IOP Conference Series: Materials Science and Engineering

PAPER • **OPEN ACCESS**

The Theoretical Research on Solving Accuracy Based on Level Set Topology Optimization

To cite this article: Dongyue Qu *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **493** 012030

IOP Conf. Series: Materials Science and Engineering **493** (2019) 012030 doi:10.1088/1757-899X/493/1/012030

The Theoretical Research on Solving Accuracy Based on Level Set Topology Optimization

Dongyue Qu, Haibing Zhang, Jianan Xu* and Qiang Chen

College of Mechanical and Electrical Engineering, Harbin Engineering University, Heilongjiang, China

*Corresponding author e-mail: xujianan@hrbeu.edu.cn

Abstract. Based on the level set topological optimization method, a mathematical model for the optimization problem is established to maximize the stiffness of the structure as the objective function, and the structure volume is used as the constraint conditions. The level set function is updated by the reaction diffusion equation; the shape and topology of the structure are optimized together. In order to solve the problem that the low accuracy of solving for level set topology optimization method, a method to improve the accuracy of the level set topology optimization method was proposes by using grid encryption and node encryption method. The validity of this method is verified by the relevant examples.

1. Introduction

The level set method was originally proposed by Osher and Sethian [1] and it is a general method to implicitly represent the evolutionary interface in the Euler coordinate system. Osher and Santosa [2][3] used the level set method to optimize the frequency of the structure; and applied this technique to model problems involving the vibration system. In China, the level set topology optimization was applied to solve problem of multi-material and multi-load structure by Mei Yulin[4], and made corresponding improvement to the lack of convergence speed and accuracy of the level set method.

This paper is aimed at improving the solving accuracy of the level set topology optimization method. Firstly, Based on the level set topological optimization method, a mathematical model for the optimization problem is established to maximize the stiffness of the structure as the objective function, and the structure volume is used as the constraint conditions. Secondly, grid encryption and node encryption methods are used to improve the solving accuracy of level set topology optimization.

2. Structural topology optimization design model of level set method

This paper mainly aims at the new method of Takayuki Yamada [5] based on the level set function based on the reaction diffusion equation. The mathematical model of topology optimization is established, which maximizes the structural stiffness as the objective function and the volume constraint is the main constraint condition. The structural design area and boundary conditions are shown in Figure 1.

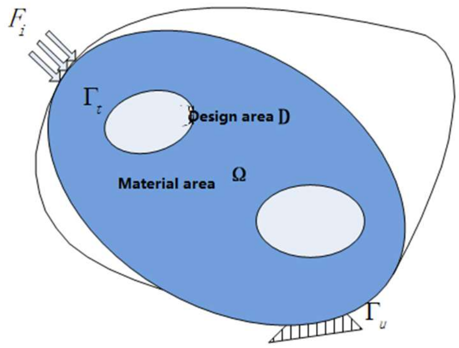


Figure 1. The structure boundary diagram of level set topology optimization method

Where Γ_u is a place of displacement constraint Γ_t is external force, the material area is Ω, the design area is D, and the detailed optimization model is as follows:

$$\begin{aligned} &\inf_{\phi} J = \int_{\Gamma} F_i u_i d\Gamma \\ &s.t. G = \int_{\Omega} d\Omega - V_{\max} \le 0 \\ &div(E_{ijkl} u_{k,l}) = 0 \; on \; \Omega \\ &u_i = 0 \qquad in \;\; \Gamma_u \\ &F_i = \overline{F_i} \qquad in \;\; \Gamma_t \end{aligned} \tag{1}$$

In the upper formula, J is the objective function of the structural topology optimization problem, $V_{\max}$ is the upper limit of the volume constraint, the E_{ijkl} is the modulus of elasticity of the material, the u_i and F_i are the displacement and external force constraints, in which the displacement on the boundary Γ_u is equal to 0, and $\overline{F_i}$ represents the given external load, and $div(E_{ijkl} u_{k,l}) = 0$ represents the effect of no other external loads on the Material area Ω.

The above equation (1) is transformed into an unconstrained augmented Lagrange formula by using the Lagrange multiplier algorithm

$$\bar{J} = \int_{\Gamma} F_i u_i d\Gamma + \int_{\Omega} u_i^* div\left(E_{ijkl} u_{k,l}\right) d\Omega + \lambda\left(\int_{\Omega} d\Omega - V_{\max}\right) \tag{2}$$

$\bar{J}$ is a new objective function and u_i^*, λ is a LaGrange multiplier. $d_t \bar{J}$ is defined as the topological derivative of the objective function for structural topology optimization. By introducing the reaction diffusion equation to update the level set function, the specific equation is as follows:

$$\begin{cases} \dfrac{\delta\phi}{\delta t} = -K\left(d_t \bar{J} - \tau \nabla^2 \phi\right) & in D \\ \phi = 0 & on \partial D \end{cases} \tag{3}$$

ϕ is a level set function, K is a proportional coefficient greater than zero, and τ is a regularization parameter, To a certain extent, the geometric complexity of the final optimized structure is affected. The ideal objective function can be obtained by the rational determination of numerical results of the $\nabla^2\phi$ is a diffusion term, which guarantees the smoothness of the level set function.

3. Theoretical research on improving solving accuracy of level set topology optimization

In the theory of finite element, in order to enhance the accuracy of the calculation results, there are two methods in general, one, with the increase of the power of the form function (the increase of the number of nodes), the accuracy of the calculated results is increased, which is called the P method[6]. The second method is to improve the accuracy of the solution only by encrypting the finite element mesh without changing the number of nodes. This method is widely used as the H method.

3.1. The H method for improving solving accuracy of level set topology optimization

The level set topology optimization program obtained through the grid encryption can realize the division and solution of the different grid sizes at the same structure size, so as to enhance the solving accuracy of the level set topological optimization. When grid thinning is done, it is necessary to modify the stiffness matrix and modify to form a unit size. The following example is taken from the first item in the stiffness matrix, as shown in concrete equation (4).

$$\begin{aligned}&\int\left(\frac{1}{2a}\left(1-\frac{2y}{b}\right)\right)^2+\left(\left(\frac{1}{8b^2}\frac{\mu}{}\right)\left(1-\frac{2x}{a}\right)\right)dxdy\\&=\frac{b}{3a}+\frac{(1-\mu)a}{6b}\end{aligned} \tag{4}$$

The parameters a and b correspond to the length and width of the element respectively, and the element stiffness matrix is shown in the following equation.

$$k_{ij}=\frac{Et}{4(1-\mu^2)}\begin{bmatrix}\frac{b}{a}\varepsilon_i\varepsilon_j\left(1+\frac{1}{3}\eta_i\eta_j\right)+\frac{1-\mu}{2}\frac{a}{b}\eta_i\eta_j\left(1+\frac{1}{3}\varepsilon_i\varepsilon_j\right) & \mu\varepsilon_i\eta_j+\frac{1-\mu}{2}\eta_i\varepsilon_j\\ \mu\varepsilon_i\eta_j+\frac{1-\mu}{2}\eta_i\varepsilon_j & \frac{b}{a}\eta_i\eta_j\left(1+\frac{1}{3}\varepsilon_i\varepsilon_j\right)+\frac{1-\mu}{2}\frac{a}{b}\varepsilon_i\varepsilon_j\left(1+\frac{1}{3}\eta_i\eta_j\right)\end{bmatrix} \tag{5}$$

Take a common example to demonstrate the effectiveness of the method and the structural size of the example is $L=60mm$, $H=30mm$. The middle position of the right side of the structure is subjected to a concentrated load of 1000N. The volume constraint of the structure is 0.5, and the specific structural schematic diagram is shown in Figure 2.

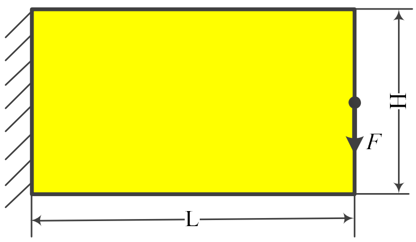
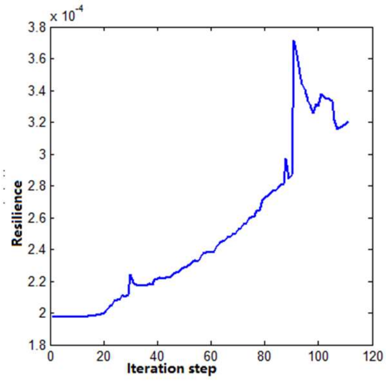


Figure 2. The structure diagram of grid encryption example

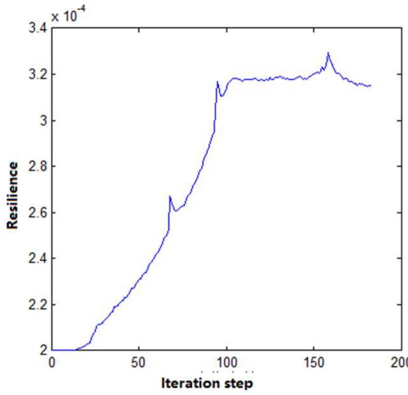
The grid is divided into 60×30 and 120×60, the results of topology optimization are respectively shown in Figures 3.

(a) The optimization results corresponding to 60*30 Grid size

(b) The optimization results corresponding to 120*60 Grid size



(c) The result curve corresponding to 60*30 Grid size



(d) The result curve corresponding to 120*60 Grid size

Figure 3. The topology optimization result of different grid size

The unit size is 1×1 in Figure 3 (a). At the end of iteration, the corresponding total strain energy of the structure is $3.2041\text{e}-004\text{N.m}$, which the same is as the calculation result of this example in ANSYS. When the size of the method is used in this chapter, the sum of the corresponding structural strain energy at the end of the iteration is 3.1472e-004N.m, which is the same as that in this calculation example in ANSYS. The result of topology optimization shows that the final result is the same in the same structure size when different mesh sizes are used, but the result is smoother and the result of strain energy is more accurate. The strain energy curve of the grid after encryption is more stable, and the strain energy is smaller and its corresponding structural stiffness is larger after encryption. As shown in Figure3(c)and(d), the final iteration curve of the strain energy after encryption is more stable. It is proved that the effectiveness of the improved level set topology optimization method in this section.

3.2. The P method for improving solving accuracy of level set topology optimization

In the theory of finite element, with the increasing of the power of the form function (the increasing of the number of nodes), the accuracy of the calculated results is increased, which is called the P method. The stiffness matrix of the unit is the same as that of the four node quadrilateral isoperimetric element stiffness matrix, only because the number of nodes is doubled in the unit, so the overall stiffness matrix becomes a matrix of 16×16,and the results are shown in the following equation.

$$k_{r,s}=\iint\frac{E}{1-\mu^2}\begin{bmatrix}\frac{\partial N_r}{\partial x}\frac{\partial N_s}{\partial x}+\frac{1-\mu}{2}\frac{\partial N_r}{\partial y}\frac{\partial N_s}{\partial y} & \mu\frac{\partial N_r}{\partial x}\frac{\partial N_s}{\partial y}+\frac{1-\mu}{2}\frac{\partial N_r}{\partial y}\frac{\partial N_s}{\partial x}\\ \mu\frac{\partial N_r}{\partial y}\frac{\partial N_s}{\partial x}+\frac{1-\mu}{2}\frac{\partial N_r}{\partial x}\frac{\partial N_s}{\partial y} & \frac{\partial N_r}{\partial y}\frac{\partial N_s}{\partial y}+\frac{1-\mu}{2}\frac{\partial N_r}{\partial x}\frac{\partial N_s}{\partial x}\end{bmatrix}drds \tag{6}$$

IOP Conf. Series: Materials Science and Engineering **493** (2019) 012030 doi:10.1088/1757-899X/493/1/012030

In the process of calculation, the expressions of the integrand are ξ、η and $\frac{\partial N_r}{\partial x}$、$\frac{\partial N_s}{\partial y}$ are more cumbersome. The Gauss numerical integration is used to approximate the integral expression [7].

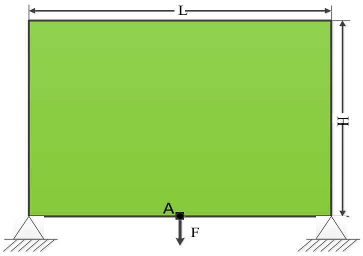
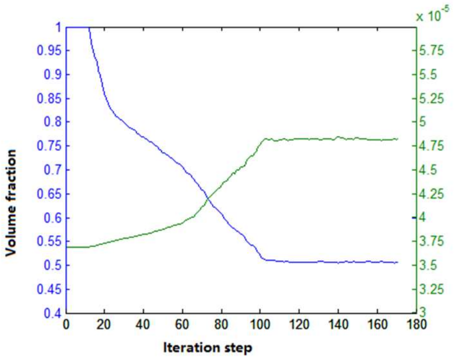


Figure 4. The structural diagram of two-dimensional simply supported beam

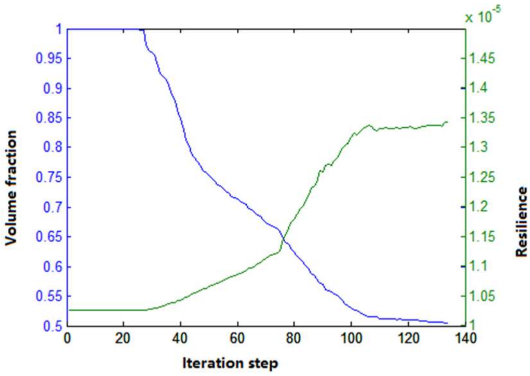
The initial optimization area of a two-dimensional simply supported beam is $L = 200\text{mm}$, $H = 60mm$ and the thickness is $t = 1mm$ in Figure. 5. The centre of the lower end of the structure is subjected to a vertical downward concentrated load of $F = 1000N$. The right and left endpoints of the lower end of the structure restrict the degree of freedom of the X and Y directions. The number of grids are divided into 60×30 , and the elastic modulus and Poisson's ratio of solid materials in the structural design domain are $E_1 = 2\times10^{11}Pa$ and $\mu = 0.3$. The elastic modulus of the void material area is, the volume constraint is 0.5 .The topology optimization results are obtained through the number of $E_{\min} = 1\times10^{-6}P\text{a}$ encrypted node nodes and the optimization results of the unencrypted first four node units are shown in Figure 5.

(a) Four node element optimization results

(b) Eight node element optimization results



(c) Four node element result curve



(d) Eight node element result curve

Figure 5. The topology optimization results of two-dimensional simply supported beam

IOP Conf. Series: Materials Science and Engineering **493** (2019) 012030 doi:10.1088/1757-899X/493/1/012030

In Figure 5 (a), the result of the four nodes element are used in the optimization. The sum of the corresponding structural strain energy at the end of iteration is $4.82\times10^{-5}N\cdot\mathrm{m}$, In Figure 5 (b), the result of the eight nodes element are used in the optimization. The sum of the corresponding structural strain energy at the end of iteration is $1.34\times10^{-5}N\cdot\mathrm{m}$ .It can be seen from the result image that under the same structure size, the number of nodes is increased, the final result of shape is the same, but the structural strain can be different. After increasing the number of nodes, the change trend of the strain energy of the corresponding results is more uniform, and the strain energy is smaller and its corresponding structural stiffness is larger. The optimized structure is imported into ANSYS for calculation, and the displacement results are shown below.

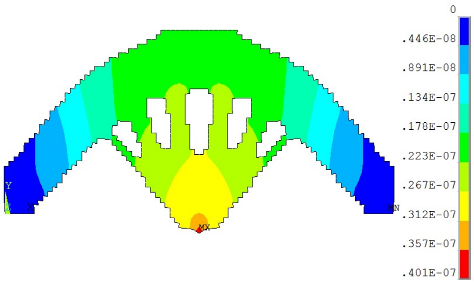


Figure 6. Four nodes element optimization of displacement cloud map

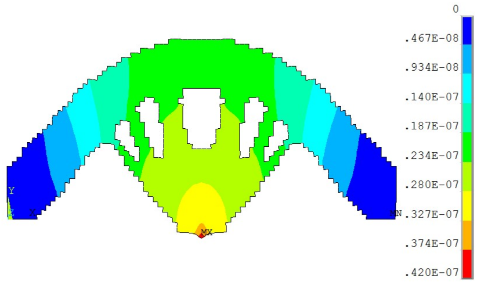


Figure 7. Eight nodes element optimization of displacement cloud map

At this point, the displacement of the middle node A of the lower end of the structure is shown below.

Table 1. Calculation results of two dimensional simple supported beams

Solution method	The displacement of A (m)
Theoretical solution	4.35e-8
Four nodes unit	4.01e-8
Eight nodes unit	4.2e-8

It can be seen from the above table that when the number of nodal points of the quadrangle element is added, the solution is more close to the theoretical solution, and it can be seen that the calculation precision has been greatly improved. There is a great significance in the practical application of the project.

4. Conclusion

In this paper, a mathematical model is established based on the level set topological optimization method of reaction diffusion equation and the optimization problem is established by maximizing the stiffness of the structure as the objective function and the volume of the structure as the constraint condition. For the problem that the low accuracy of solving for the level set topological optimization method, the cause of the problem is analyzed, and grid encryption and node encryption methods are used to improve the solving accuracy of the level set topology optimization, which not only improves the accuracy of the solving, but also makes the optimized structure more rigid and more reliable. This method provides a theoretical basis for solving the problem that the low accuracy of solving for the level set topology optimization in the three-dimensional practical problem.

Acknowledgments
The research of high-performance electric deck crane.

IOP Conf. Series: Materials Science and Engineering **493** (2019) 012030 doi:10.1088/1757-899X/493/1/012030

References

[1] S. Osher, J.A. Sethian, Front propagating with curvature dependent speed, algorithms based on the Hamilton–Jacobi formulations, J. Comput. Phys. 78(1988) 12–49.
[2] Sethian J A, Wiegann A. Structural boundary design via level set and immersed interface methods[J]. Journal of Computational Physics, 2000, 163(2) 489-528.
[3] Osher SJ, Santosa F. Level set methods for optimization problems involving geometry and constraints, I. Frequencies of a two-density inhomogeneous drum, J. Journal of Computational Physics, 2001, 171(1) 272-288.
[4] Mei Y L. Topological Optimization Level Set Method and Its Application in Rigid Structure, Flexible Mechanism and Material Design. University of Technology,2003.
[5] Otomori M, Yamada T, Izui K, et al. Matlab code for a level set-based topology optimization method using a reaction diffusion equation ,J. Structural & Multidisciplinary Optimization, 2014, 51(5):1159-1172.
[6] Wang Xucheng. Finite element method. Tsinghua University Press, 2003,pp.162-171.
[7] Wang Xinrong. Ansys finite element basic tutorial. Electronic Industry Press, 2011,pp.69-75