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Jordan — Gauss successive elimination method in solution of pneumohydraulic floatation water purification problems

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Abstract. The emphasis of the article is a potential usage of Jordan – Gauss successive elimination method for reduction of a differential equation system (DES) to one differential equation and determining its solution providing all other solutions of DES. This approach is different from standard method of determining of eigen vectors and eigen values of matrix of DES, which are necessary to determine fundamental system of solutions of homogeneous differential equations of n-th order with constant coefficients denoted in norm form. In the article analytical solution is searched for in general form and can be used for other similar mathematical problems.

1. Introduction

Floatation methods are widely used for practical tasks of wastewater treatment and separation of fine suspensions in processes of mineral processing [1-10]. In this connection efficiency enhancement of floatation process may contribute to major economic benefit. Application of effective mathematical methods plays an important role at solution of many problems concerning separation processes, such as floatation process [11-30].

For example, usage of Jordan – Gauss successive elimination method for reduction of differential equation system (DES) to one differential equation and its determining its solution providing all other solutions of DES.

2. Basic concepts and suggestions

Let's consider the nature of the proposed approach in greater detail. Matrix **A** of differential equation system (DES) is given:

A:=matrix(n,n,[]):with components:

```
k1:=1/2*10^(-3):k2:=1/10*10^(-6):k3:=1/10*10^(-3):
k4:=1/10*10^(-5):
k5:=1/10*10^(-5):k6:=1/2*10^(-6):
A[1,1]:=-k1-k5:A[1,2]:=k2:A[1,3]:=k6:
A[2,1]:=k1:A[2,2]:=-k2-k3:A[2,3]:=k4:
A[3,1]:=k5:A[3,2]:=k3:A[3,3]:=-k4-k6:
```

according to which let's write down normal homogeneous DES of n-th order in vector-matrix form:

$$\frac{dX(t)}{dt} = A \cdot X \quad (1)$$

Let's define the order of DES and vector of solutions

n:=3: X:=vector(n,[seq(y[j](t),j=1..n)])

DES (1) according to [2] can be transformed to the following form

$$Z = B[s] \cdot X \quad (2)$$

Let's express vector X from (2) if inverse matrix $B^{-1}[s]$ exists

$$X = B^{-1}[s] \cdot Z \quad (3)$$

Expression (3) allows to reduce DES (1) to one differential equation (DE) of n-th order for any variable $x[s](t)$, $s = 1..n$. It can be shown that inverse matrix $B[s]$^(–1) always exists if det(A)<>0. Therefore research or solution of DES (1) should be started from determining of det(A). In case if det(A) = 0, it is necessary to find principal minor *Am in matrix A and write down new DES (4), which is equivalent to previous system (1)*

$$\frac{dX_m(t)}{dt} = Am \cdot Xm \quad (4)$$

Without loss of generality it can be assumed that principal minor *Am is placed in first* *m* rows and columns of matrix *A,* and basis solutions of (1) are first *m* components of vector *X*, which means $Xm = (x[1](t), x[2](t), ..., x[m](t))$, $m <= n$. In this case components that are left $x[m+1](t), ...,x[n](t)$ are linearly connected with basis ones. This relation is defined from linear dependence equations of *A*-matrix rows of DES (1):

$$\sum_{j=1}^{m} A_{j,i} \cdot u_{j,k} = A_{k+m,i}, \quad i = 1..n, \quad k = 1..n - m \quad (5)$$

$$\sum_{j=1}^{m} \frac{d}{dt} x_j(t) \cdot u_{j,k} = \frac{d}{dt} x_{k+m}, \; k = 1..n - m$$

Where $\{u[j,k]\}$, $j = 1...m$, $k = 1...n– m$ matrix of coefficients defining linear dependence of rows of matrix *A*. Definition of matrix $Aa = \{A[j,i]\}$, $j = 1...m$, $i = 1...n$ allows to denote previously shown dependencies in the following vector-matrix form in Maple package

$$Aa^T \cdot col(u,k) = col(A^T, k+m), \quad (6)$$

$$\dot{X}m \cdot col(u,k) = \dot{x}_{k+m}, \; k = 1..n - m \quad (7)$$

The last of them (7) defines relation between basis and non-basis solutions of DES. It is important to note that such transformations decrease the order of initial DES and remarkably simplify the process of its solution by reduction to one differential equation. In Maple package coefficient matrix $\{u[j,k]\}$ can be defined by means of standard tools of the package, among them is Jordan — Gauss method.

Further it is necessary to define new DES, which doesn't contain linearly dependent rows and columns and its rank is rank(Ap) = m, and then decrease its order. Decrease of order of DES, which is important for solution of differential equations [3], is carried out the following way: first m equations (5) are differentiated termwise, while other variables are excluded:

$$\dot{x}_{m+k} = \sum_{j=1}^{m} \dot{x}_j \cdot u_{j,k}, \; k = 1..n - m \quad (8)$$

then a system of m equations with m variables is obtained, and it is necessary to introduce new variables v

$$\dot{x}_i = v_i,\ \ddot{x}_i = \dot{v}_i,\ i = 1..m \qquad (9)$$

The result of termwise differentiation (5) is as follows

$$\frac{d^2}{dt^2} x_j(t) = \sum_{i=1}^{m} A_{j,i} \cdot \dot{x}_i + \sum_{i=m+1}^{n} A_{j,i} \cdot \dot{x}_i,\ j = 1..n$$

Thus, by usage of new variables, new DES is obtained

$$\dot{v}_j = \sum_{p=1}^{m} \left[A_{j,p} + \sum_{k=1}^{n-m} A_{j,m+k} \cdot u_{p,k} \right] \cdot v_p,\ j = 1..m \qquad (10)$$

This DES will be equivalent to the initial one in case of fulfillment of conditions (8), defined by new variables

$$v_{m+k} = \sum_{j=1}^{m} v_j \cdot u_{j,k},\ k = 1..n-m$$

Consequently, DES coefficients (10) are defined by formulae:

$$Ap[j,p] := A[j,p] + \sum_{k=1}^{n-m} A[j,m+k] \cdot u[p,k],\ j = 1..m,\ p = 1..m \qquad (11)$$

Here, as it has been mentioned above, determinant of matrix *Ap*is not equal to zero, while its rank is *m* which can be confirmed by direct verification.

Here the process of DES (10) solution can be started. Let's introduce basis vector *V* of new variables $v[j](t)$, $j = 1 .. m$and derivatives of *V*

dV:=map(diff,[seq(v[j](t),j=1..m)],t)

The formula defining right-hand side of DES (10) is **SD:=evalm(Ap&*V).** DES in new variables, which relation to old variables is defined by formulas (9)

sys1:=seq(dV[k]=SD[k],k=1..m); fcn1:={seq(v[j](t),j=1..m)}:

System of differential equations defining one of the new variables, for example, $v[s](t)$, $s = 1$, is

$$\begin{pmatrix} v_s{}' \\ v_s{}'' \\ \ldots \\ v_s{}^{(m)} \end{pmatrix} = \begin{pmatrix} row(Ap,s) \cdot Ap^0 \\ row(Ap,s) \cdot Ap^1 \\ \ldots \\ row(Ap,s) \cdot Ap^{m-1} \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \\ \ldots \\ v_m \end{pmatrix},\ s = 1..m \qquad (12)$$

The system (12) allows to obtain a differential equation for each variable $v[s](t)$ and solve it.

When inverse matrix *Bb* is known, from (12) differential equation defining components of vector *V* by the following formula is obtained

$$\begin{pmatrix} v_1 \\ v_2 \\ \ldots \\ v_m \end{pmatrix} = Bb[s] \cdot \begin{pmatrix} v_s{}' \\ v_s{}'' \\ \ldots \\ v_s{}^{(m)} \end{pmatrix},\ s = 1. \qquad (13)$$

or inserting expressions (9) into (13), formula defining calculations of components of vector *Xm* belonging to initial DES (5) is obtained as follows

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dots \\ \dot{x}_m \end{pmatrix} = Bb[s] \cdot \begin{pmatrix} \ddot{x}_s \\ \dddot{x}_s \\ \dots \\ x_s^{(m+1)} \end{pmatrix}, \; s = 1..m \qquad (14)$$

Thus, for s = 1 differential equation of order m+1 is obtained for defining component $x[1](t)$. Let's firstly solve equation (13) using derivatives vector

dVV:=[seq(diff(v[1](t),t$j),j=1..m)]; evalm(Bb&*dVV)

and transform the equation as follows

ur:=V[1]=evalm(row(Bb,1)&*dVV);

Let's set up a task for solution of *ur*

resh:=dsolve(ur); rhs(resh);v[1](t):

On integrating solution $v[1](t)$ $xm[1](t)$ is obtained.

Solutions $v[1](t)$ and $xm[1](t)$ differ by a constant, for example, H1.

Let's define a vector of constants h, which is necessary for notation of DES solutions

$$xm_1 := t \rightarrow \sum_{i=1}^{m+1} h_i e^{rs_i t}$$

Other solutions are defined on the base of previously found $xm[1](t)$ with addition of a constant, occurring while integration and decreasing the order of differential equation by one.

Let's introduce a vector of solutions of the initial DES with new name *xM.* Consistency conditions (8), defining relation between non-basis variables and basis variables, are also applied to solutions of the initial DES and provide (n-m) solutions more with imperative introduction of constants $hn[i]$, $i = 1...n - m$ on the same basis

Let's calculate these solutions:

$$xM_{m+k} := \left(\sum_{j=1}^{m} xM_j U_{j,k} \right) + hn_k$$

Calculated $xM[i]$, i=1...n are valid with an accuracy to constants, but cannot be considered as solutions of the initial DES. "Excessive" constants $hb[k]$, $hn[i]$ should be expressed through $h[3]$, according to substitution of solutions $xM[i]$ in the initial DES.

As the result equations defining relations between constants are obtained, from which $(n - 1)$ constants are expressed through constant $h[3]$:

$$p6 := \left[\left[hb_1 = \frac{15020}{1003} h_3, \; hn_1 = \frac{1018025}{1003} h_3 \right] \right]$$

Defined values of constants are used in vector of solutions *xM*.

Solutions $y[i](t)$, i = 1..n, obtained in general form are valid for the initial DES.

Vectors *h*, *hb*stay unknown because they depend on vector of initial conditions*Cn* of the initial *DES.* That particular vector is defined and on the base of this vectorthe constants $h[i]$,$hb[j]$ are calculated, as it is shown below.

Let's introduce a vector of initial (or boundary conditions if they are determined) condition for each solution from $y[i](t)$, $i = 1...n$ for the initial DES:

Cn:=vector(n,[]):

Let's determine relation between initial conditions *Cn*[*i*], *i*=1...*n* and constants *h*[*i*], *hb*[j]. In order to do this *n*equations are defined by using $t = 0$ in the system determined by vector *xM*

s4:=seq(Cn[i]=simplify(subs(t=0,xM[i])),i=1..n):

from which we determine

s2:=solve({s4},[seq(h[i],i=1..m+1),seq(hb[j],j=1..n-m-1)]):

3. Results processing and discussion

Let's proceed to plot construction for DES. Particular values for initial conditions are defined:

Cn[1]:=100:Cn[2]:=0:Cn[3]:=0:

and constants are calculated

$$s3 := \left[\left[h_1 = 100.024,\ h_2 = -0.123,\ h_3 = 0.098\right]\right]$$

Determined constants are used in obtained solutions defined by vector *xM*.

Obtained solutions are expressed in digitalized form and plots are constructed (figures 1-4).

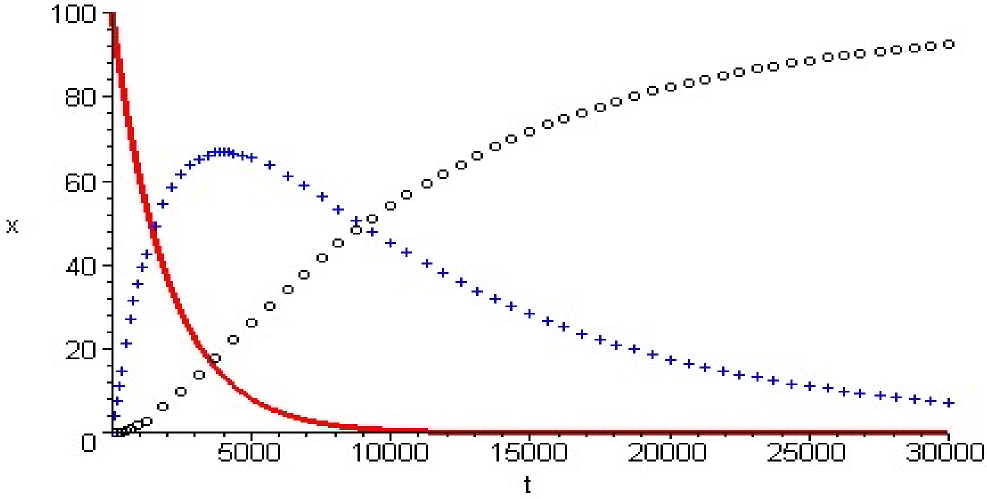

Figure 1. Solutions plot

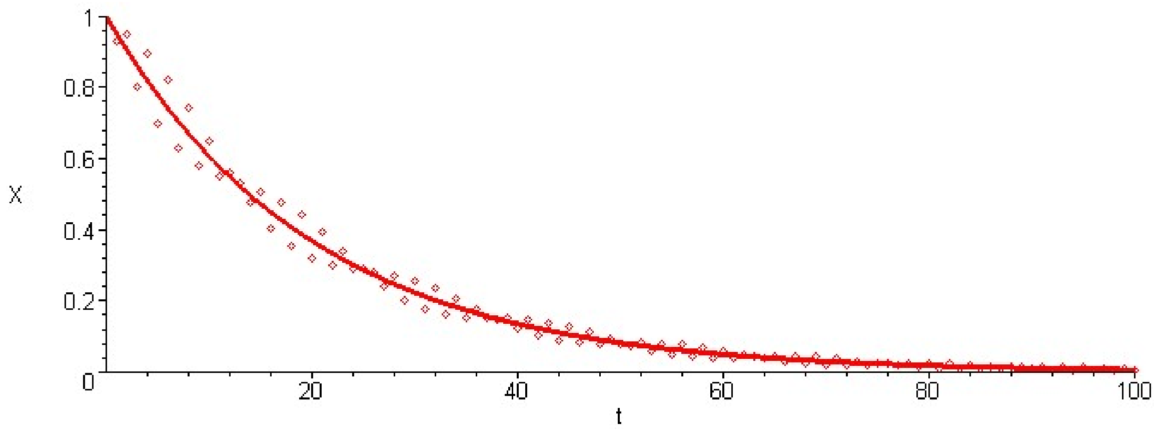

Figure 2. Analytical kinetic dependence of floatation process of fat-containing wastewaters purification: solid curve (red) — extraction of pollution particles out of water (%) to time (sec) and its experimental approve shown in discrete points.

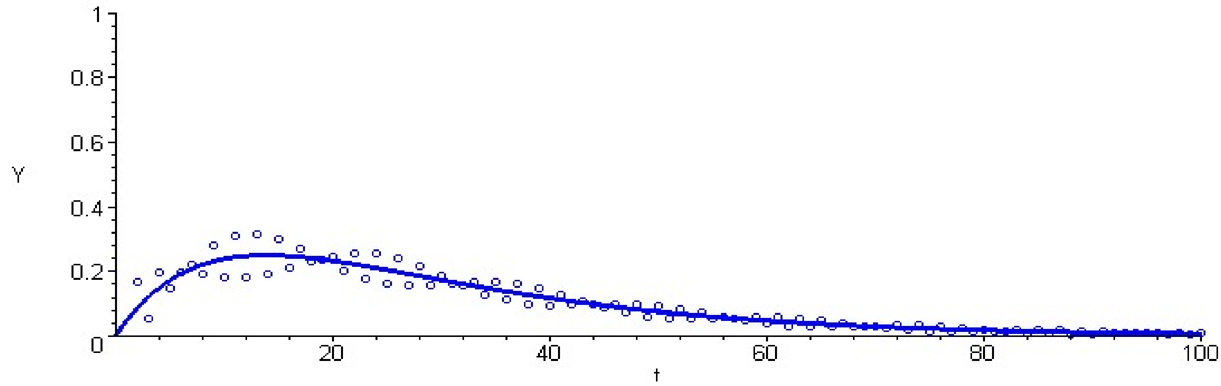

Figure 3. Analytical kinetic dependence of floatation process of fat-containing wastewaters purification: solid curve (blue) — formation of floatation complexes particle-bubble (%) to time (sec) and its experimental approve shown in discrete points.

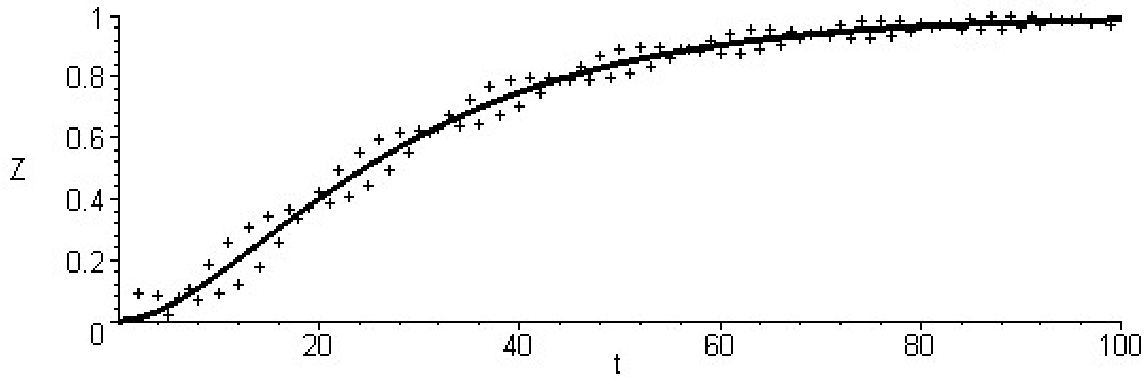

Figure 4. Analytical kinetic dependence of floatation process of fat-containing wastewaters purification: solid curve (black) — formation froth product (%) to time (sec) and its experimental approve shown in discrete points.

Each curve shown on figure 1 was approved by results of experiments conducted during floatation purification of fat-containing wastewaters. Plots of experimental calculations are shown below on figures 2-4 in reduced scale, which, however, allows to estimate accuracy of coincidence between them and theoretical curves and to draw conclusion about adequacy of mathematical model used.

Difference between experimental and theoretical data does not exceed 5-7%, which is affordable for process of wastewater floatation purification. Obtained data allows to estimate dimensions of flotators with adequate accuracy on the base of general parameter which is floatation time.

4. Conclusion

Thus, proposed approach allows to obtain basic kinetic dependencies of floatation process of separation non-homogeneous systems, in particular wastewaters and fine suspensions. Proposed approach is approved during the process of design of floatation units for purification of wastewaters of different manufactures. Upon that, the results of tests show that theoretical data are close enough to experimental data.

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