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## Novel Scaling Approach for ISAL via the Fractional Fourier Transform

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# Novel Scaling Approach for ISAL via the Fractional Fourier Transform

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**Abstract.** Inverse Synthetic Aperture LADAR (ISAL) can obtain high resolution images of remote targets. In order to measure the dimension of the target, the ISAL image must be scaled. Many existing scaling methods require isolated strong scattering centers which are not easily found in ISAL images. To solve this problem, a novel ISAL scaling method is proposed. The method estimates the Doppler frequency modulation rate of each range row by the fractional Fourier transform (FrFT) and image entropy. The linear relationship between the frequency modulation rate and the range is used to obtain an estimate of the target equivalent rotational angular velocity which is the key parameter for scaling. The outfield experimental results confirm the effectiveness of the method.

## 1. Introduction

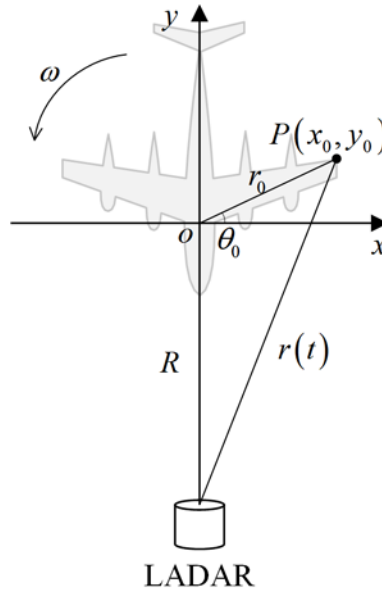
Inverse synthetic aperture LADAR (ISAL) is a combination of synthetic aperture technology and coherent laser technology. ISAL enables high resolution imaging of remote targets. The resolution is not limited by the diffraction limit and is independent of distance [1-4]. The imaging principle of ISAL is similar to that of inverse synthetic aperture radar (ISAR). The resolution of the distance direction is achieved by transmitting wideband modulated signal. The range resolution can be expressed as  $c/2B$ , where  $c$  is the speed of light and  $B$  is the signal bandwidth. The azimuth resolution is achieved by the relative motion between the target and the LADAR sensor, and the resolution can be expressed as  $\lambda/2\theta$ , where  $\lambda$  represents the laser wavelength and  $\theta$  represents the equivalent rotation angle of the target relative to the LADAR sensor. Since the motion state of the non-cooperative target is usually difficult to obtain, the exact value of  $\theta$  cannot be directly obtained. Therefore, the resolution of the azimuth is difficult to obtain, which brings difficulty in obtaining the shape and size of the target. The estimation of the angle  $\theta$  or the equivalent angular velocity becomes a necessary task. Existing methods can be divided into two categories, based on auxiliary sensors, and based on echo data. The first category obtains motion parameters of a target through additional measuring devices to calculate the equivalent rotation angle [5,6]. However, the accuracy cannot meet the requirements of ISAL scaling. The second category uses only echo data for estimation. Many of the reported algorithms require multiple isolated strong scattering points in the image, which is easier to satisfy for microwave imaging [7-9]. However, since the wavelength of the laser is much smaller than that of the microwave, the scattering characteristics are different from those of the microwave. ISAL images are more continuous and lack isolated scattering centers. Therefore, the above algorithms are difficult to apply in ISAL scaling. Here, a new scaling method for ISAL imaging has been proposed. This method



does not depend on isolated strong points in the image, and is tested through an outdoor experiment. The experimental results verify the effectiveness of the method.

## 2. Algorithm principle

The observed geometry of ISAL is shown in Fig. 1. The relative motion of the target and the LADAR sensor can be divided into two components, translation and rotation. Translation does not contribute to ISAL imaging. It needs to be compensated in the imaging process. Therefore, only the rotational component is considered [10,11].



**Figure 1.** ISAL imaging geometry.

As shown in Fig. 1, at the time  $t_0$ , the coordinate of a scattering point  $P$  on the target are  $(x_0, y_0)$ , the distance from the point  $P$  to the equivalent rotation center  $O$  is  $r_0 = \sqrt{x_0^2 + y_0^2}$ , and the angle between the  $PO$  and the  $x$ -axis is  $\theta_0$ . Define  $R$  as the distance from the rotation center of the target to the sensor, and  $\omega$  is the equivalent rotational angular velocity. Then in the far field condition, the instantaneous range between the point  $P$  and the sensor can be approximated as

$$\begin{aligned} r(t) &= R + r_0 \sin(\theta_0 + \omega t) \\ &= R + x_0 \sin \omega t + y_0 \cos \omega t. \end{aligned} \quad (1)$$

Then the Doppler frequency can be expressed as

$$f_D(t) = \frac{2}{\lambda} \frac{dr(t)}{dt} = \frac{2\omega}{\lambda} (x_0 \cos \omega t + y_0 \sin \omega t). \quad (2)$$

Since the equivalent rotation angle of the target in ISAL imaging is very small (less than 1 mrad),  $\cos \omega t \approx 1$ ,  $\sin \omega t \approx \omega t$ , and the Doppler frequency can be approximated as

$$f_D(t) \approx \frac{2\omega}{\lambda} (x_0 + y_0 \omega t). \quad (3)$$

The Doppler frequency modulation rate can be expressed as

$$K_D = \frac{df_D(t)}{dt} = \frac{2\omega^2 y_0}{\lambda}. \quad (4)$$

It can be seen that the Doppler frequency modulation rate  $K_D$  is proportional to the range coordinate  $y_0$ , and the proportionality coefficient is  $2\omega^2/\lambda$ . Therefore, through estimating the Doppler frequency modulation rate of the echo, the value of the equivalent rotational angular velocity  $\omega$  can be obtained. But another problem is that ISAL cannot obtain the absolute value of the range coordinate  $y_0$ , only the relative amount can be obtained. Thus, the rotational angular velocity  $\omega$  cannot be estimated with the Doppler frequency modulation rate in only one range row. In order to solve this problem, Eq. 4 is transformed into

$$\omega = \sqrt{\frac{\lambda}{2}}k, \quad k = \frac{K_D}{y_0} \quad (5)$$

Where  $k$  represents the proportional coefficient of the Doppler frequency modulation rate  $K_D$  and the range coordinate  $y_0$ . The estimation of this coefficient requires only the relative value of  $y_0$ . It should be noted that the sign of  $\omega$  does not affect ISAL scaling, so the sign in the equation is ignored.

If the Doppler frequency modulation rates in at least two range rows can be obtained, the above-mentioned proportional coefficient  $k$  can be estimated. Since there is almost no isolated strong point in ISAL images, in the proposed method, the Doppler frequency modulation rates is estimated by the fractional Fourier transform (FrFT). According to Eq. 4, the echo signals have the same Doppler frequency modulation rate in the same range row. The Doppler frequency modulation rate in  $n$ th range row is defined as  $K_{D,n}$ . When the transform order  $\alpha = -\text{arccot}(2\pi K_{D,n})$ , the  $n$ th range row can be completely focused by FrFT. For other transformation orders, there will be some defocus [12]. The degree of the image focus can be evaluated by image entropy, which is expressed as

$$E(I_n) = -\sum_{k=1}^K D(k) \ln D(k), \quad \text{where } D(k) = \frac{|I_n(k)|^2}{\sum_{k=1}^K |I_n(k)|^2} \quad (6)$$

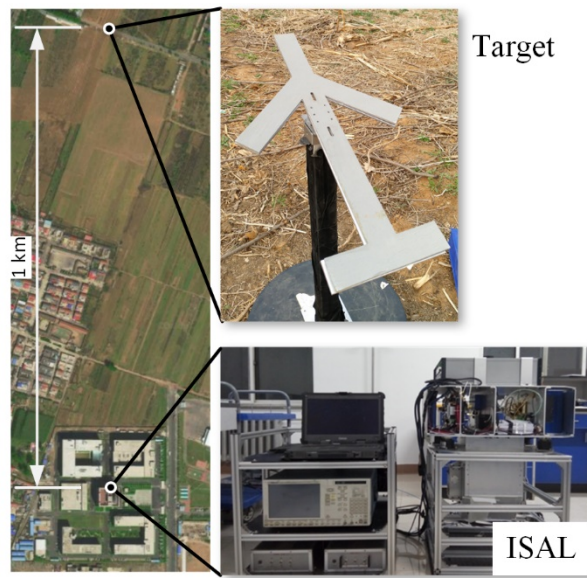
Where  $I_n(k)$  represents the intensity of the  $k$ th resolution unit in the  $n$ th range row. A better focused range row has a smaller image entropy. A range row is transformed with different transformation orders and the image entropies are calculated. The transform order with the smallest image entropy corresponds to the estimate of the frequency modulation rate of the current range row. The estimation of the equivalent angular velocity can be achieved by performing the above processing on different range rows.

### 3. Experiment and discussion

The experiment was conducted outdoors, and the scene is shown in Fig. 2. The target is an aluminum plate in the shape of an airplane, the dimensions of which are shown in Fig. 4(a). The target is 1 km away from the LADAR and placed on an electric turntable with an angular velocity of 52.36 mrad/s. The transmitted laser signal is the linear frequency modulated continuous wave. Other experiment parameters are shown in Table 1.

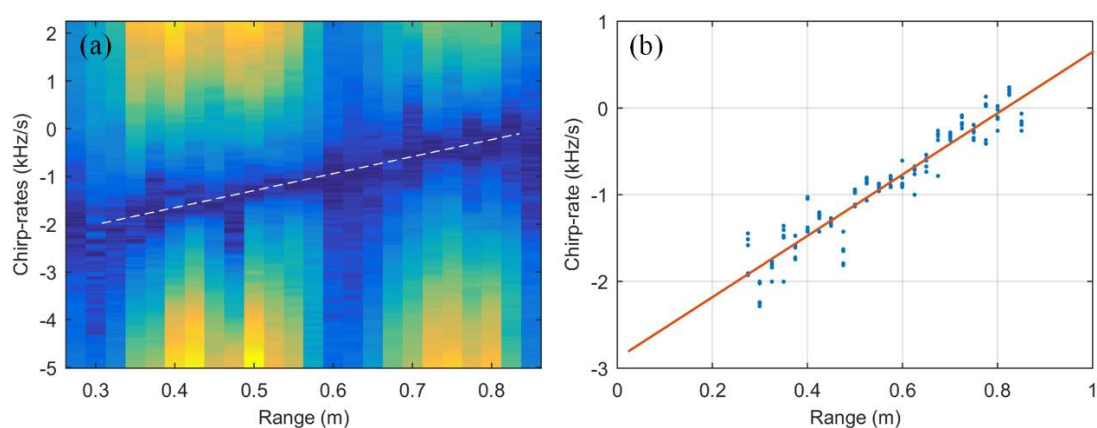
**Table 1.** Main parameters in experiment.

Parameters	Values
Signal bandwidth	1550 nm
Signal bandwidth	6 GHz
Chirp time	60 us
Pulse repetition frequency	16.67 KHz
Laser power	10 W
Equivalent angular velocity	52.36 mrad/s



**Figure 2.** Experiment scene of the ISAL.

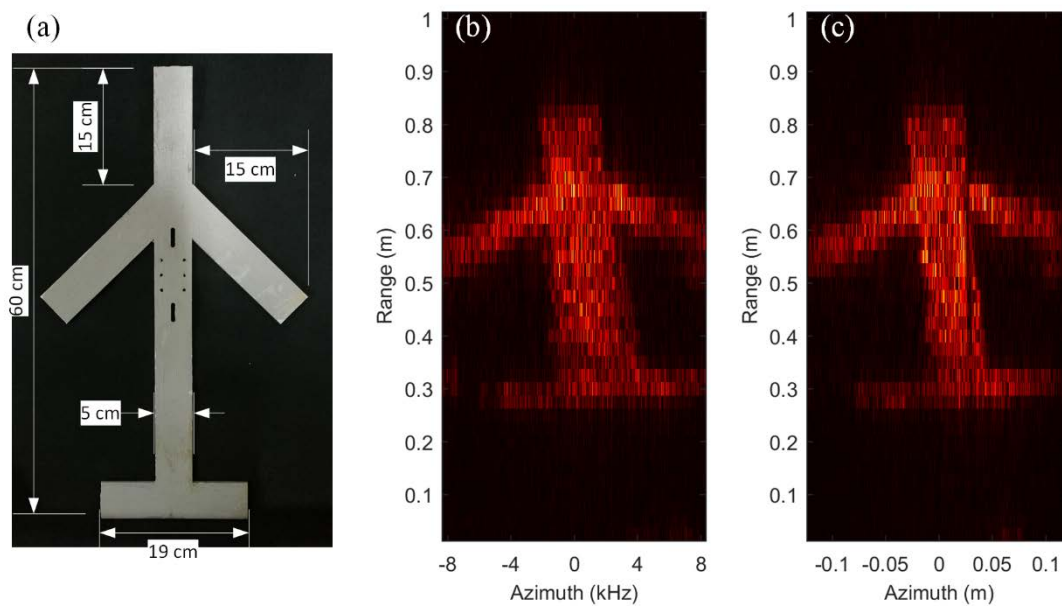
The results of two-dimensional imaging directly using echo data are shown in Fig. 4(b). It can be seen that the shape of the target can be distinguished. However, since the equivalent angular velocity of target is unknown, the azimuth has not yet been scaled, and its unit is still kHz, which means that the image cannot reflect the size of the target. The data is processed using the proposed method. For the data in the experiment, the echo energy of the target is distributed at a range of 0.275 to 0.85 m. It is only necessary to perform FrFT of different orders for this range of echoes and calculate image entropy. The processing result is shown in the Fig. 3(a), where the horizontal axis represents different distance lines, the vertical axis is the modulation frequency (corresponding to different FrFT orders), and the color in the image represents the value of the image entropy (the darker the color represents the smaller image entropy). As with the previous analysis, different range rows achieve minimum image entropy at different transformation orders. Moreover, the optimal transformation order is significantly linear with the range coordinate, as indicated by the dashed line in the figure.



**Figure 3.** (a) Image entropy of range rows for different transformation Orders.  
(b) The fitting result of the proportional coefficient  $k$ .

What needs to be done now is to estimate the slope of the dashed line. There are many ways to accomplish this, and the difference is insignificant. In our processing, we first determine the best transform order for each focus line. These data points are then fitted by least squares. It should be noted that since ISAL imaging is coherent, the image is affected by speckle noise, which causes the

image entropy to be not smooth with the change of the transform order. In other words, although the general trend is not affected, the small jitter makes the minimum image entropy not represent the best focus. In order to reduce the influence of this jitter, in each range row, the five transformation orders corresponding to the minimum image entropy are selected. The case of the selected data points and the fitting results are shown in Fig. 3(b), where the blue dots indicate all the data points selected, and the red line is the result of the fit. Bring the fitting result into Eq. 5 and obtain the equivalent angular velocity of 51.69 mrad/s which is 1.3% different from the set value. Finally, the image after scaling is shown in Fig. 4(c). It can be seen that the dimensions of target in the image matches the true size, and the image of target in Fig. 4(c) achieves better focus than that in Fig. 4(b).



**Figure 4.** (a) The photo and dimensions of the target. (b) Unscaled image. (c) Scaled image.

#### 4. Conclusion

In this paper, a novel azimuth scaling method for ISAL data is proposed. The method uses the linear relationship between the Doppler frequency modulation rates and the range to estimate the equivalent rotational angular velocity of the target. Since the estimation of the Doppler frequency modulation rates is achieved by FrFT and image entropy in processing, this method does not require isolated strong scattering points in the image. It is more suitable with ISAL imaging than other methods that have been reported. Finally, the results of the outdoor experiment verify the effectiveness of the method algorithm in the actual scenario.

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