

PAPER • OPEN ACCESS

Stability Region of Integrating Process based on Routh- Hurwitz Stability Criteria

To cite this article: B Ugon and J Nandong 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **495** 012048

View the [article online](#) for updates and enhancements.



IOP | ebooks™

Bringing you innovative digital publishing with leading voices to create your essential collection of books in STEM research.

Start exploring the **collection** - download the first chapter of every title for free.

Stability Region of Integrating Process based on Routh-Hurwitz Stability Criteria

B Ugon¹ and J Nandong²

¹ Department of Foundation in Engineering and Science, Faculty of Engineering and Science, Curtin University, Miri 98009, Sarawak, Malaysia

² Department of Chemical Engineering, Faculty of Engineering and Science, Curtin University, Miri 98009, Sarawak, Malaysia

Email: bejay@curtin.edu.my

Abstract. This paper presents a new tuning method for P and PID controller in a double-loop feedback structures. The stability region is first determined by Routh-Hurwitz stability criteria. Simple tuning rules are established to determine the optimal P/PID parameter values within the stability regions, to meet desired performance-robustness criteria. The effectiveness of the newly established tuning rules are evaluated via numerical study and compared with those based on the same existing PID tuning methods.

1. Introduction

Routh-Hurwitz criteria for stability analysis can be used to determine how many closed-loop system poles are on the left half-plane, and/or on the right half-plane of a complex plane. The method also yields stability information without the need to solve for the close-loop system poles, i.e., the characteristics equation. In the present research, we use the Routh-Hurwitz criteria to establish stability regions of PID controller for a class of second-order integrating processes with time-delays. In process industry, many real systems possess integrating behaviours, such as a high purity distillation column, and storage tank with a pump at the outlet. Integrating processes are non-self-regulating, which means that when subjected to an input change, the process will not attain a new steady state condition. These processes are often difficult to control compared with self-regulating processes

Note that several different PID tuning methods have been proposed by different researcher in order to control integrating systems, e.g., methods that related to stability analysis. Among them are method proposed by Luyben [1]-[2], Chidambaram [3]. A computational method by Hermite-Biehler theorem [4], [5] on stabilizing parameter region of PI and PID controller was also proposed. Others methods include stability boundary locus approach by Tan et al. [6], which determines the PID parameter plane that will stabilize the system, Nyquist plot approach by Söylemez et al. [7], and Munro and Söylemez [8] based on finite value real-axis crossing the Nyquist plot of the system with test compensator with real values of frequency and corresponding gains.

Based on the PID stability theorem developed by Seer and Nandong [9] we propose to develop stability regions for P and PID controllers applied to a double-loop control structure (Figure 1). Since PID stability theorem in [9] is based on unstable process model, while in this research we use integrating process as process model. So we use the theorem in [9] as a guide to find new parameter regions for PID controller for the integrating process. First, we determine the controller gain (of a P-



only controller) in the inner feedback loop within, which a tuning value is obtained. Upon completing the tuning of the inner loop, an augmented plant, i.e., pre-stabilized system is formed based on which the outer (PID) controller is designed. Second, the overall system stability is to be addressed via the Routh stability criteria [10]. Stability regions are obtained using the PID stability theorem; tuning values of the PID controllers are obtained within the established stability regions.

2. Integrating Process Stability

2.1. PID Controller Design

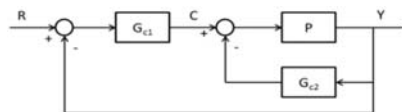


Figure 1. The double-loop structure of feedback control scheme.

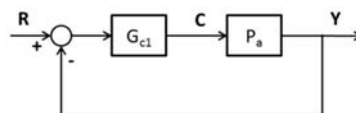


Figure 2. The equivalent single-loop structure of double-loop control scheme.

Figure 1 the block diagram of double-loop control structure. There are two controllers involved: the inner controller (G_{c2}) is used to provide pre-stabilization of the process or system P. Meanwhile, the outer controller (G_{c1}) is used to provide an overall performance specification for the system. The double-loop control structure in Figure 1 can be reduced to a standard single-loop control structure shown in Figure 2. Here, P_a denotes the pre-stabilized plant of system P.

Let us consider a class of process given by the second-order integrating plus dead-time model as follows

$$P(s) = \frac{K_p e^{-\theta s}}{s(\tau s + 1)} \quad (1)$$

Hence, K_p , τ and θ represent the process gain, time constant and dead-time for the integrating process in (1) respectively.

The double-loop consists of two controllers: a primary controller G_{c1} and secondary controller G_{c2} . G_{c1} is chosen to be a PID controller given by

$$G_{c1}(s) = K_c \left(1 + \frac{1}{\tau_I s} + \tau_D s \right) \quad (2)$$

where K_c , τ_I and τ_D denote the controller gain, reset time and derivative time respectively.

Note that, for the purpose of conducting stability analysis on the secondary loop, the dead-time term is approximated using the first-order Taylor series, i.e.:

$$e^{-\theta s} \cong 1 - 2\alpha s, \quad \alpha = \frac{\theta}{2} \quad (3)$$

2.2. Inner-loop Controller Design

Figure 2 shows that the augmented plant (transfer function) from C to Y is given by

$$P_a(s) = \frac{G_{c2}}{1 + G_{c2}P(s)} \quad (4)$$

From equation 4, the closed loop characteristics equation for the secondary loop is given as follow

$$1 + G_{c2}P(s) = 0 \quad (5)$$

By approximating the dead-time as in (3), the close-loop characteristics equation (5) can be expressed in a polynomial form

$$\tau^2 + s + K_{c2}K_p(1 - \alpha s) = 0 \quad (6)$$

Based on the necessary condition of Routh stability, for the inner loop stability, the secondary loop gain must be bounded

$$0 < K_{c2}K_p < \frac{1}{2\alpha} \quad (7)$$

where the secondary loop gain is $K_{L2} = K_{c2}K_p$. Equivalently, the range in (7) can be written in the following form

$$K_{L2} = r_{p2} \left(\frac{1}{2\alpha} \right), \quad r_{p2} \in (0,1) \quad (8)$$

Here, r_{p2} is a dimensionless parameter with a value lying between 0 and 1. This parameter can be used for tuning the inner controller gain.

The augmented plant (4) can now be expressed as a second-order process model as follow

$$p_a(s) = \frac{K_p^0 e^{-\theta s}}{a_2 s^2 + a_1 s + 1} \quad (9)$$

It can be readily shown that, $K_{c2}K_p = r_{p2} / (2\alpha) = K_{L2}$, which is a positive scalar parameter used for tuning the value of loop gain $K_{c2}K_p$. The scaling parameter must lie in the range of $r_{p2} \in (0,1)$ to ensure closed-loop stability of the inner loop. Note that, K_p^0 , a_2 and a_1 are given in the forms of

$$K_p^0 = \frac{2K_p\alpha}{r_{p2}} \quad (10)$$

$$a_2 = \frac{2\tau\alpha}{r_{p2}} \quad (11)$$

$$a_1 = \frac{2\alpha(1 - r_{p2})}{r_{p2}} \quad (12)$$

2.3. Overall Controller

The closed-loop set-point tracking transfer function based on Figure 2 can be written as

$$P_O(s) = \frac{G_{cl}P_a(s)}{1 + G_{cl}P_a(s)} \quad (13)$$

By selecting, the primary controller as in (2), the overall close-loop characteristic equation is

$$1 + G_{cl}P_a(s) = 0 \quad (14)$$

Let $K_c K_p^0 = K_{L1}$ and (13) be written as follow

$$B_3 s^3 + B_2 s^2 + B_1 s + B_0 = 0 \quad (15)$$

where the coefficients of the equations are

$$\begin{aligned} B_3 &= \tau_I (a_2 - 2K_{L1} \tau_D \alpha) \\ B_2 &= \tau_I [a_1 + 2K_{L1} (\tau_D - \alpha)] \\ B_1 &= \tau_I + K_{L1} (\tau_I - 2\alpha) \\ B_0 &= K_{L1} \end{aligned} \quad (16)$$

In the Routh array, the necessary elements to be determined are as follows

$$b_1 = \frac{B_2 B_1 - B_3 B_0}{B_2} \quad (17)$$

$$c_1 = B_0 = K_{L1} \quad (17a)$$

From the necessary condition, the coefficient of s^3 , s^2 and s lead to an upper limit (18) and two lower limits (19)-(20) for the primary loop gain:

$$K_{L1} < \bar{K}_{L1} = \frac{a_2}{2\alpha\tau_D} \quad (18)$$

$$K_{L1} > \underline{K}_{L1} = -\frac{a_1}{\tau_D - 2\alpha} \quad (19)$$

$$K_{L1} > \underline{K}_{L1} = -\frac{\tau_I}{\tau_I - 2\alpha} \quad (20)$$

The above concludes that according to the necessary condition, the close-loop is stable if and only if the loop gain in (18) is bounded

$$0 < K_{L1} < \frac{a_2}{2\tau_D\alpha} \quad (21)$$

Let us rearrange (21), so that it becomes

$$K_{L1} = r_{p1} \left(\frac{a_2}{2\tau_D\alpha} \right) ; r_{p1} \in (0,1) \quad (22)$$

where r_{p1} is another dimensionless tuning parameter.

2.4. Sufficient Condition for Closed-loop Stability

The element of b_1 from the Routh array (16) provides a set for sufficient criterion of Routh stability, from which one can establish another lower limit of τ_I given by

$$\tau_I > \tau_{i\min} = \left(\frac{K_{L1}}{1 + K_{L1}} \right) \left(2\theta + \frac{a_2 - K_{L1}2\tau_D\alpha}{a_2 + K_{L1}(\tau_D - 2\alpha)} \right) \quad (23)$$

Equation (23) is valid provided that $\tau_D > \alpha$. With this condition, we suggest the tuning for the value of derivative time be done via two simple equations

$$\tau_D = 2r_d\alpha, \quad r_d > 1 \quad (24)$$

and

$$\tau_I = r_i \max\{2\alpha, \tau_{i\min}\}, \quad r_i > 1 \quad (25)$$

In summary, we have 4 dimensionless tuning parameters based on [9], for the double-loop control system with a combination of P and PID controllers. These are r_{p2} , r_{p1} , r_i and r_d .

3. Result and Discussion

Example 1

Let us consider the second-order integrating process with dead-time (1), and compare the proposed method with that from the work in Jin and Liu [11]:

$$P(s) = \frac{0.2e^{-s}}{s(4s+1)} \quad (26)$$

The performance of the proposed double-loop P/PID scheme is compared with that of the single-loop PID scheme, tuned using two different methods: method by Jin and Liu [11] and MoReRT method [12]. The tuning values are shown in table 1.

TABLE I. CONTROLLER PARAMETER VALUES FOR EXAMPLE 1

Method	K_c	τ_I	τ_D	K_{c2}
Proposed	4.210 5	3.7	3.8	2.5
MoReRT	3.75	12	2.667	-
Jin and Liu ^a	2.378	13.741	2.835	-

$$^a \text{Set point filter} = \frac{1}{\tau_I \tau_d s^2 + \tau_I s + 1}$$

In the example 1, the proposed method shows better performance in terms of Integral Absolute Error (IAE) value than the others two methods as shown in Figure 3. We set the values as $r_{p1} = 0.8$, $r_{p2} = 0.5$, $r_i = 3.7$ and $r_d = 3.8$. The introduction of set point filter indeed increases the performance in term of IAE as shown in figure 3. Figures 3 shows that the nominal response for 1 unit setpoint change followed by 1 unit in output disturbance change. The proposed double-loop control method shows the smallest IAE. Please note that, a 20% modeling error is also introduced (to simulate a perturbed condition) to the deadtime in (26). Figure 4 shows the response under this perturbed condition. The proposed double-loop control still outperforms the methods of Jin and Liu [11] and MoReRT method [12].

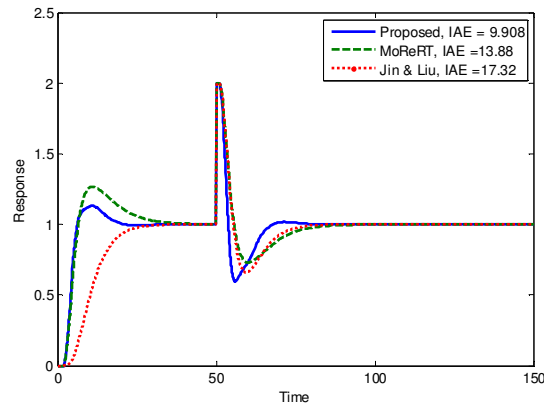


Figure 3. Step response proposed method, [11] method and [12] method under normal condition for the close-loop system example 1.

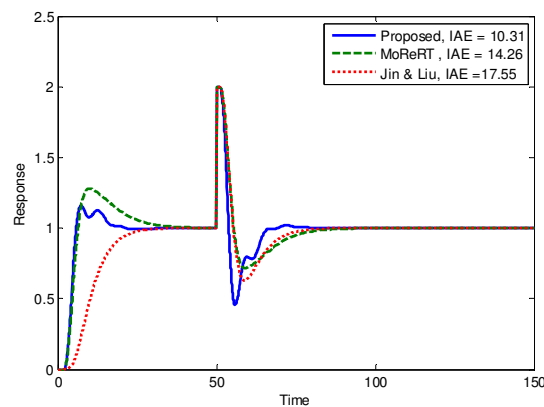


Figure 4. Closed-loop responses for proposed method, [11] method and [12] under perturbed condition (20% modeling error) in deadtime.

Example 2

In this example, consider an integrating process with dead-time and inverse response as in [13]. We compare the performance of the double-loop scheme with some existing methods reported in Tyreus and Luyben [2], and in Jeng and Lin [13]. We set the dimensionless parameter values as $r_{p1} = 0.1$, $r_{p2} = 0.025$, $r_i = 29$ and $r_d = 14$. The controller parameters are shown in Table 2.

Under the nominal condition, the responses shown in Figure 5 indicate that the proposed scheme outperforms the methods of Jeng and Lin [13] and Tyreus and Luyben [2], i.e., the proposed scheme results in the smallest IAE value. This shows that the P/PID tunings based on the rigorous analysis of stability regions can result in better selection of parameter values.

Figure 6 shows the responses under the modelling error of 20% in process gain and deadtime. It is worth noting that, the proposed double-loop scheme still performs better than the other methods; in fact, the proposed

scheme shows very little degradation in its performance in the presence of the aforementioned modeling error.

$$P(s) = \frac{0.547(-0.418 + 1)se^{-0.1s}}{s(1.06s + 1)} \quad (27)$$

TABLE II. CONTROLLER PARAMETER VALUES FOR EXAMPLE 2

Method	K_c	τ_I	τ_D	τ_f	K_{c2}
Proposed	1.3842	12.7955	1.4	-	0.457
Jeng and Lin [13]	1.608	3.518	1.06	0.029	-
Tyreus and Luyben [2]	1.610	5.75	1.15	-	-

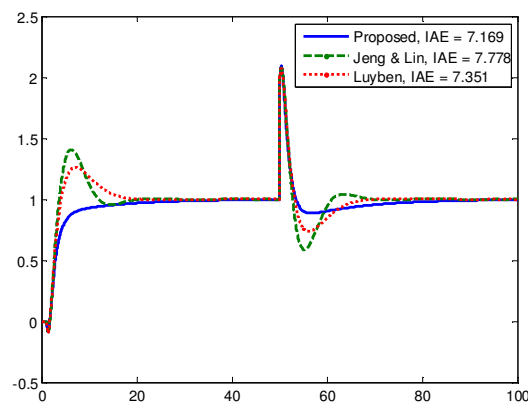


Figure 5. Closed-loop responses under nominal condition for example 2.

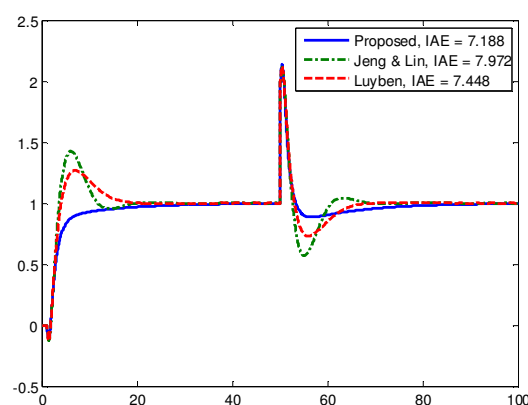


Figure 6. Closed-loop responses for example 2 under 20% modeling error in process gain and dead-time.

The stability analysis of P and PID controllers in the double-loop structure has been conducted for a class of second-order integrating process with deadtime. The stability regions have been established

based on the Routh-Hurwitz criteria with the aid of PID theorem. Simple P and PID controller tunings for the inner and outer loops respectively have been proposed. Simulation results have shown that the proposed scheme using double-loop structure can outperform the standard single-loop PID control scheme tuned using some of the best methods reported in the literature. In future research, it is worth to explore the application of the proposed double-loop scheme for different types of process models.

Acknowledgment

This work is partly supported by Curtin University Malaysia, under the Intelligent System, Design and Control (ISDCON) research cluster.

References

- [1] W. L. Luyben, "Identification and Tuning of Integrating Processes with Deadtime and Inverse Response," *Ind. Eng. Chem. Res.*, vol. 42, pp. 3030–3035, 2003.
- [2] B. D. Tyreus and W. L. Luyben, "Tuning PI Controllers for Integrator/Dead Time Processes," *Ind. Eng. Chem. Res.*, vol. 31, no. 11, pp. 2625–2628, 1992.
- [3] V. Venkatasankar and M. Chidambaram, "Design of P and PI controllers for unstable first-order plus time delay systems," *Int. J. Control*, vol. 60, no. 1, pp. 137–144, 1994.
- [4] M. Ho, A. Datta, and S. P. Bhattacharyya, "Linear Programming Characterization of All Stabilizing PID Controllers," *Proc. Am. Control Conf.*, no. June, pp. 3922–3928, 1997.
- [5] K. M. Tsang, & A. Besharati Rad, K. M. Tsang, and A. Besharati Radt, "A new approach to auto-tuning of PID controllers," *INT. J. Syst. SCI.*, vol. 26, no. 3, pp. 639–658, 1995.
- [6] N. Tan, I. Kaya, C. Yeroglu, and D. P. Atherton, "Computation of stabilizing PI and PID controllers using the stability boundary locus," *Energy Convers. Manag.*, vol. 47, no. 18–19, pp. 3045–3058, 2006.
- [7] M. T. Söylemez, N. Munro, and H. Baki, "Fast calculation of stabilizing PID controllers," *Automatica*, vol. 39, no. 1, pp. 121–126, 2003.
- [8] N. Munro and M. T. Soylemez, "Fast Calculation of Stabilizing PID Controllers for Uncertain Parameter Systems," *IFAC Proc. Vol.*, vol. 33, no. 14, pp. 549–554, 2000.
- [9] Q. H. Seer and J. Nandong, "Stabilization and PID tuning algorithms for second-order unstable processes with time-delays," *ISA Trans.*, vol. 67, pp. 233–245, 2017.
- [10] D. E. Seborg, T. F. Edgar, and D. A. Mellichamp, *Process Dynamics and Control*. New York: John Wiley & Sons, Inc., 1989.
- [11] Q. B. Jin and Q. Liu, "Analytical IMC-PID design in terms of performance/robustness tradeoff for integrating processes: From 2-Dof to 1-Dof," *J. Process Control*, vol. 24, no. 3, pp. 22–32, 2014.
- [12] V. M. Alfaro and R. Vilanova, "Robust tuning and performance analysis of 2dof pi controllers for integrating controlled processes," *Ind. Eng. Chem. Res.*, vol. 51, no. 40, pp. 13182–13194, 2012.
- [13] J. C. Jeng and S. W. Lin, "Robust proportional-integral-derivative controller design for stable/integrating processes with inverse response and time delay," *Ind. Eng. Chem. Res.*, vol. 51, no. 6, pp. 2652–2665, 2012.