

PAPER • OPEN ACCESS

Estimation of Yield Surface on Iron Powder Compaction for Finite Element Analysis using Drucker-Prager Cap Model

To cite this article: R Ogawa *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **501** 012021

View the [article online](#) for updates and enhancements.

Estimation of Yield Surface on Iron Powder Compaction for Finite Element Analysis using Drucker-Prager Cap Model

R Ogawa ¹, T Yoshimura ¹, N Yagi ¹, K Mizuta ¹, and Y Taniguchi ^{2,*}

¹ Student of National Institute of Technology, Nara College, Nara 639-1080, Japan

² Department of Mechanical Engineering, National Institute of Technology, Nara College, Nara 639-1080, Japan

* Corresponding Author: taniguchi@mech.nara-k.ac.jp

Abstract. Shear failure yield line in Drucker-Prager yield surface for conventional atomized iron powder has been estimated with critical stress state and uniaxial failure stress state by single shear test and uniaxial failure test for green compact. Lateral force measurement test was also conducted using newly devised instrumented die compaction system to identify compaction yield characteristics as Cap yield surface. By those estimations, materials input parameters for Drucker-Prager Cap model have been determined successfully. Finite element analysis of simple closed-die compaction has been conducted and good correspondence in variation of compaction pressure with densification has been recognized.

1. Introduction

Powder metallurgy, PM method is one suitable method to produce metal parts with high accuracy and high cost performance as it can process metal parts with certain mechanical properties in complex shape such as gear, sprocket, pulley, and synchronizer hub. In the metal powder compaction process using closed-die system, which applies to mass production for various PM parts, failure of green compact such as slip-crack often occurs especially for compaction of multi-step part. Since slip-crack usually caused by improper powder flow on the boundary of high density part and low density part during consolidation, it is important to optimize compression velocity in tools for each compaction step including ejection of solidified green compact [1]. Application of finite element numerical simulation method reduces time and cost in design and trials of the metal powder die compaction processes, as it is able to predict magnitude of density distribution and shear stress concentration occurrence. A number of yield criteria with constitutive equation such as Shima-Oyane model and Drucker-Prager model have been suggested to describe consolidation behaviour that treats powder as continuum materials [2-4]. Materials input parameters necessary in use of numerical simulations are usually estimated by several material testing, such as uniaxial closed-die compaction test, triaxial compaction test and direct shear test, which are commercially used in soil mechanics [5]. However, degrees of die compaction pressure for usual metal powders such as iron for sintered alloy are at least 10 times higher in comparison with soil mechanics and pharmaceutical powder compaction industry. Triaxial compaction method is most preferable to estimate yield surface and constitutive behaviour of granular materials as it can possible to measure both of elastic and plastic properties. Nevertheless, it is hard to develop for metal powders due to high strength in consolidation, so a few multiaxial compaction testing system with high rigidities exists. Practical way of determining key material parameters at low cost is using instrumented die compaction



system with mathematical treatment to convert the applied stresses to a multiaxis stress yield properties [6].

Application of numerical simulation method including shear failure criterions is most effective to predict crack occurrence during compaction, unloading and ejection stage. Drucker-Prager Cap, *DPC* model provides both shear yield behaviour and volumetric deformation with hydrostatic stress for granular materials. When stress state is encountered for critical state corresponding to a high deviatoric stress but a moderate hydrostatic stress, consolidated powder fractures as weak mechanical links between particles cannot resist applied shear stress. Such stress states on compression-shear plane with several compact densities provide critical state line, *CSL*, which is division point in both consolidation and failure occurrence. Although single shear testing is most simple method to estimate shear failure characteristics of granular materials, it was not applicable for metal powder, because at high pressure region the direction of shear plane does not correspond to the shear direction [7]. We have developed improved single shear testing method for metal powder to measure stress state in shear failure plane on high density state [8]. As results, *CSL* of iron powder compact with corresponding density ratio has been successfully estimated as described in previous work [9].

In this study, material input parameters in *DPC* model is estimated to carry out numerical simulation of iron powder compaction process using commercially available finite element method (FEM) solver ANSYS. Firstly, *CSL* estimated in previous work is converted in the stress plane with first invariant stress I_1 and second invariant deviatoric stress J_2 . Then shear failure lines in *DPC* model is estimated with *CSL* and uniaxial failure stress state of green compact. For the identification of compaction yield surface in *DPC* model, lateral force measurement was conducted using newly devised instrumented die compaction system. Thus the whole of yield surface is able to determine by combining both stress state, shear failure occurrence and compaction proceeding. Elastic properties are also identified by measuring unloading characteristics, so variations of volumetric plastic strain during compaction were calculated and compaction hardening characteristics has been identified. Finally, FEM simulation of cylindrical die compaction using commercial available software ANSYS has been carried out and validated.

2. Definition of yield surface in DPC model

2.1. Shear failure line

Yield criterion of *DPC* model in ANSYS is a function of stress invariants I_1 , J_2 and J_3 . Shear failure function Y_s is given by:

$$Y_s(I_1, \sigma_c) = \sigma_c - A \exp(BI_1) - \alpha I_1 \quad (1)$$

Where σ_c is the cohesion related hardening parameter with consolidation. A , B and α are material parameter which defines shear yield stress respectively. Equation (1) means that shear failure envelope is simplified as linear relationship with σ_c and α for $A = B = 0$ as shown in figure 1. So, shear yield surface is expressed as conical shape having center axis of mean stress σ_m in three principal stress space. α means slope of shear failure line.

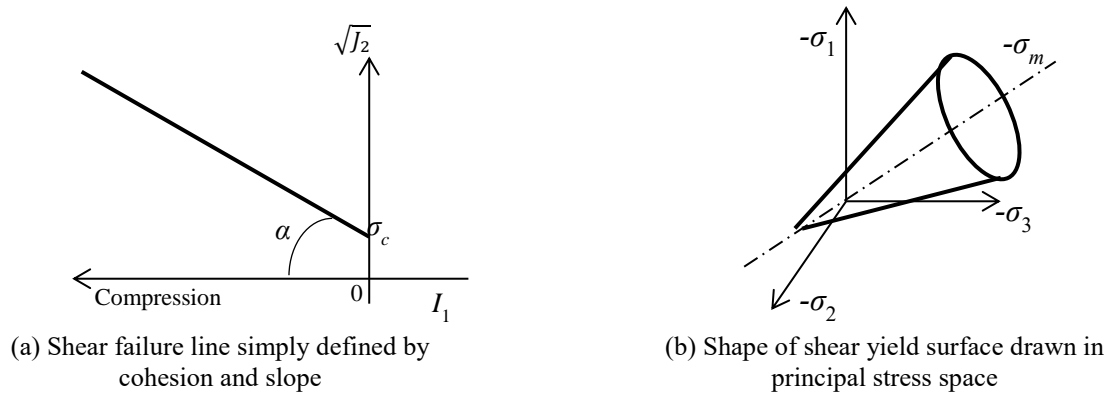


Figure 1. Definition of shear failure criterion in DPC model

2.2. Compaction yield line

The Compaction cap function Y_c is formulated using the shear failure function as:

$$Y_c(I_1, K_0, \sigma_0) = 1 - H(K_0 - I_1) \left(\frac{I_1 - K_0}{R_c Y_s(K_0, \sigma_0)} \right)^2 \quad (2)$$

where: H means Heaviside function, R_c = Ratio of elliptical I_1 to J_2 axis, and K_0 = Key flag indicating the current transition point at which the compaction cap surface and shear failure surface intersect. σ_0 expresses subsequent yield surface which is defined as evolution of cohesion. Note that the Heaviside function roles as follows;

1. When I_1 is greater than K_0 , the compaction cap takes no effect on yielding. It means that only shear failure function Y_s is effective.
2. When I_1 is less than K_0 , the yielding may only happen in the compaction cap portion, which is shaped by both the shear failure function Y_s and cap function Y_c as expressed by equation (2).

2.3. Hardening functions

The cap hardening law is defined by describing the evolution of the parameter X_0 , which is the current intersection point of the compaction cap and the I_1 axis. In other words, hardening is expressed by increasing volumetric plastic strain ε_v^p with consolidation proceeding that is defined as:

$$\varepsilon_v^p = W_1^c [\exp\{D_1^c(X_0 - X_i)\} - 1] \quad (3)$$

where: X_i = Initial value of X_0 at which the cap function takes effect. W_1^c = Maximum possible plastic volumetric strain which is defined from densification curve of the powder material. D_1^c = Hardening parameter. So that the evolution of K_0 provides the isotropic hardening of compaction cap as state variable, which can be implicitly described using X_0 and σ_0 given below:

$$K_0 = X_0 + R_c Y_s(K_0, \sigma_0) \quad (4)$$

2.4. DPC model in $I_1 - \sqrt{J_2}$ plane

According to above yield functions with some assumptions in theory of plasticity, DPC yield function is described as follows;

$$F(I_1, J_1, J_2, J_3, K_0, \sigma_0) = J_2 - Y_c(I_1, K_0, \sigma_0) Y_t(I_1, \sigma_0) Y_s^2(I_1, \sigma_0) \quad (5)$$

where Y_t is expansion cap function defined similar to equation (2). The yield surface with subsequent yield locus can be drawn schematically in $I_1 - \sqrt{J_2}$ plane as shown in figure 2.

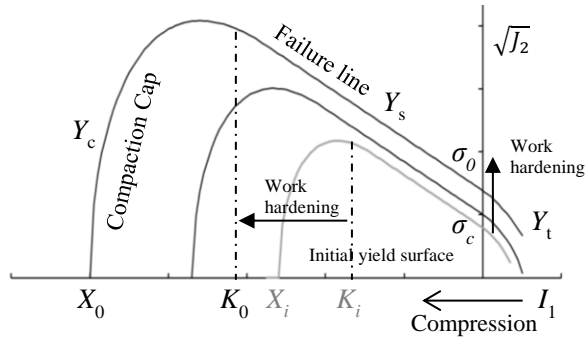


Figure 2. Yield locus of DPC model:

Consists of three segments which are different yield functions, unite to express both of shear failure and consolidation characteristics in powder materials. Work hardening implemented by state variable K_0 with volumetric plastic strain ε_v^p and evolutions of cohesion σ_0 defines subsequent yield condition.

Thus, the materials input parameters are summarized as follows; R_c : Compaction cap parameter, X_i : Compaction cap yield pressure, σ_c : Cohesion yield parameter, α : Shear envelope linear coefficient, W_1^c : Limiting value of volumetric plastic strain, D_1^c : Hardening parameter respectively.

3. Experimental procedures

3.1. Estimation of input parameters

Three kind of material testing is suggested in this study, single shear test, uniaxial failure test, and uniaxial closed-die compaction test with lateral force monitoring using newly devised instrumented die compaction system. Uniaxial failure test of green compact provides a point of stress state in failure by uniaxial principal stress ($\sigma_1, \sigma_2 = \sigma_3 = 0$). The uniaxial failure stress state is obtained as $\sqrt{J_2} = \sigma_1/\sqrt{3}$ and $I_1 = \sigma_1$ at current compact density. Single shear test of green compact provides CSL . Since CSL has been identified in previous work [9], we are able to estimate K_0 directly. Therefore, slope of failure lines α and evolution of cohesion σ_0 with densification proceedings can be estimated by equation (1).

Lateral force measurement provides a point of stress state in densification by simple triaxial principal stresses ($\sigma_1, \sigma_2 = \sigma_3 = \sigma_L$, σ_L : Lateral pressure) during compaction. Therefore the stress state is given by:

$$I_1 = \sigma_1 + 2\sigma_L \quad (6)$$

$$\sqrt{J_2} = |\sigma_1 - \sigma_L|/\sqrt{3} \quad (7)$$

3.2. Experimental conditions

Conventional atomized iron powder (Kobe Steel, Ltd. 300M) with 1.0 mass % zinc stearate which is an additive as interparticle lubricant has been used. It is necessary to prepare green compact specimens with proper density for both of uniaxial failure test and single shear test. The specimens in its density ratio ρ , defined as the ratio of the compact density to material density, of 0.70, 0.75, 0.80 and 0.85 was used. The shape of the specimens is cylindrical in 10 mm diameter with 15 mm height for uniaxial failure test, and 20 mm diameter within 10 mm height for single shear test.

Principle of the newly devised instrumented die compaction system is shown in figure 3. The green compact which shape is square column with 8 mm \times 8 mm sectional dimension can be compacted. Compaction load and lateral force during compaction including unloading stage were measured by load cells. Punch strokes were adjusted to obtain green compacts of 0.70, 0.75, 0.80 and 0.85 in density ratio after ejection respectively. Weight of powder was fixed in 3.5 g so that about 8 mm square cube shape is obtained at $\rho = 0.85$. Compression punch velocity is 0.5 mm/s and punch strokes is stopped for 5 sec before unloading with 0.1 mm/s in each conditions.

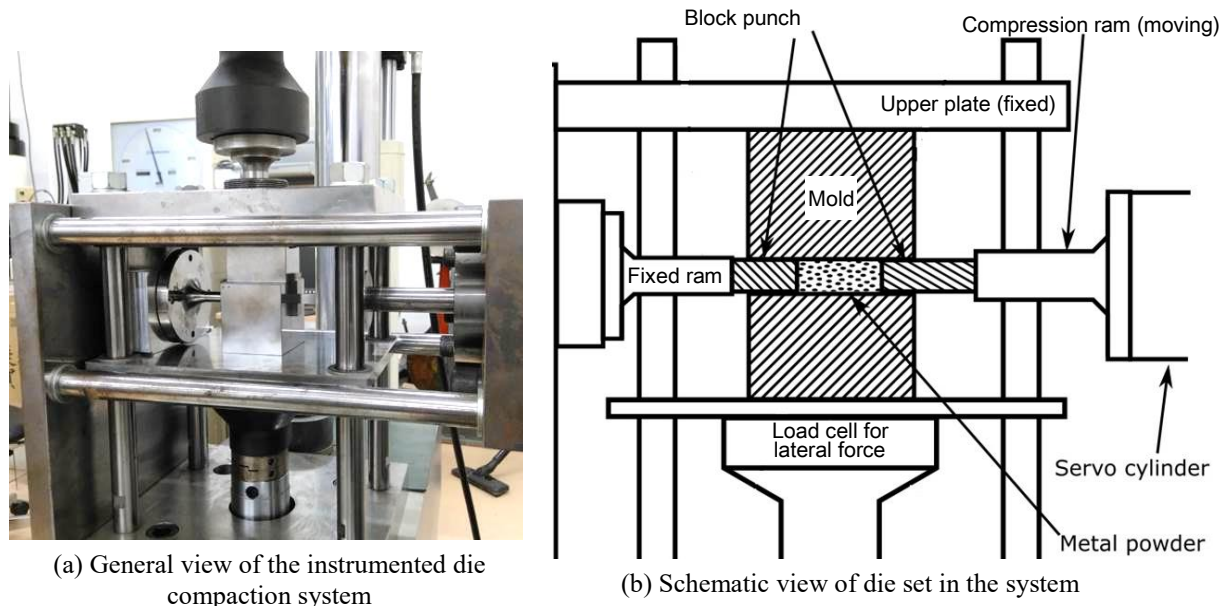
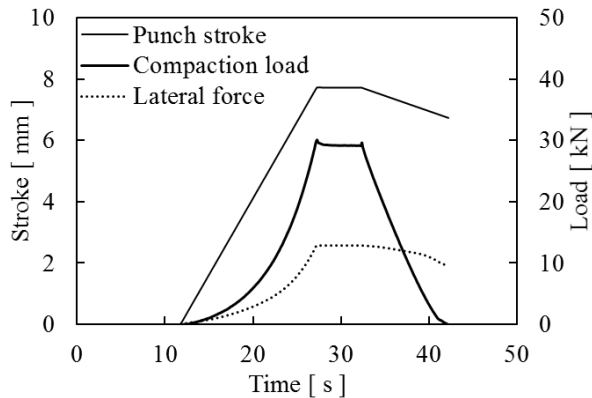


Figure 3. Principle of instrumented die compaction system

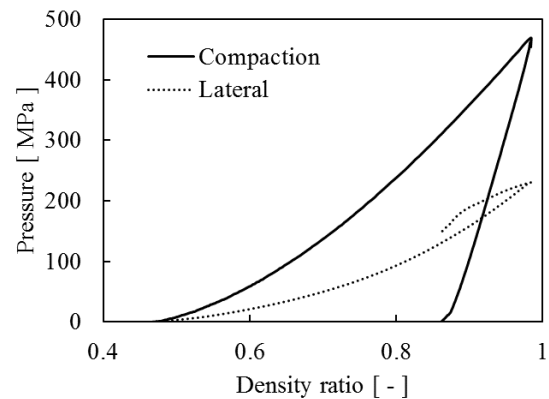
4. Results and discussions

4.1. Estimation of compaction cap surface by lateral force measurement

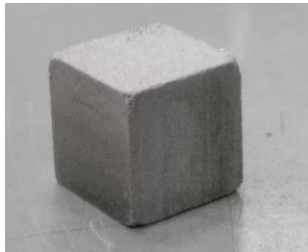
Figure 4 shows results of instrumented die compaction test. Lateral force/pressure clearly remains as residual components after unloading as shown in figure 4(a) and (b). As results variation of $\sqrt{J_2}$ which is non-negative value reverses its direction at the point of stress state $\sigma_1 = \sigma_2 = \sigma_3$ during unloading stage as shown in figure 4(d). Further unloading is accompanied by increasing shear stress component with elastic recovery. It is considered that plastic volumetric expansion occurs eventually when $\sqrt{J_2}$ changes beyond the secondary peak, then σ_1 becomes 0. These loading properties seems to be the same for each density ratio. Stress state corresponding to the compaction cap surface is indicated at the maximum compaction stress for each density ratio.



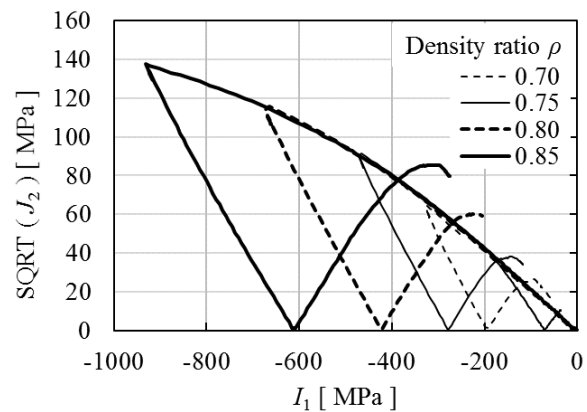
(a) Variations of punch stroke and forces on operation time (Density ratio $\rho = 0.85$)



(b) Variations of compaction and lateral pressure with densification



(c) Photograph of green compact after ejection (Density ratio $\rho = 0.85$)



(d) Densification curve mentioned in $I_1 - \sqrt{J_2}$ plane

Figure 4. Results of instrumented die compaction test

4.2. Estimation of yield surface of DPC model

Concerning results of the estimation of shear failure yield line, the stress states which on the yield lines are plotted and yield surface has been drawn as shown in figure 5. Materials parameters, $R_c = 3.8$ and $\alpha = 0.2$ with proper X_i and σ_c values which are suitable for each plots has been identified. It is considered that estimation of yield surface has been successfully achieved because the all plots are seems to be compatible with the yield functions although these plots were measured by different experiment respectively. This fact is well supported by unloading curve that ends on the *CSL*.

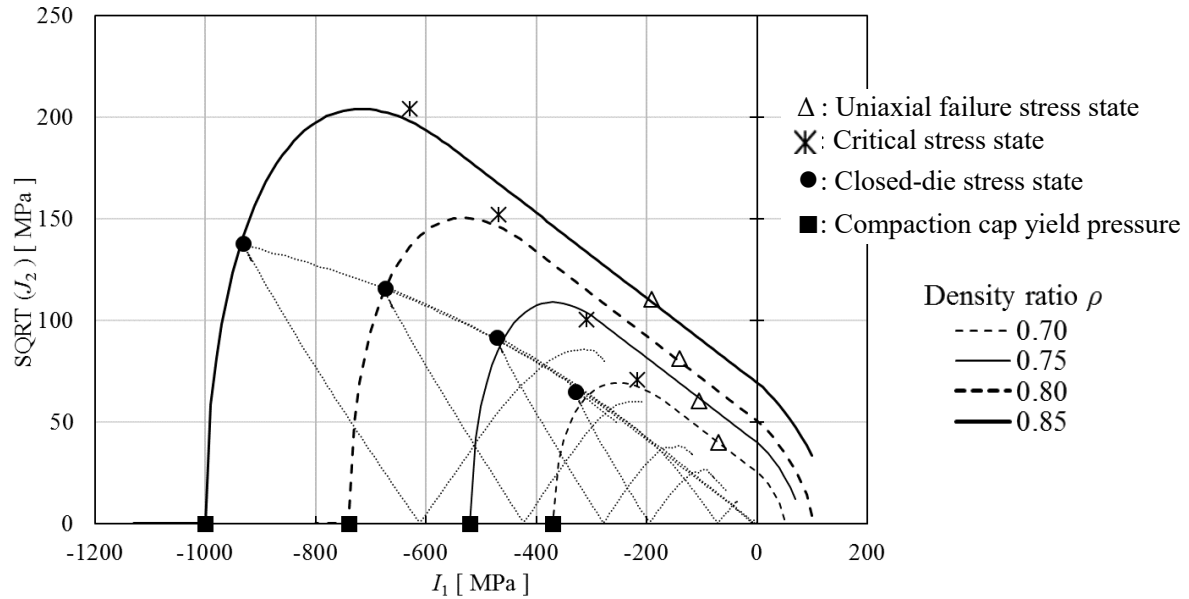


Figure 5. Estimated DPC surfaces for each green compact density

4.3. Cap hardening parameters

Cap hardening parameters has been identified by equation (3) as $W_1^c = 0.6$ and $D_1^c = 0.0045$. Figure 6 shows relationship between I_1 and ε_v^p with experimental result. Unfortunately, there is deviation because the influence of changes in elastic properties is not considered in this study.

4.4. FEM results and validation in densification curve

FEM analysis of cylindrical closed-die compaction has been carried out with the elastic modulus of 4245 MPa as constant value. Figure 7 shows equivalent plastic strain distributions inside of the green compact. Note that one-half of the shape has been shown as axi-symmetric analysis. It seems that the distribution tendencies are fairly simulated as strain concentrate at top corner is clearly recognized in contrast with lower strain observed at bottom corner due to single action compression.

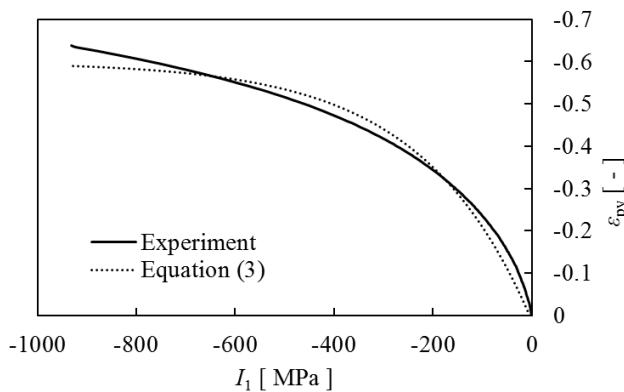


Figure 6. Relationship between I_1 and ε_v^p

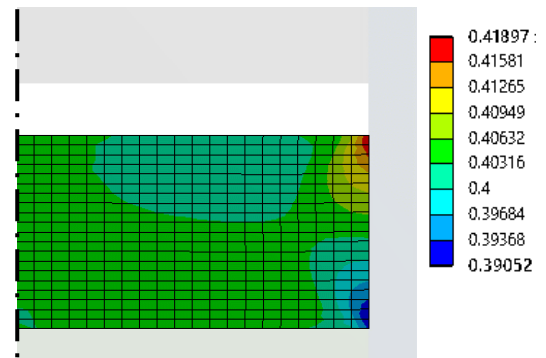


Figure 7. Equivalent plastic strain distributions by FEM analysis

Figure 8 shows variations of compaction pressure with mean density ratio increase. According to the deviation in ε_v^p , it is hard to simulate densification curve at high pressure region precisely. Unloading properties is quite different because of elastic modulus that changes with density in actual compaction.

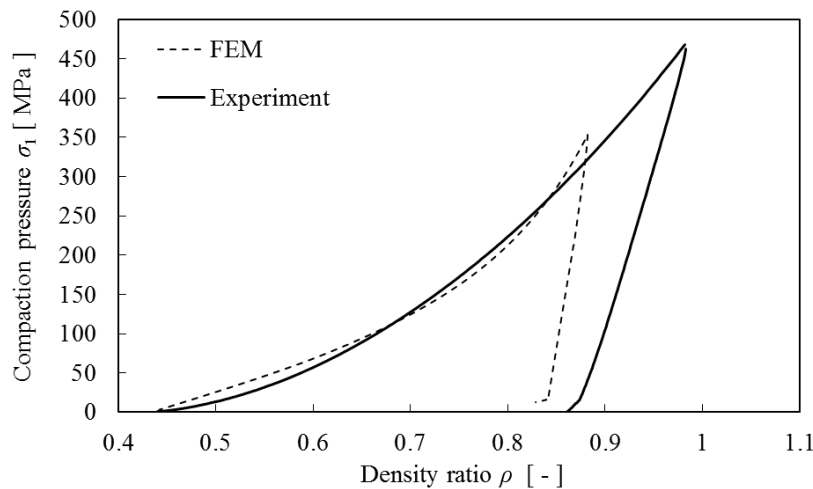


Figure 8. Variations of compaction pressure with density ratio increase

However, it is possible to predict compaction pressure and plastic strain distributions when density ratio is less than 0.88 as shown in figure 7 and figure 8. It seems that estimation of elastic properties becomes effective to optimize cap hardening parameters more precisely.

5. Conclusions

Materials input parameters of iron powder on shear failure yield and compaction cap yield functions for *DPC* model have been estimated from several material testing, uniaxial failure, single shear and lateral force measurement. It has been recognized that the all plots mentioned in $I_1 - \sqrt{J_2}$ plane are compatible with the yield functions so *DPC* yield surface has been successfully identified. Finite element analysis of simple closed-die compaction has been conducted and plastic strain distributions are fairly simulated. Unloading properties should be investigated to optimize cap hardening parameters more precisely.

Acknowledgments

A part of this work was financially supported by JSPS KAKENHI Grant No. 15K13851.

References

- [1] Mosbah P, Shima S, Habraken A and Charlier R 1999 *J. Japan Society of Powder and Powder Metallurgy* **46(7)** 696-704
- [2] Shima S and Oyane M 1976 *Int. J. Mech. Sci.* **18** 285-291
- [3] Shima S and Saleh M 1993 *Mechanics of Materials* **16(1-2)** 73-81
- [4] Drucker DC and Prager W 1952 *Quarterly of Appl. Math.* **10** 157-165
- [5] Sinka I, Cocks A, Morrison C and Lightfoot A 2000 *Powder Metall.* **43(3)** 253-262
- [6] Mosbah P, Bouvard D, Ouedraogo E and Stutz P 1997 *Powder Metall.* **40(4)** 269-277
- [7] Shima S, Kotera K and Nishimura K 1999 *Adv. Tech. of Plasticity* **2** 1333-1338
- [8] Taniguchi Y, Dohda K and Wang Z 2004 *Trans. Japan Society of Mech. Eng.* **70(696)** 2555-2560 (in Japanese)
- [9] Taniguchi Y, Shiota K and Kodama K 2015 *Proc. 6th TSME Int. Conf. on Mech. Eng. (Phetchaburi)* AMM020