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Adaptive Control design for a 3D reservoir model under water coning

Mehdi Safari [1*], Mohammad Javad Ameri [2]

[1] Department of Petroleum engineering, Curtin University, Malaysia

[2] Department of Petroleum engineering, Amirkabir University of Technology, Iran

*Mehdi.safari@curtin.edu.my

Abstract. Prevention of water production is a serious concern in the oil fields as it reduces the oil production rate and makes some troubles for surface facilities. Some researchers suggested boundary control approach to delay the water/gas cone break through. According to the physical nature of petroleum reservoirs, uncertainty is an undeniable fact. Therefore, the control law that has been developed by the researchers is not implementable for practical applications. To this end, the paper proposes to use the adaptive control approach for the reservoir which is described by a nonlinear parabolic PDE with uncertainty in permeability. For this reason, based on direct Lyapunov stability method and adaptive control theory, an adaptive control law is developed for the closed-loop system. Next, a mathematical proof is used to show that the law always guarantees the global stability of the reservoir. Also, simulation results are presented to demonstrate the effectiveness and the performance quality of the presented approach in a general model. Using the proposed approach, production engineers are able to control an oil reservoir in order to prevent water coning without any knowledge about its real permeability.

Keywords: Unknown permeability; Water coning; Adaptive control approach; Lyapunov function

1. Introduction

Rising water-oil contact (WOC) into the perforations of a production well in a cone shape is called water coning [1]. Schematic of the phenomenon is shown in Figure 1. It reduces the oil production rate and makes some troubles for surface facilities and finally leads to early shutdown of the well. So prevention of water coning is an important challenge in the oil production units. Oil production rate is the only controllable factor that may affect water coning.

Muskat and Wyckoff [2] reported the first study about the coning in 1934. They suggested the parameters which may affect the cone movement.

Konieczek [3], based on some assumptions, developed a nonlinear partial differential equation (PDE) as coning equation. After that, a lot of attention was paid to this nonlinear PDE by researchers in the field. An early study on the available potential of using smart well technology was done by Brouwer et al. [4]. A well that is equipped with monitoring sensors and down-hole inflow control valves (ICVs) and instrumentations is called smart well. By controlling the production rates the upward movement of cone-shape can be delayed.

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The first boundary control law for gas coning problem was proposed by [5]. Ronen [6] proved that Sagatun's controller and all other positive flow rate controllers make the one dimensional system stable.

Agus et al. [7-9] investigated the conventional control of the system. Safari et al. proposed an efficient boundary control in one dimension that guaranteed exponential stability of the system [10]. According to the physical nature of petroleum reservoirs, uncertainty is an undeniable fact. Therefore, the control law that has been developed by the researchers cannot be implemented in practical applications. Safari et al. [11] designed an adaptive control law in one dimension that guaranteed asymptotic stability of model. But no one has proposed an adaptive control law for 3D reservoir model. In this paper, we focus on the design of an adaptive control for the 3D model in the presence of uncertainty such that it guarantees the global stability of the system.

2. Problem formulation

Dynamic behavior of oil-water interface is described by a nonlinear PDE as follow [3, 8, 11].

$$u_t = k\nabla.(u\nabla u) \tag{1}$$

$$u(0,t)\nabla u(0,t).d\vec{n} = \frac{1}{k}Q(t) \tag{2}$$

$$\nabla u(1,t).d\vec{n} = au(0,t) \tag{3}$$

Where the system state $u(x,t)$ depicts the thickness of oil column at a spatial point $x=(x_1,x_2)\in A$ at time t. k denotes the reservoir permeability that is an unknown parameter here. a is a known constant. Following is the initial condition

$$u(x,0)=u_0 \qquad \forall x\in[0,1]\times[0,1]$$

The objective is preventing the reservoir from water production.

To develop the model of oil-water interface the following assumptions are considered; a rectangle that is isotropic and homogeneous with respect to porosity and permeability, no capillary pressure (a well-defined Water-Oil contact (WOC)), constant PVT (pressure-volume-temperature) properties of the fluid (isothermal condition), a strong bottom aquifer with constant pressure in time and space (Horizontal aquifer pressure gradient is neglected) and Darcy's flow from the gravity-based pressure gradient is the only responsible for fluid movement [11].

3. Adaptive boundary control

Using the Lyapunov method a control law for the well production rate $Q(t)$ is derived as follows.

Theorem

For the given system (1)-(3), if the following adaptive control law is applied

$$Q(t)=\frac{2\hat{k}u(1,t)^2\nabla u(1,t).d\vec{n}}{u(0,t)}+\alpha u(0,t)^{\beta} \tag{4}$$

where α and $0\le\beta\le 1$ are constant control gains and $\hat{k}(t)$ is an estimated feedback gain updated using the following adaptation law

$$\dot{\hat{k}}=\gamma u^2(0,t)\nabla u(0,t).d\vec{n} \tag{5}$$

Then, the zero solution of the system state (5) is globally stable.

Proof.

The Lyapunov function is chosen as follows:

$$V(t) = \iint_S u(x,t)^2 dA + \frac{1}{\gamma}(k-\hat{k})^2 \quad (6)$$

Taking time derivative of the Lyapunov function and substituting system equations (1)-(3) yields

$$\dot{V}(t) = \iint_S 2k\nabla.(u\nabla u)udA + \frac{2\dot{\hat{k}}}{\gamma}(k-\hat{k}) \quad (7)$$

Taking integral by parts yields

$$\dot{V}(t) = 2ku(1,\mathrm{t})^2\nabla u(1,t).d\vec{n} - 2ku(0,\mathrm{t})^2\nabla u(0,t).d\vec{n} - \iint_S 2ku\left|\nabla u\right|^2 dA + \frac{2\dot{\hat{k}}}{\gamma}(k-\hat{k}) \quad (8)$$

where $\vec{n}$ is the outward unit normal vector.

Substituting (2) and (3) into (8) yields

$$\dot{V}(t) = 2kau(1,\mathrm{t})^2 u(0,t) - Qu(0,\mathrm{t}) - \iint_S 2ku\left|\nabla u\right|^2 dA - \frac{2\dot{\hat{k}}}{\gamma}(k-\hat{k}) \quad (9)$$

Therefore, $\dot{V}(t)$ can be rewritten as

$$\dot{V}(t) = -Qu(0,\mathrm{t}) + (-u(1,\mathrm{t})^2\nabla u(1,\mathrm{t}).d\vec{n} + \frac{\dot{\hat{k}}}{\gamma})2(-k+\hat{k}) + 2\hat{k}u(1,\mathrm{t})^2\nabla u(1,\mathrm{t}).d\vec{n} - \iint_S 2ku\left|\nabla u\right|^2 dA + \alpha u(0,t)^{1+\beta} - \alpha u(0,t)^{1+\beta} \quad (10)$$

After canceling the first, third and fifth terms of right hand side, the following expression is obtained (4). Also canceling the second term results in (5).

4. Model description

In this section, numerical simulation results are presented to verify and demonstrate the effectiveness of the proposed adaptive control laws. The finite difference scheme is used to solve the system with time step size of 10 days. For the adaptive control law (Equation 4) initial permeability is chosen as k_1= 10 md. Properties of the synthetic model are reported in Table 1. Geometry of the model and coning issue is shown in Figure 1.

Table 1. Synthetic model properties

parameters	Value	Unit
Horizontal permeability	20	md
Vertical permeability	8	md
Porosity of model	0.2	fraction
Viscosity of oil	0.44	cp
Density of oil	794.38	Kg/m^3
Density of water	1000	Kg/m^3
Length of Reservoir	700	m
Thickness of Reservoir	80	m
Initial oil column	70	m

Grid block number	35*35*90	-

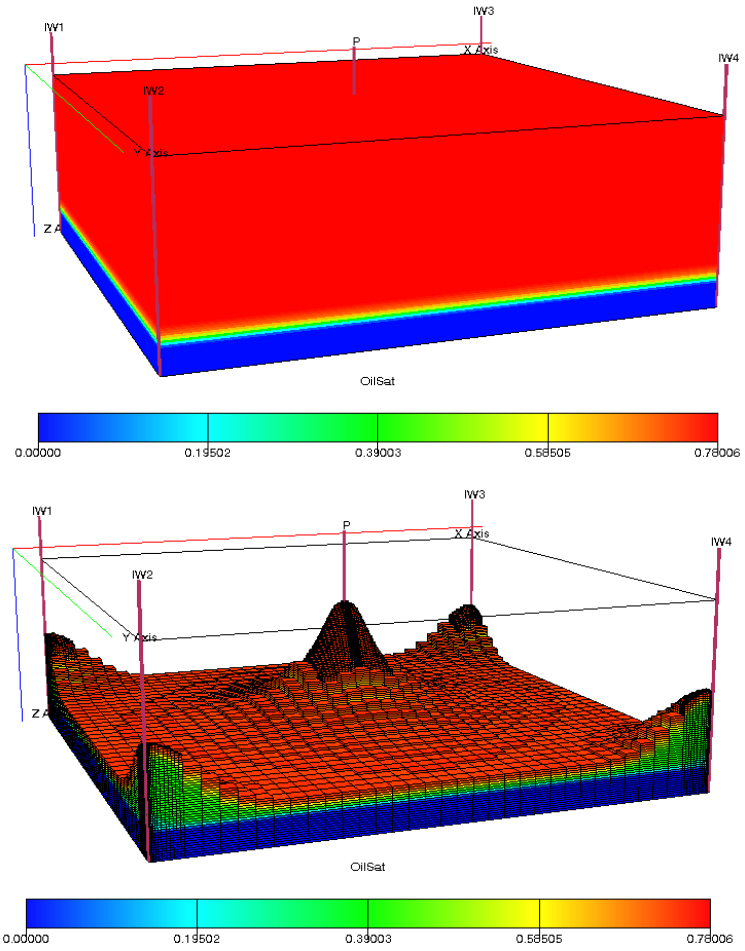


Figure 1. Geometry of the model and coning phenomenon

5. Simulation study

Figure 2 shows that the well flow rate is controlled automatically. As shown, the production controlled rate started at 400 m^3/day, and then it decreased slightly and finally reached 50 m^3/day.

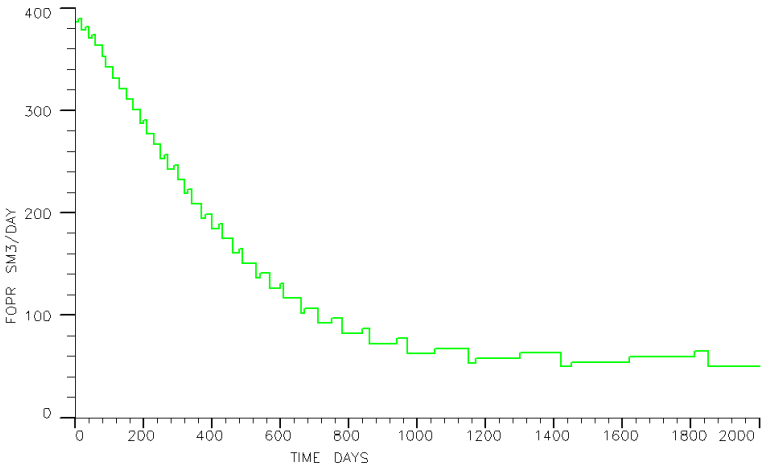
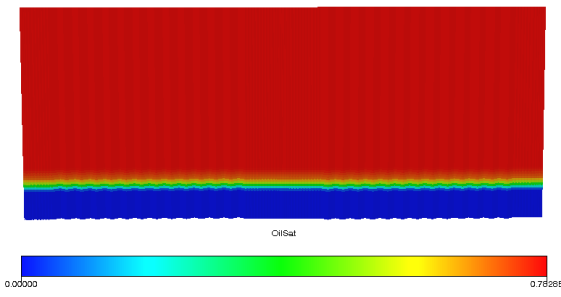
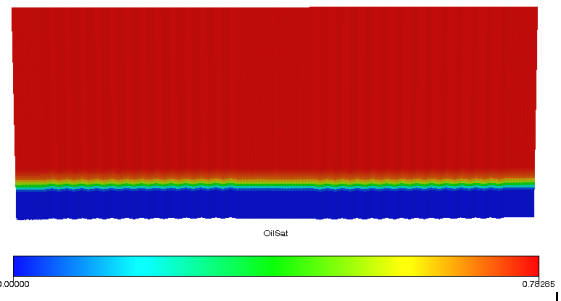
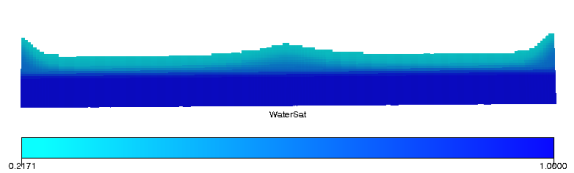
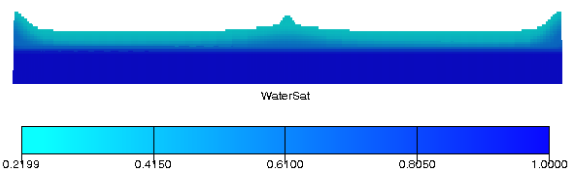
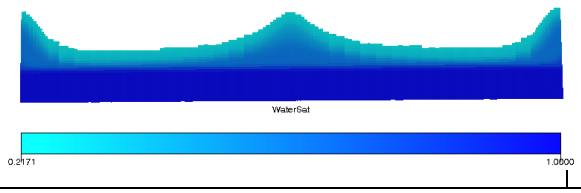
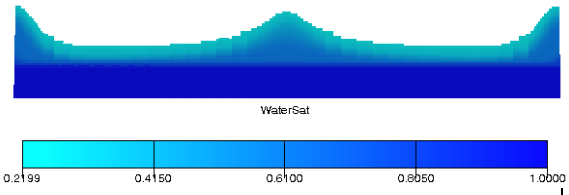
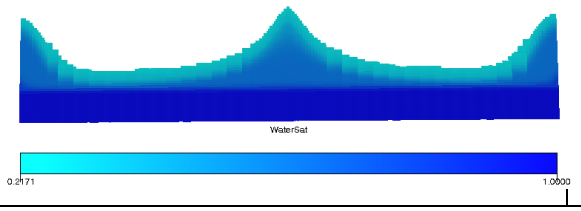
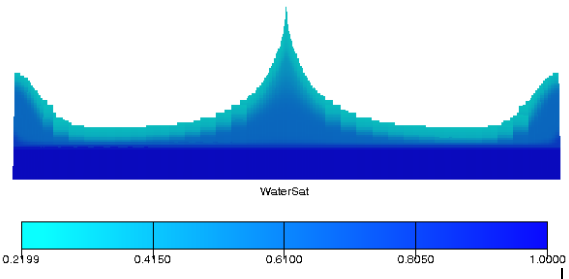


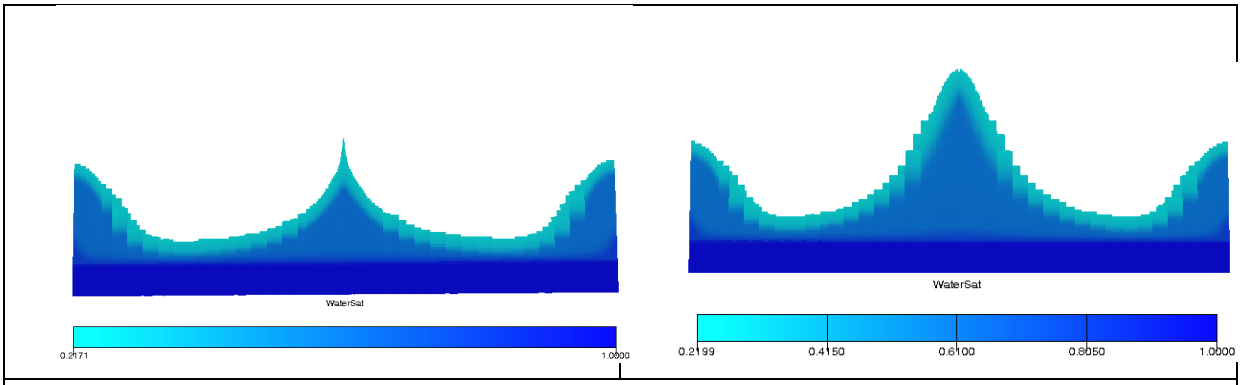
Figure 2. The controlled field oil production rate

Figure 3 depicts the shape of cone along the entire of the model during the production life cycle. It demonstrates that the cone extension decreases versus time due to the selected closed-loop control model. This occurs because as soon as the cone height starts to increase, the controller commences to close the chock valve, so oil production rate reduces (Figure 2).

As shown in this figure, system state approaches zero in the whole spatial domain. It means that the entire oil column can be produced in the presence of water coning in the reservoir, and consequently the goal is achieved.

From Figure 4, it is found that the unknown parameter can be updated to help achieving the problem goal. So, it can be concluded that based on the approach, it is possible to control a system with unknown parameters.

Controlled case	Uncontrolled case
	
Before production	
	
Time step=10	
	
Time step=30	
	
Time step=60	



Time step=200

Figure 3. cross section of water cone between injection and production wells during production life cycle for controlled and uncontrolled cases

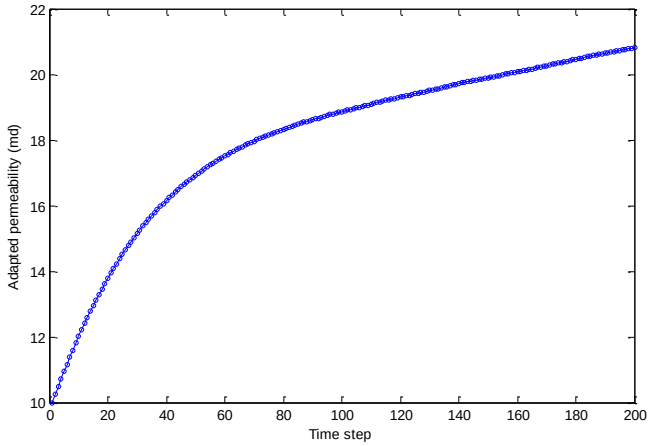


Figure 4. parameter adaptation versus simulation time

Figure 5 depicts well water cut. As shown, no water is produced within simulation time, demonstrating the stability of the system.

Figure 6 shows the production well bottom hole pressure. As production rate decreases bottom hole pressure increases.

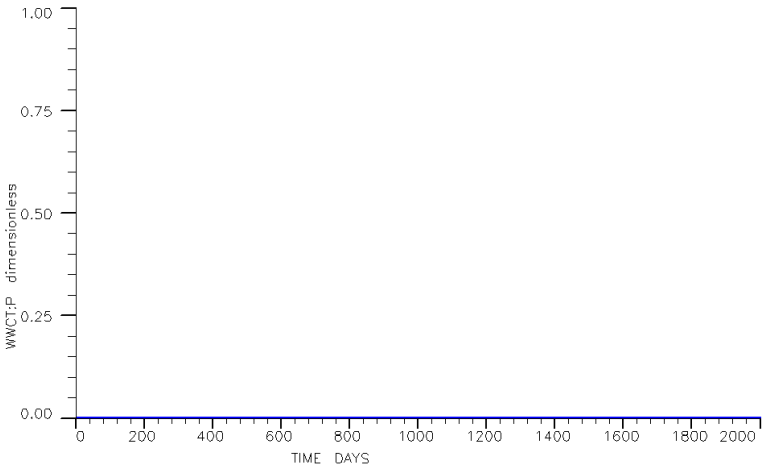


Figure 5. Lyapunov function versus time

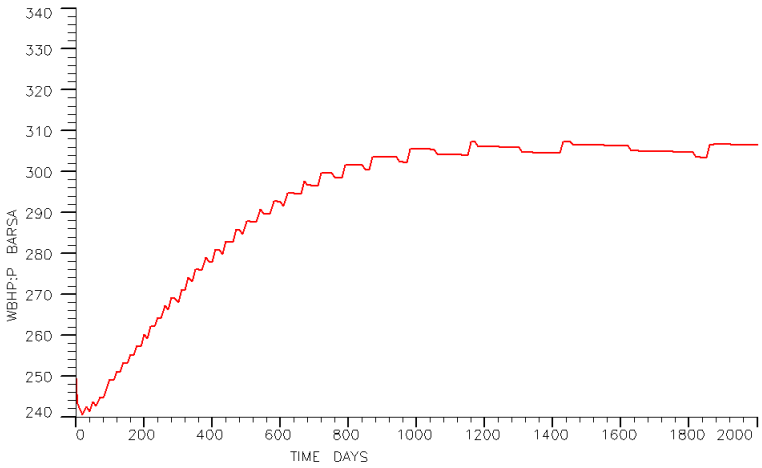


Figure 6. Lyapunov function versus time

6. Conclusion

This study presents an adaptive control law and a parameter estimation update law for a 3D reservoir model under water coning issue. It was mathematically proved that the governing equation of water coning phenomenon is globally stable using the aforementioned control and adaptation laws. Also, based on this approach, oil reservoirs under water coning issue are controlled without knowledge about real permeability.

Nomenclature

Thickness of oil column	u
spatial parameter	x
Time	t
Permeability	k
Normal surface vector	$d\vec{n}$
flow rate	Q
External boundary constant	a
Initial length	u_0
Estimated permeability	$\hat{k}(t)$
Controller gains	α & β
Constant	γ
Area	A
Domain	S
Minimum of u	u_w

References

[1] Ahmed, T., *Reservoir engineering handbook*. 3rd Edition ed. 2006, Burlington: Gulf Professional Publishing.

[2] Muskat, M. and R. Wycokoff, *An approximate theory of water-coning in oil production.* Transactions of the AIME, 1935. **114**(01): p. 144-163.

[3] Konieczek, J. *The concept of critical rate in gas coning and its use in production forecasting*. in *SPE Annual Technical Conference and Exhibition*. 1990. New Orleans: Society of Petroleum Engineers.

[4] Brouwer, D.R., et al., *Recovery Increase through Water Flooding with Smart Well Technology*, in *SPE European Formation Damage Conference*. 2001, Society of Petroleum Engineers: The Hague.

[5] Sagatun, S.I., *Boundary control of a horizontal oil reservoir.* SPE Journal, 2010. **15**(04): p. 1,020-1,027.

[6] Ronen, J., Boundary Control of the Gas Coning Problem. 2010, Institutt for teknisk kybernetikk.

[7] Hasan, A., *Optimization and Control of Petroleum Reservoirs.* 2013.

[8] Hasan, A., B. Foss, and S. Sagatun, *Flow control of fluids through porous media.* Applied mathematics and computation, 2012. **219**(7): p. 3323-3335.

[9] Hasan, A., B. Foss, and S. Sagatun, *Optimization of oil production under gas coning conditions.* Journal of Petroleum Science and Engineering, 2013. **105**: p. 26-33.

[10] Safari, M., M.J. Ameri, and A. Naderifar, *An Efficient Boundary Control for Porous Media Equation: Motivated by Water Coning Problem.* The Canadian Journal of Chemical Engineering.

[11] Safari, M., M.J. Ameri, and A. Naderifar, *Adaptive Control Design for a Nonlinear Parabolic PDE: Application to Water Coning.* The Canadian Journal of Chemical Engineering.