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## Study on Nonlinear Dynamics Characteristics of Shape Memory Alloy Composite Doubly-Clamped Beam

To cite this article: Fang Liu *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **493** 012130

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# Study on Nonlinear Dynamics Characteristics of Shape Memory Alloy Composite Doubly-Clamped Beam

Fang Liu<sup>1</sup>, Zhiwen Zhu<sup>2</sup>, Chao Li<sup>1</sup>, Jia Xu<sup>2,\*</sup>

<sup>1</sup>Department of Mechanics, Tianjin University, Tianjin, China

<sup>2</sup>Tianjin Key Laboratory of Nonlinear Dynamics and Control, Tianjin, China

\*Corresponding author e-mail: xujia@tju.edu.cn

**Abstract.** In this paper, the nonlinear constitutive relation of shape memory alloy is proposed. Based on this, the nonlinear dynamic model of a shape memory alloy composite doubly-clamped beam is established. Multi-scale method is used to obtain an approximate solution of the system. Finally, the theoretical results were verified by experiments. The experimental results show that the nonlinear characteristics of the system response in the high-order mode are caused by the hysteresis nonlinear characteristics of shape memory alloy. The results of this paper is helpful for the applications of shape memory alloy composite structure in engineering.

## 1. Introduction

Shape memory alloys (SMA) are widely used in engineering and daily life due to their excellent shape memory characteristics and super-elasticity. The dynamic properties of shape memory alloy materials have become an important research direction in nonlinear dynamics field. Many scholars have studied the nonlinear dynamic behaviors of shape memory alloy structures, including shape memory alloy beams [1], shape memory alloy supports [2], shape memory alloy oscillators [3], shape memory alloy film [4] and so on. Among them, the study of the vibration characteristics of shape memory alloy beams is of great significance to its application and practice in engineering. However, the mechanism models used in some of the earlier studies of shape memory alloys are not suitable for kinetic theoretical analysis.

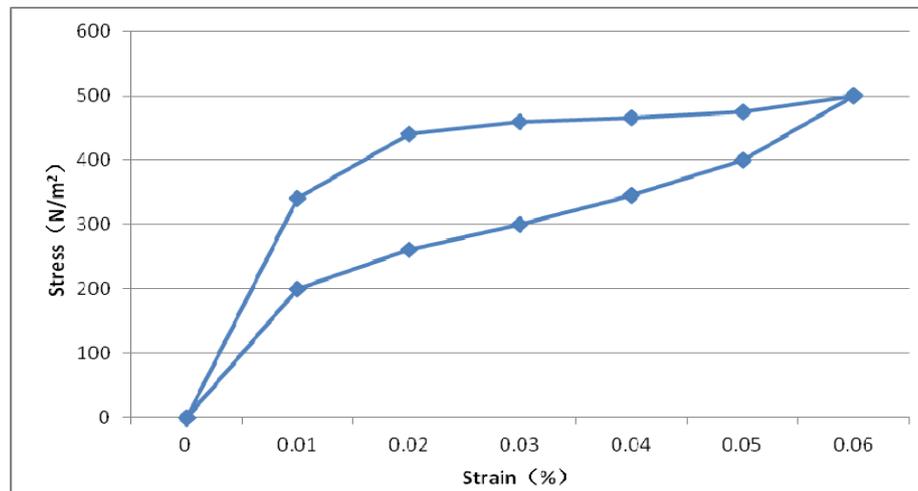
At present, a lot of research work has been carried out in the vibration of shape memory alloy beams. Hashime et al. [5] discretized the continuous beam, modeled and analyzed the shape memory alloy beam under the asymmetric assumption. Machado et al. [6] applied coupled shape oscillators to describe some of the dynamics of structures. Tsai et al. [7] discussed the dynamic stability of reinforced composite beams with additional shape memory alloy wires. However, when the above methods were applied to study the nonlinear dynamic characteristics of shape memory alloy beams, the models are too complicated to study the dynamics of the beam conveniently, which increases the difficulty of research. Moreover, some theoretical researches have been simplified a lot and the hysteretic loop of SMA material during loading and unloading is neglected, which cause the accurate conclusion cannot be obtained.

In this paper, the nonlinear constitutive relation of shape memory alloy is proposed. Based on this, the nonlinear dynamic model of a shape memory alloy composite doubly-clamped beam is established.



Multi-scale method is used to obtain an approximate solution of the system. Finally, the theoretical results were verified by experiments.

## 2. Modeling of shape memory alloy

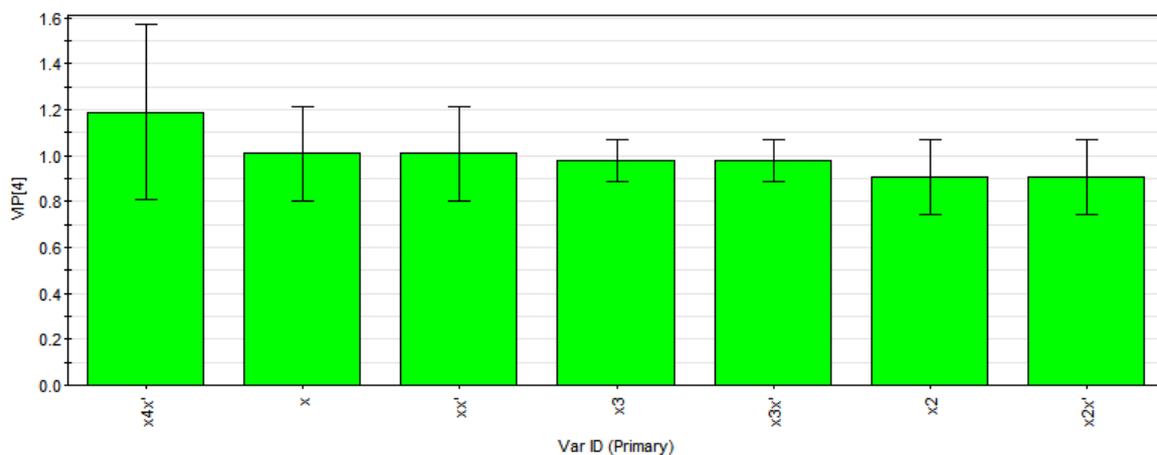


**Figure 1.** Stress-strain curves of shape memory alloy.

The stress-strain curves of shape memory alloy under the same temperature condition are shown in Fig. 1. Obviously, this curve has a hysteretic nonlinear phenomenon. In this paper, the Van der Pol hysteresis model is improved to describe the stress-strain curve of shape memory alloy. The Van der Pol lag model was modified to describe the stress-strain curve of a shape memory alloy. The equation is as follows:

$$\sigma_x = b_1 \varepsilon_x + b_2 \varepsilon_x^2 + b_3 \varepsilon_x^3 + (b_4 \varepsilon_x + b_5 \varepsilon_x^2 + b_6 \varepsilon_x^3 + b_7 \varepsilon_x^4) \dot{\varepsilon}_x \quad (1)$$

Where  $\sigma_x$  is the stress,  $\varepsilon_x$  is the strain,  $b_i (i=1-7)$  are the nonlinear coefficients determined by the hysteretic loop.



**Figure 2.** Analysis result of principal component.

The basic data  $(x, \sigma)$  are processed by EXCEL software to obtain the specific data required (including  $\varepsilon_x$ ,  $\varepsilon_x^2$ ,  $\varepsilon_x^3$ ,  $\varepsilon_x \dot{\varepsilon}_x$ ,  $\varepsilon_x^2 \dot{\varepsilon}_x$ ,  $\varepsilon_x^3 \dot{\varepsilon}_x$ ,  $\varepsilon_x^4 \dot{\varepsilon}_x$ ), and then the data are imported into SIMCA-P software for principal component analysis. The results of principal component analysis based on experimental data are shown in Figure 2.

Because the weights of each terms are relatively large, all variables are retained, so that the final relationship of stress-strain is expressed as follows:

$$\sigma_x = b_1 \varepsilon_x + b_2 \varepsilon_x^2 + b_3 \varepsilon_x^3 + (b_4 \varepsilon_x + b_5 \varepsilon_x^2 + b_6 \varepsilon_x^3 + b_7 \varepsilon_x^4) \dot{\varepsilon}_x \quad (2)$$

### 3. Dynamic Analysis and Experiment of Shape Memory Alloy Composite Doubly-clamped Beam

The structure of a shape memory alloy composite doubly-clamped beam is shown in Figure 3. The composite beam consists shape memory alloy (SMA) and no.45 steel.

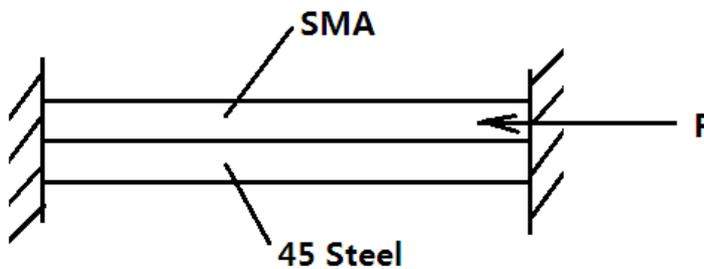


Figure 3. Structure of SMA composite beam.



Figure 4. The system of fixed beam.

According to the Hamilton principle, we have:

$$\ddot{q} + (2\eta + d_1 q^2 + d_2 q^4 + d_3 q^6) \dot{q} + d_4 q + d_5 q^3 + d_6 q^5 = d_7 F \quad (3)$$

The multi-scale method is used to study the non-autonomous system of equation (3) where  $F = F_1 \cos \Omega t$ . The damping force and the right end of the equation are all small, multiplied by  $\varepsilon$ .

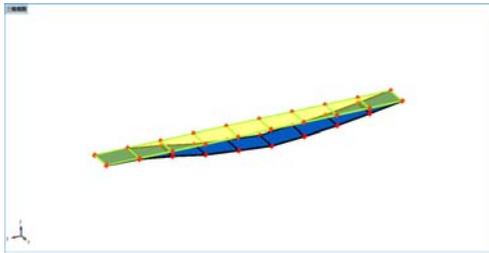
$$\ddot{q} + d_4 q = -\varepsilon [(2\eta + d_1 q^2 + d_2 q^4 + d_3 q^6) \dot{q} + d_5 q^3 + d_6 q^5] + \varepsilon d_7 F_1 \cos \Omega t \quad (4)$$

Where  $\Omega = \omega + \varepsilon \sigma$ .

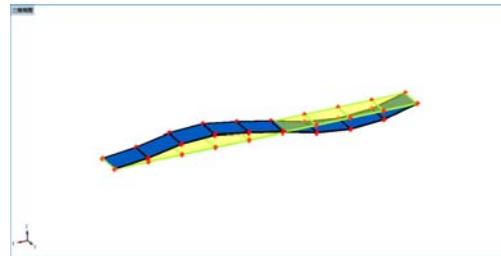
The first-order approximate solution of the system is:

$$q = a \cos(\omega t + \theta) + \varepsilon \left[ \left( \frac{id_1}{32\omega} a^3 + \frac{id_2}{32\omega} a^5 + \frac{15id_3}{512\omega} a^7 + \frac{d_5}{32\omega^2} a^3 + \frac{5d_6}{128\omega^2} a^5 \right) \cos 3(\omega t + \theta) \right. \\ \left. + \left( \frac{id_2}{384\omega} a^5 + \frac{id_3}{256\omega} a^7 + \frac{d_6}{384\omega^2} a^5 \right) \cos 5(\omega t + \theta) + \frac{id_3}{3072\omega} a^7 \cos 7(\omega t + \theta) \right] \quad (5)$$

The experiment device of shape memory alloy composite doubly-clamped beam is shown as Figure 4. The experimental vibration pattern is shown in Figures 5-6:



**Figure 5.** First-order mode diagram.



**Figure 6.** Second-order mode diagram.

Comparing the theoretical results with the experimental results, we find that the system response is linear approximately in the low-order modes (1, 2 orders), but differs greatly from the linear modes in the higher-order modes (3, 4 orders). It is caused by the hysteretic nonlinearity of the shape memory alloy.

#### 4. Conclusion

In this paper, the nonlinear constitutive relation of shape memory alloy is proposed. Based on this, the nonlinear dynamic model of a shape memory alloy composite doubly-clamped beam is established. Multi-scale method is used to obtain an approximate solution of the system. Finally, the theoretical results were verified by experiments. The experimental results show that the system response is linear approximately in the low-order modes, but it differs greatly from the linearity in the high-order mode. This difference is caused by the hysteretic nonlinearity of the shape memory alloy. The results of this paper is helpful for the applications of shape memory alloy composite structure in engineering.

#### Acknowledgments

The authors gratefully acknowledge the support of Natural Science Foundation of China (NSFC) through grant no.11872266 and 51875396, and Aviation Science Foundation of China (ASFC) through grant no.2016ZA48001, and Tianjin Research Program of Application Foundation and Advanced Technology through grant no.16JCYBJC18800.

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