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To cite this article: A V Leonard *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **489** 012054

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Features of the mathematical model for the control forces calculation of the four-wheeled robot

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Abstract. The article discusses the issue of determining program forces in drives of a four-wheel robotic platform - the issue of opening the uncertainty of the equations system. The motion laws are synthesized on the basis of application of the calculus of variations. The method of determining the program forces is based on the compilation of equations of the dynamics of a rigid body motion and additional equations - "zero-functions" (displacements) of the points of the application of redundant reactions.

1. Introduction

Formation of the laws of motion, both for walking machines and for traditional vehicles, can be based on methods of the calculus of variations – minimization of criteria in the form of linear convolutions of integral indicators with weighting coefficients [1, 2]. In general, the formula of the criteria is a functional as follows:

$$I = \sum_{i=1}^n k_i \int_0^{\tau} f_i \left(x, \dot{x}, \ddot{x}, \dots, y, \dot{y}, \ddot{y}, \dots, \varphi, \dot{\varphi}, \ddot{\varphi}, \dots, x_1, \dots, x_j, \dots, y_1, \dots, y_j, \dots, p_j, \dots \right) dt \rightarrow \min, \quad (1)$$

where k_i – the weights that are set by experts or determined by the robot as a result of the learning algorithm (other conditions); x, y – the coordinates of the robot's characteristic point when it moves along a plane; x_j, y_j – the coordinates characteristic points of the obstacles (moving or static); φ – the angular coordinate defining the orientation of the robot body on the plane; p_j – the parameters (in particular, the radii of the circles, describing the obstacles); τ – the time of performing the task by robot. In this case, the integrands are written in the form, which contains variables or their linear combinations in powers of two. It's necessary that criteria have the ordinary minimum and method is worked too. And then, the conditions of criterion's minimum are:



$$\begin{cases} \frac{\partial \Phi}{\partial \varphi} - \frac{d}{dt} \left(\frac{\partial \Phi}{\partial \dot{\varphi}} \right) + \frac{d^2}{dt^2} \left(\frac{\partial \Phi}{\partial \ddot{\varphi}} \right) - \frac{d^3}{dt^3} \left(\frac{\partial \Phi}{\partial \ddot{\ddot{\varphi}}} \right) + \dots = 0, \\ \frac{\partial \Phi}{\partial x} - \frac{d}{dt} \left(\frac{\partial \Phi}{\partial \dot{x}} \right) + \frac{d^3}{dt^3} \left(\frac{\partial \Phi}{\partial \ddot{x}} \right) - \frac{d^3}{dt^3} \left(\frac{\partial \Phi}{\partial \ddot{x}} \right) + \dots = 0, \\ \frac{\partial \Phi}{\partial y} - \frac{d}{dt} \left(\frac{\partial \Phi}{\partial \dot{y}} \right) + \frac{d^2}{dt^2} \left(\frac{\partial \Phi}{\partial \ddot{y}} \right) - \frac{d^3}{dt^3} \left(\frac{\partial \Phi}{\partial \ddot{y}} \right) + \dots = 0, \end{cases} \quad (2)$$

$$\Phi = \sum_{i=1}^n k_i f_i \left(x, \dot{x}, \ddot{x}, \dots, y, \dot{y}, \ddot{y}, \dots, \varphi, \dot{\varphi}, \ddot{\varphi}, \dots, x_1, \dots, x_j, \dots, y_1, \dots, y_j, \dots, p_j, \dots \right), \quad (3)$$

Here are the program laws of robot's motion in differential form. After that, the problem of integrating the obtained system of equations is solved for given boundary conditions. To obtain the values of the motion laws, the numerical method of integration is mainly used. The method of integration is included in the method of multi-criteria optimization to achieve exact values of the parameters set at the boundaries. The question of numerical integration at boundaries is a separate task and will be discussed in details in another article. As a result of solving the boundary task, tables of program laws values are obtained. In order to adapt the proposed approach to the control of walking machines [3-5], control refinement based on the variational calculus is implemented on a laboratory model of a wheeled platform (see figure 1).

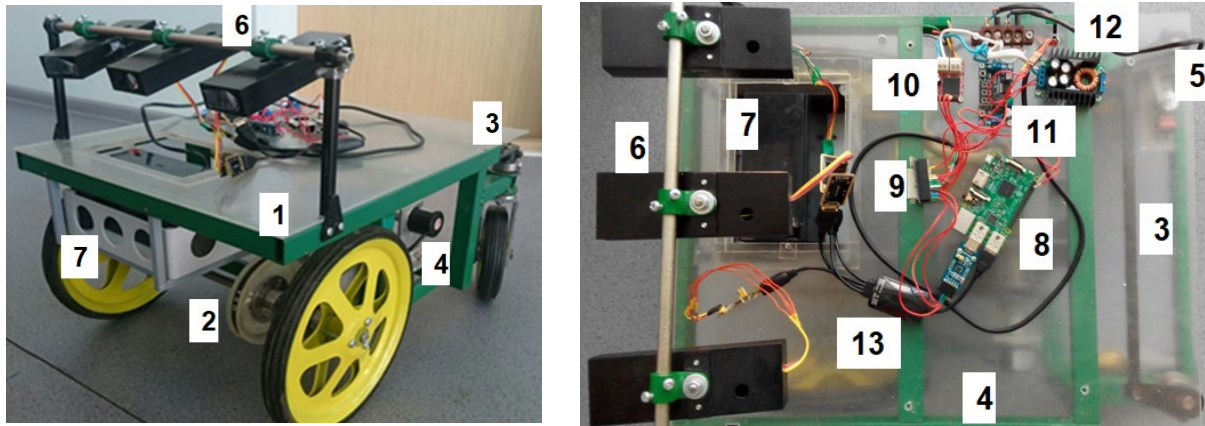


Figure 1. Robotic wheeled platform.

In Figure 1: 1 – wheeled platform; 2 – drive course movement; 3 – rotational drive; 4 – angular encoder; 5 – servo; 6 – a group of laser range finders; 7 – 12 V battery; 8 – on-board computer Raspberry Pi 3; 9 – Arduino Nano controller; 10 – Driver amplifier; 11 – voltage converter 5 V; 12 – servo current limiter; 13 – USB hub.

2. Determination of additional force equations

To implement the motion laws obtained from the extremum conditions (2) of integral target functions (1), it is necessary to take into account the features of robot design, the organization features of its drive and steering. Taking this into account, the features of the robot are reduced to the definition of a force factors system (see figure 2) describing the effect of the environment and actuators on the device. Ultimately, a system of algebraic equations is compiled, from which the programmed drive and steering forces necessary to implement the synthesized motion laws are determined [6, 7]. Using

the example of a robotic platform with four wheels, the number of required force factors determining the dynamics of the robot's movement is reduced to ten in advance: $F_1^r, F_2^r, N_1, N_2, N_3, N_4, F_1^n, F_2^n, F_3^n, F_4^n, a, L = L^* = L_{if}$. For the closure of the dynamic equations system of robot's motion four equations are missing. These equations can be obtained from another hypotheses that relate to the object of research.

Three possible ways to obtain additional forces ratios include:

- The rejection of the hypothesis of hardness of wheels and the support base - taking into account their elastic (ductile) properties.
- Applying control to the supports reactions.
- Taking into account the elastic properties of the platform design.

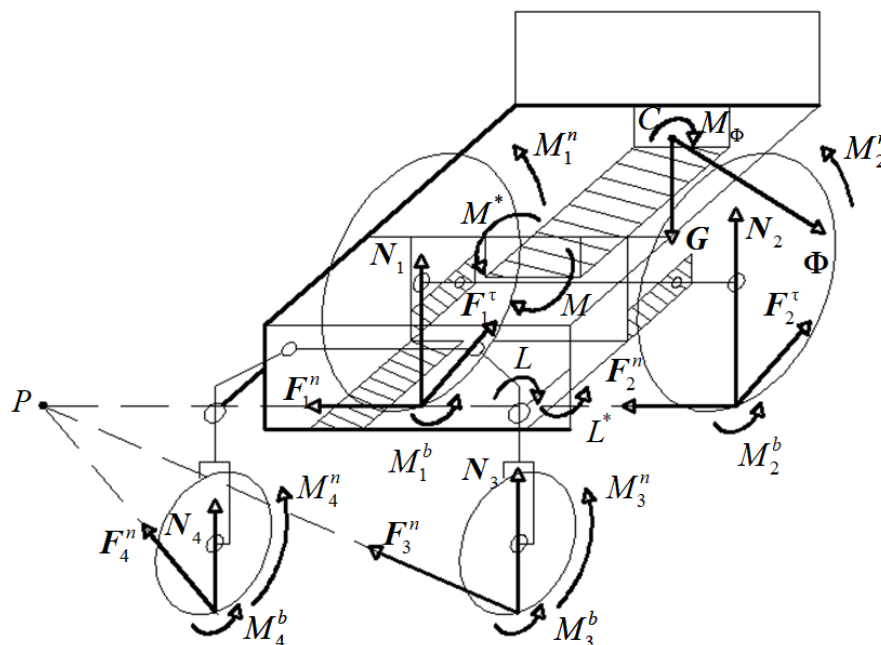


Figure 2. A complete system of force factors determining the movement of the platform on the plane.

For example, the first case (soft wheels and base) requires preliminary identification of the wheel stiffness and installation of four sensors into the control system (excluding the effect of vibrations) that can measure the vertical deformations of four wheels rubber with high precision (in fractions of millimeters). Since the robot control system is already configured specifically: servo, angle encoder, inertial navigation device, terrain laser scanners, the first approach of determining unknown reactions is not considered.

The second approach also requires changes in the robot's design - installation of pneumatic wheels with sensors and devices for distribution and continuous maintenance of pressure integrated into the control system. In the case of recording four additional relations between forces and pressures, the system of equations of motion dynamics will close and it will be possible to determine all unknowns. The obtained pressure values together with the control moments must be reproduced by the robot in the process of implementing the program motion.

The third way to obtain additional relations between forces does not require changes in the physical design of the robot and modernization (complication of the control system). This approach is based on the application of the method of forces - the method of opening the static uncertainty of structures through the deformations recording (functions equal to zero / canonical equations). A robot moving

with small accelerations (with real deformations close to zero) can be considered to be in equilibrium under the action of the considered force factors system (see figure 2). To accurately compile canonical equations by the forces method, it is necessary to take into account the gravity of each core element of the structure. Release from internal relations in the core elements of the frame will lead to the compilation of about 50 canonical equations. To simplify the mathematical model for calculating the internal forces of the platform, it is assumed that the platform consists of rods with the same cross section and material, and the gravity force of the entire platform is concentrated in one point belonging to the frame. As a result, after analyzing the conditions of the kinematic immutability of an equivalent construction (discarding redundant bonds), a calculation scheme is obtained for compiling additional 5 equations to 6 known and independent of them (see figure 3).

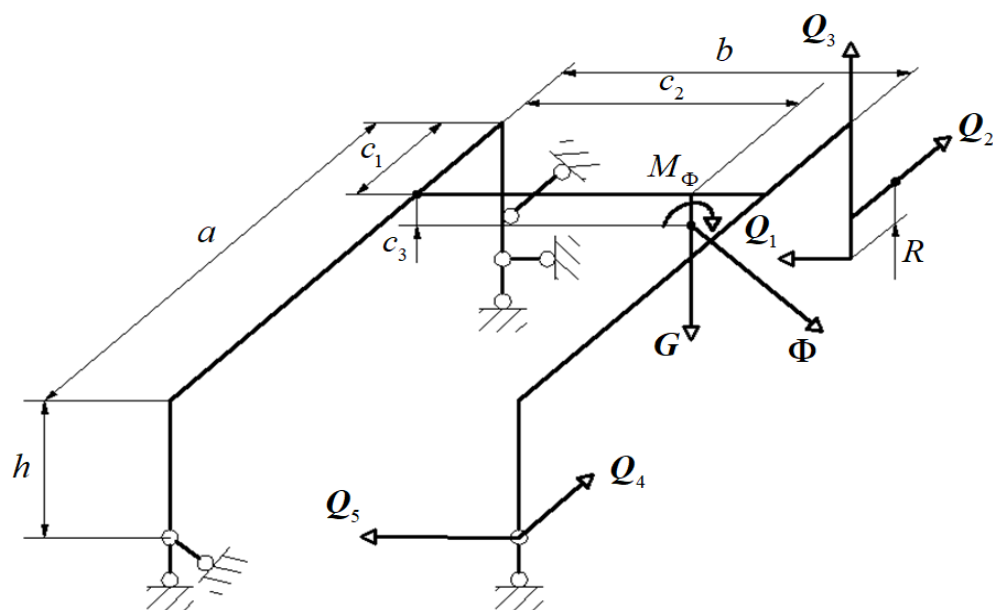


Figure 3. Calculation scheme for obtaining additional forces equations.

For the convenience of compiling canonical equations, the designation of unknown forces varies:

$Q_1 = F_2^n$, $Q_2 = F_2^r$, $Q_3 = N_2$, $Q_4 = T_3$, $Q_5 = T_4$, and: $F_3^n = \sqrt{T_3^2 + T_4^2}$. Since the equivalent construction is at rest, the displacement of the points of application forces along the directions of these forces from the all forces action must be zero:

$$\begin{cases} \Delta_1(Q_1, Q_2, Q_3, Q_4, Q_5, G, \Phi, M_\Phi) = 0, \\ \Delta_2(Q_1, Q_2, Q_3, Q_4, Q_5, G, \Phi, M_\Phi) = 0, \\ \Delta_3(Q_1, Q_2, Q_3, Q_4, Q_5, G, \Phi, M_\Phi) = 0, \\ \Delta_4(Q_1, Q_2, Q_3, Q_4, Q_5, G, \Phi, M_\Phi) = 0, \\ \Delta_5(Q_1, Q_2, Q_3, Q_4, Q_5, G, \Phi, M_\Phi) = 0. \end{cases} \quad (4)$$

Then, using the principle of independence of the forces action, the sums of displacements are recorded:

$$\begin{cases} \Delta_{1Q_1} + \Delta_{1Q_2} + \Delta_{1Q_3} + \Delta_{1Q_4} + \Delta_{1Q_5} + \Delta_{1G} + \Delta_{1\Phi} + \Delta_{1M} = 0, \\ \Delta_{2Q_1} + \Delta_{2Q_2} + \Delta_{2Q_3} + \Delta_{2Q_4} + \Delta_{2Q_5} + \Delta_{2G} + \Delta_{2\Phi} + \Delta_{2M} = 0, \\ \Delta_{3Q_1} + \Delta_{3Q_2} + \Delta_{3Q_3} + \Delta_{3Q_4} + \Delta_{3Q_5} + \Delta_{3G} + \Delta_{3\Phi} + \Delta_{3M} = 0, \\ \Delta_{4Q_1} + \Delta_{4Q_2} + \Delta_{4Q_3} + \Delta_{4Q_4} + \Delta_{4Q_5} + \Delta_{4G} + \Delta_{4\Phi} + \Delta_{4M} = 0, \\ \Delta_{5Q_1} + \Delta_{5Q_2} + \Delta_{5Q_3} + \Delta_{5Q_4} + \Delta_{5Q_5} + \Delta_{5G} + \Delta_{5\Phi} + \Delta_{5M} = 0. \end{cases} \quad (5)$$

Since each partial displacement is proportional to the force of its generating, then:

$$\begin{cases} \delta_{11}Q_1 + \delta_{12}Q_2 + \delta_{13}Q_3 + \delta_{14}Q_4 + \delta_{15}Q_5 = -\Delta_{1G} - \Delta_{1\Phi} - \Delta_{1M}, \\ \delta_{21}Q_1 + \delta_{22}Q_2 + \delta_{23}Q_3 + \delta_{24}Q_4 + \delta_{25}Q_5 = -\Delta_{2G} - \Delta_{2\Phi} - \Delta_{2M}, \\ \delta_{31}Q_1 + \delta_{32}Q_2 + \delta_{33}Q_3 + \delta_{34}Q_4 + \delta_{35}Q_5 = -\Delta_{3G} - \Delta_{3\Phi} - \Delta_{3M}, \\ \delta_{41}Q_1 + \delta_{42}Q_2 + \delta_{43}Q_3 + \delta_{44}Q_4 + \delta_{45}Q_5 = -\Delta_{4G} - \Delta_{4\Phi} - \Delta_{4M}, \\ \delta_{51}Q_1 + \delta_{52}Q_2 + \delta_{53}Q_3 + \delta_{54}Q_4 + \delta_{55}Q_5 = -\Delta_{5G} - \Delta_{5\Phi} - \Delta_{5M}, \end{cases} \quad (6)$$

where δ_{ik} – partial displacement of the i -th point of force's application from the action of a single value force in the direction of the k -th force's factor. The proportionality coefficients are determined through Mohr integrals:

$$\delta_{ik} = \int_l \frac{M_{Kk}M_{Ki}}{GJ_K} dl + \int_l \frac{M_{xk}M_{xi}}{EJ_x} dl + \int_l \frac{M_{yk}M_{yi}}{EJ_y} dl + \int_l \frac{N_kN_i}{EF} dl + \int_l \frac{k_x L_{xk}L_{xi}}{GF} dl + \int_l \frac{k_y L_{yk}L_{yi}}{GF} dl, \quad (7)$$

where M_{Ki} – internal torsional moment; M_{xi} – internal bending moment relative to the main axis of inertia of the cross section of the core element of the structure parallel to the support base; M_{yi} – internal bending moment relative to the main axis of inertia of the cross section of the core element, perpendicular to the support base; N_i – internal tensile force; L_{xi} – internal shear force parallel to the section axis x ; L_{yi} – internal shear force parallel to the section axis y .

All the above, internal force factors arise from the action of the i -th unit force. Given the orthogonality of forces, the system of canonical equations will take the following form:

$$\begin{cases} \delta_{11}Q_1 + \delta_{15}Q_5 = -\Delta_{1\Phi} - \Delta_{1M}, \\ \delta_{22}Q_2 + \delta_{24}Q_4 = -\Delta_{2\Phi}, \\ \delta_{33}Q_3 = -\Delta_{3G}, \\ \delta_{42}Q_2 + \delta_{44}Q_4 = -\Delta_{4\Phi}, \\ \delta_{51}Q_1 + \delta_{55}Q_5 = -\Delta_{5\Phi} - \Delta_{5M}. \end{cases} \quad (8)$$

Mohr integrals are calculated using the Vereshchagin method. For this purpose, all the diagrams of internal force factors are constructed from the action of each external force factor (reactions and active forces). Analysis of internal forces begins with the definition of a complete system of external forces (reactions). For example: the equilibrium equations determine the reactions from the action of a single force in the direction of the force Q_1 (see Figure 4):

$$\begin{aligned}
\Sigma M_{DD_1} &= 0 : F_1^n a - 1a = 0 \Rightarrow F_1^n = 1; \\
\Sigma Y_i &= 0 : -1 + F_1^n - F_4^n f_y = 0 \Rightarrow F_4^n = 0; \\
\Sigma X_i &= 0 : F_4^n f_x - F_1^\tau = 0 \Rightarrow F_1^\tau = 0; \\
\Sigma M_{D_1 A_1} &= 0 : (F_1^n - 1 - F_4^n f_y)h + N_3 b = 0 \Rightarrow N_3 = 0; \\
\Sigma M_{A_1 B_1} &= 0 : (N_3 + N_4)a - F_1^\tau (h - R) = 0 \Rightarrow N_4 = 0; \\
\Sigma Z_i &= 0 : N_1 + N_3 + N_4 = 0 \Rightarrow N_1 = 0.
\end{aligned} \tag{9}$$

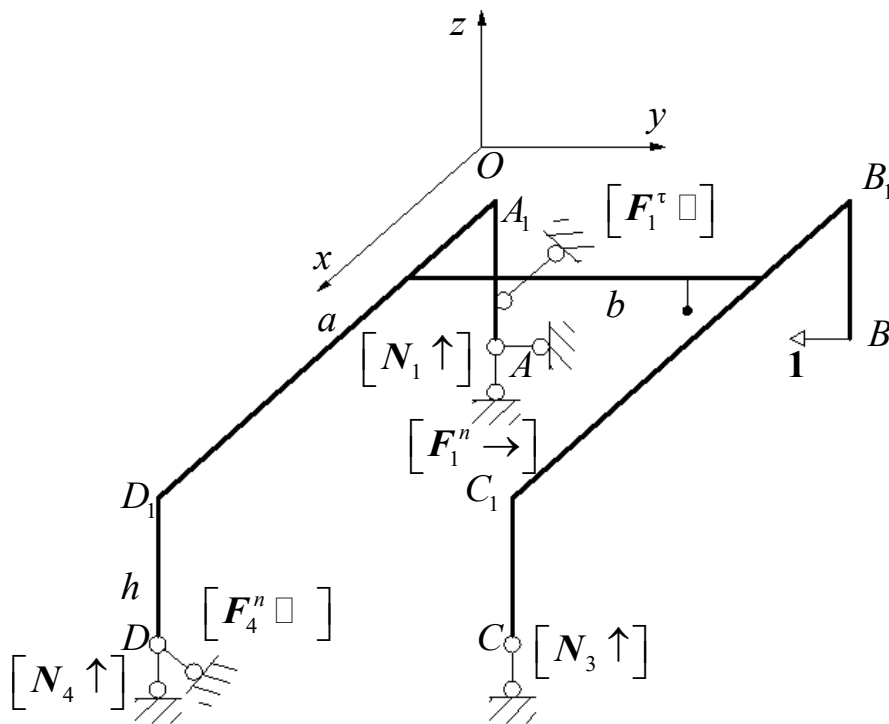


Figure 4. The scheme for calculating reactions from the action of a single force in the direction of Q_1 .

From the analysis of equations (9) it follows that the structure is in equilibrium under the action of only two: external and internal unit forces. Since the structure is symmetrical and symmetrical with the load, to determine the δ_{11} internal forces diagrams are constructed for a half frame (see figure 5). For the second half of the diagram will be symmetric.

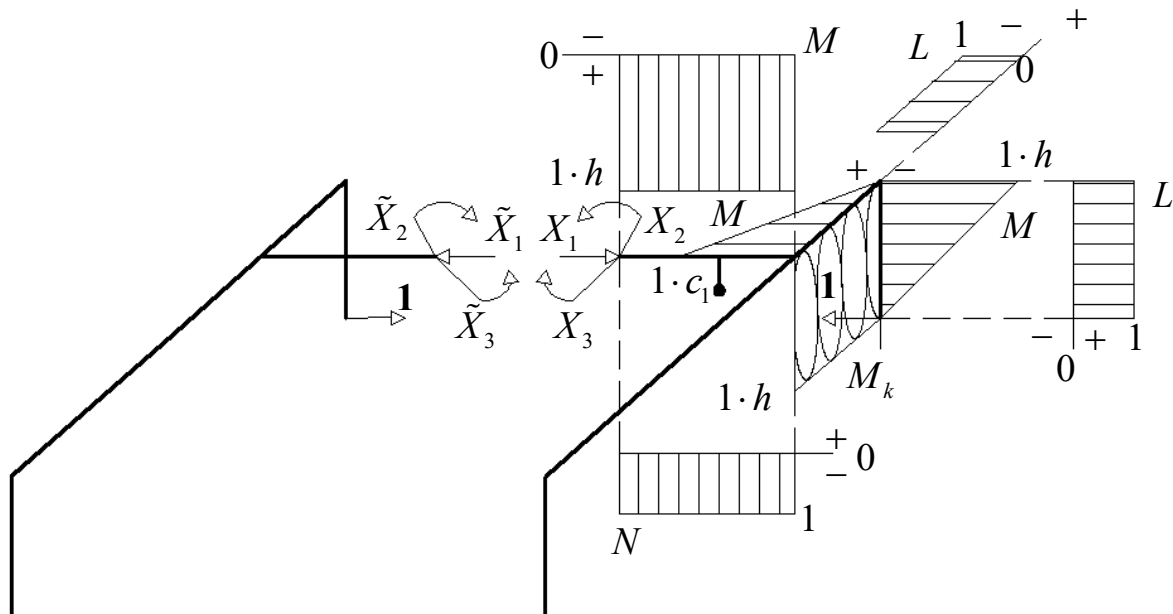


Figure 5. – Diagrams of internal efforts from the action of a single force in the direction Q_1

Considering symmetry diagrams of internal forces, δ_{11} is:

$$\delta_{11} = 2 \frac{1}{GJ_n} [1hc_1 1h] + 2 \frac{1}{EJ} \left[\frac{1}{2} hh \frac{2}{3} h + \frac{1}{2} c_1 c_1 \frac{2}{3} c_1 + \frac{b}{2} hh \right] + 2 \frac{1}{EF} \left[\frac{b}{2} \frac{1}{2} \right] + 2 \frac{k}{GF} [1h1 + 1c_1 1]. \quad (10)$$

3. Conclusion

The proposed approach of compiling additional equations (by the method of forces) allows one to determine software control forces for multiwheel transport robotized systems that move along a fairly rigid foundation without adjusting the pressure in the wheels (with a small deformation of the rubber-casing shell). The mathematical model does not describe fully the design features of the robot, so the inaccuracy of the forces calculation can be compensated (partially or completely) with the control system for the values of the parameters coming from the sensors.

Acknowledgments

The research was supported by Russian Foundation for Basic Research (Project No. 16-38-60086 mol_a_dk).

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