is $[45/-45]_{5S}$ with high shear modulus and W2 is $[0/90]_{5S}$ with high extensional modulus. A uniform distributed load $p_z = 125 \text{ N/m}$ was applied to the upper flange.

Table 1. Properties of Glass/Epoxy UD lamina [5].

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	t (mm)
53.78	17.93	8.96	0.25	0.13

4.1. Laminate and cross section properties,

Effective modulus of the laminates used for the flanges and the web of the beams can be calculated using equations (3) and (4). Relevant results are shown in table 2. The values of the in-plane modulus and the flexural modulus of each laminate are approximately the same. For the laminate F4, E_x^f is noticeably less than E_x because the 0-degree layers are placed toward the middle of the laminate. The layers then become less efficient for bending resistance.

Table 2. Effective in-plane and flexural modulus.

Laminate	Layup	E_x (GPa)	E_x^f (GPa)	G_{xy} (GPa)	G_{xy}^f (GPa)
F1	$[20/-70/-70/-70/20]_{2S}$	28.42	28.92	10.92	10.94
F2	$[20/20/20/20/20]_{2S}$	38.72	38.72	9.99	9.99
F3	$[0/90/0/90/0]_{2S}$	39.64	40.07	8.96	8.96
F4	$[90/45/-45/0/0]_{2S}$	35.64	29.87	11.78	12.06
W1	$[45/-45]_{5S}$	24.98	24.93	16.02	15.93
W2	$[0/90]_{5S}$	36.05	38.75	8.96	8.96

The equivalent bending stiffness and shear stiffness are required for the calculation of the beam deflection. They can be determined using equations (6) and (7) respectively. Table 3 shows the equivalent stiffness of 8 I-beams obtained from a combination of the flanges and webs. The modulus E_x of the flange has strong influence on the bending stiffness \bar{EI}_{yy} . The E_x of laminate F2 is 36.24% greater than the E_x of laminate F1. This makes the bending stiffness of the beam F2W1 higher than the beam F1W1 by 31.97%. The modulus of the web has less effect on the bending stiffness. While E_x of laminate W2 is greater than W1 by 44.32%, the bending stiffness of the beam F1W2 is higher than F1W1 by only 5.14%. For shear deformation, the shear modulus G_{xy} of the web has direct effect on the shear stiffness \bar{S} of the beam.

Table 3. Equivalent bending stiffness and shear stiffness of the cross section.

Beam	Flange	Web	\bar{EI}_{yy} (N-m ²)	\bar{S} (N)
F1W1	$[20/-70/-70/-70/20]_{2S}$	$[45/-45]_{5S}$	5,793	2.083×10^6
F2W1	$[20/20/20/20/20]_{2S}$	$[45/-45]_{5S}$	7,645	2.083×10^6
F3W1	$[0/90/0/90/0]_{2S}$	$[45/-45]_{5S}$	7,812	2.083×10^6
F4W1	$[90/45/-45/0/0]_{2S}$	$[45/-45]_{5S}$	7,090	2.083×10^6
F1W2	$[20/-70/-70/-70/20]_{2S}$	$[0/90]_{5S}$	6,091	1.165×10^6
F2W2	$[20/20/20/20/20]_{2S}$	$[0/90]_{5S}$	7,944	1.165×10^6
F3W2	$[0/90/0/90/0]_{2S}$	$[0/90]_{5S}$	8,112	1.165×10^6
F4W2	$[90/45/-45/0/0]_{2S}$	$[0/90]_{5S}$	7,389	1.165×10^6

4.2. Finite element analysis (FEA)

The beams were modelled with 2D shell element. After a mesh convergence study, the element size was chosen to be 2.5 x 2.5 mm. Each beam model has a fixed supports at one end and is free at the other end. The geometry, material, stacking sequences, and loading are described in the previous section. Figure 4 shows the beam's FEA model with coarse mesh to make the image clear.

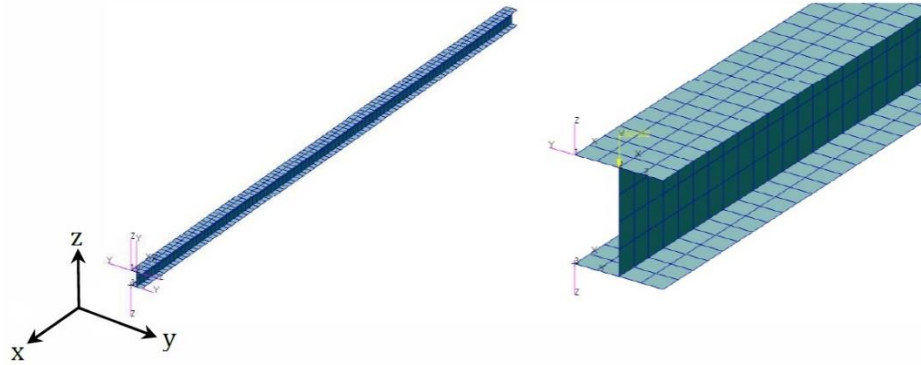


Figure 4. FEA Model of the composite cantilever I-beams.

4.3. Maximum deflection at free end

Deflection at the free end of the beams are presented in table 4. The CBT method does not take into account the shear deformation. Only the bending deformation (w_b) was calculated using equation (5). The FSDT method predicted the deflection using both bending and shear deformation from equation (5). The results from FEA are also included. Figure 5 shows a typical deformation result where the maximum deflection occurs at the free end of the beam. For the beam configuration in the present work, the shear deformation is negligible as the deflection values determined by CBT and FSDT are practically the same. The FEA method consistently predicts greater deflection compared to the FSDT. However, both method are still in good agreement with the maximum difference less than 11%.

Table 4. Deflection at free end of composite cantilever I-beams.

Beam	Flange	Web	Tip Deflection (mm)		
			CBT	FSDT	FEA
F1W1	[20/-70/-70/-70/20] _{2S}	[45/-45] _{5S}	105.36	105.55	116.00
F2W1	[20/20/20/20/20] _{2S}	[45/-45] _{5S}	79.84	80.03	88.80
F3W1	[0/90/0/90/0] _{2S}	[45/-45] _{5S}	78.13	78.32	86.80
F4W1	[90/45/-45/0/0] _{2S}	[45/-45] _{5S}	86.09	86.28	95.40
F1W2	[20/-70/-70/-70/20] _{2S}	[0/90] _{5S}	100.20	100.54	110.00
F2W2	[20/20/20/20/20] _{2S}	[0/90] _{5S}	76.83	77.16	85.50
F3W2	[0/90/0/90/0] _{2S}	[0/90] _{5S}	75.24	75.58	83.60
F4W2	[90/45/-45/0/0] _{2S}	[0/90] _{5S}	82.60	82.93	91.40

The modulus E_x of the flange has strong influence on the deflection. The E_x of laminate F2 is 36.24% greater than the E_x of laminate F1. As a result, FSDT predicts the deflection of beam F2W1 less than F1W1 by 24.18%. The modulus of the web has little effect on the deflection. While E_x of laminate W2 is greater than W1 by 44.32%, the deflection of the beam F1W2 is lower than F1W1 by only 4.75%.

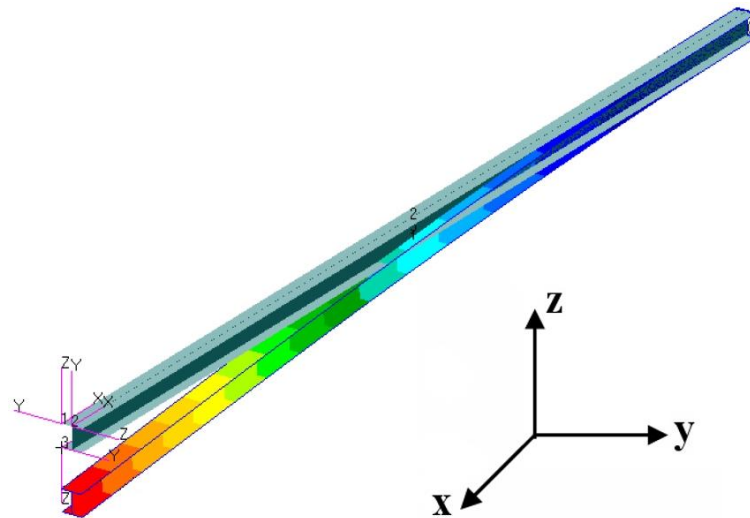


Figure 5. Deflection of composite cantilever I-beams from FEA.

5. Conclusion

Deflection of composite cantilever I-beams under a uniform transverse load was investigated. The deflection of the free end were determined using classical beam theory (CBT), first-order shear deformation theory (FSDT), and finite element analysis (FEA).

For the beam configuration in the present work, the shear deformation is negligible as the deflection values determined by CBT and FSDT are practically the same. The deflection results from FSDT and FEA are in good agreement. The FEA method consistently predicts greater deflection compared to the FSDT. However, the difference is less than 11%.

The effective longitudinal modulus (E_x) of the flanges has strong influence on the bending stiffness and the beam deflection. Understanding important roles of materials and beam construction can help improving the design of I-beams. The validated FEA procedure can be extended to the analysis of realistic spars, which is based on an I-cross section plus curve, taper, and twist along the length.

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