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Normal vibrations and the stability of plates with moving borders

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Abstract. In this paper vibrations of a thin elastic moving plate are investigated. The plate is fixed between vibrating mill rolls. Cases of cylindrical and general bending are considered. Natural frequencies and natural modes of the plate are obtained numerically. It is shown that a resonance is possible in this system under certain boundary conditions.

1. Introduction

Plates are widely applied in modern construction and engineering. Investigating the problem of movable plates between mill rolls is related to work of the rolling mill. Sometimes the rolling strip may be deflected during its movement. This occurs when the system frequencies are equal to the natural frequencies of the elastic plate. Detailed study of this problem is needed in order to avoid structural failures. The problem of plate and membrane vibrations is investigated in a lot of papers, e.g. [1], [2]. Paper [1] is devoted to the study of vibrations and the resonance of plates under constant tension. Paper [2] is devoted to free vibrations of the rectangular membrane.

In the current paper, the problem of the vibrations of the plate with moving borders under constant tension is considered. The expression of plate deflection is obtained and a case of the resonance is shown.

2. Setting of the problem

Consider a rectangular plate moving along the axis x at a constant velocity V as shown in figure 1. The plate is fixed between rotating mill rolls. It is assumed to be under constant tension T_0 along the axis x .

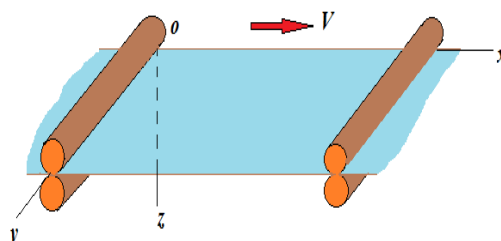


Figure 1. Moving plate fixed between rolls.



In the case of cylindrical bending the motion equation in the moving coordinate system ξ related to the plate can be written as follows:

$$\frac{\partial^4 w}{\partial \xi^4} - \frac{T_0}{D} \frac{\partial^2 w}{\partial \xi^2} = -\frac{m}{D} \frac{\partial^2 w}{\partial t'^2}, \quad (1)$$

here w is the transverse displacement, ξ is the coordinate that is co-directional with the x axis, D is the bending stiffness of the plate, m is the mass per unit length of plate, t' is the time.

Transform to a fixed coordinate system $(x; t)$ by using formulas:

$$x = \xi + Vt'; \quad t = t'; \quad \frac{\partial}{\partial \xi} = \frac{\partial}{\partial x}; \quad \frac{\partial}{\partial t'} = V \frac{\partial}{\partial x} + \frac{\partial}{\partial t}.$$

The equation of motion in dimensionless variables takes the following form:

$$\frac{\partial^4 w^*}{\partial x^{*4}} - r^2 \frac{\partial^2 w^*}{\partial x^{*2}} + p^2 \left(\frac{\partial^2 w^*}{\partial x^{*2}} + 2 \frac{\partial^2 w^*}{\partial x^* \partial t^*} + \frac{\partial^2 w^*}{\partial t^{*2}} \right) = 0, \quad (2)$$

where $p^2 = \frac{12(1-\nu^2)mV^2L^2}{Eh^3}$, $r^2 = \frac{12(1-\nu^2)T_0}{LEh^3}$, $x^* = \frac{x}{L}$, $t^* = \frac{Vt}{L}$, $w^* = \frac{w}{h}$, L is the distance between mill rolls, h is the plate thickness. For convenience we shall drop the sign $*$. We assume that the plate is fixed along two opposite edges parallel to the x axis:

$$w(x, t)|_{x=0} = 0, \quad \frac{\partial w}{\partial x}|_{x=0} = 0, \quad w(x, t)|_{x=1} = 0, \quad \frac{\partial w}{\partial x}|_{x=1} = 0.$$

We shall look for a solution in the form of harmonic vibrations $w(x, t) = Y(x)e^{-i\omega t}$. The boundary conditions for $Y(x)$ take the following form: $Y(0) = Y(1) = 0, Y'(0) = Y'(1) = 0$. Substituting this expression into equation (2), we obtain a forth-order differential equation for $Y(x)$

$$\frac{d^4 Y}{dx^4} - r^2 \frac{d^2 Y}{dx^2} + p^2 \left(\frac{d^2 Y}{dx^2} + 2(-i\omega) \frac{dY}{dx} + (-i\omega)^2 Y \right) = 0.$$

Its characteristic equation is of the following form:

$$\lambda^4 + \lambda^2(p^2 - r^2) + 2(-i\omega)p^2\lambda + (-i\omega)^2 p^2 = 0.$$

Denote the characteristic roots by $\lambda_k(\omega)$, $k = \overline{1, \dots, 4}$. Consider the case when these roots are distinct. Then general solution $Y(x)$ takes the form:

$$Y(x) = \sum_{k=1}^4 C_k e^{\lambda_k(\omega)x}. \quad (3)$$

Boundary conditions along the edges perpendicular to x axis provide a system of homogeneous equations in C_k :

$$\sum_{k=1}^4 C_k = 0, \quad \sum_{k=1}^4 e^{\lambda_k} C_k = 0, \quad \sum_{k=1}^4 \lambda_k C_k = 0, \quad \sum_{k=1}^4 \lambda_k e^{\lambda_k} C_k = 0. \quad (4)$$

For the non-trivial solution, the determinant of the coefficient matrix of this set of equation must vanish. On equating this determinant to zero, the frequency equation for the plate can be obtained.

$$F(\omega) = \begin{vmatrix} 1 & 1 & 1 & 1 \\ e^{\lambda_1} & e^{\lambda_2} & e^{\lambda_3} & e^{\lambda_4} \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \lambda_1 e^{\lambda_1} & \lambda_2 e^{\lambda_2} & \lambda_3 e^{\lambda_3} & \lambda_4 e^{\lambda_4} \end{vmatrix} = 0. \quad (5)$$

For given values of the parameters p and r , the real roots of (5) can be found numerically. For example, if $E = 2 \cdot 10^{11}$ Pa, $\nu = 0.33$, $h = 0.05$ m, $m = 770 \frac{\text{kg}}{\text{m}}$, $V = 2.35 \frac{\text{m}}{\text{s}}$, $L = 10$ m, $T_0 = 1000$ N, then the corresponding dimensionless frequencies $\omega_k = 123.259, 339.926, 666.44805, 1101.70935, 1645.79038, 2298.6876730 \dots$. From ω_k values, the values C_k can be obtained from (4). Then the $Y_k(x)$ can be found by relation (3).

2.1. Setting of the inhomogeneous problem

We shall assume plate is fixed along two opposite edges parallel to the x axis and the deflection is a time-dependent function. Physically this arises because of roll instability and certain other reasons. In this case boundary conditions can be expressed as follows:

$$w(x, t)|_{x=0} = w_0(t), \quad \frac{\partial w}{\partial x}|_{x=0} = 0, \quad w(x, t)|_{x=1} = w_1(t), \quad \frac{\partial w}{\partial x}|_{x=1} = 0.$$

Perform a change of variable in order to obtain homogeneous boundary conditions:

$$w(x, t) = u(x, t) + (-2w_1(t) + 2w_0(t))x^3 + (3w_1(t) - 3w_0(t))x^2 + w_0(t).$$

This enables us to reduce (2) to the following form:

$$\frac{\partial^4 u}{\partial x^4} - r^2 \frac{\partial^2 u}{\partial x^2} + p^2 \left(\frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 u}{\partial t^2} \right) = f(x, t). \quad (6)$$

Assuming that the plane is in a state of equilibrium and immovable at the initial moment, initial conditions can be expressed as follows:

$$u(x, t)|_{t=0} = 0, \quad \frac{\partial u(x, t)}{\partial t}|_{t=0} = 0.$$

The solution to equation (6) can be taken in the form:

$$u(x, t) = \sum_{k=1}^{\infty} Y_k(x) T_k(t). \quad (7)$$

Recall that $Y_k(x)$ is a natural mode.

Substituting (7) into equation (6), multiply this one by $Y_k(x)$ and integrate from zero to one. After some reduction, we obtain a differential equation for $T(t)$ subject to the initial conditions $T_k(0) = 0$, $T'_k(0) = 0$.

$$\frac{Y_k^2(x)}{2} \Big|_0^1 (2i\omega p^2 T_k(t) + 2p^2 T'_k(t)) + (\omega^2 p^2 T_k(t) + p^2 T''_k(t)) \int_0^1 Y_k^2(x) dx = \int_0^1 Y_k(x) f(x, t) dx \quad (8)$$

Solving this equation, obtain functions $T_k(t)$ and form of solution $u(x, t) = \sum_{k=1}^{\infty} Y_k(x) T_k(t)$.

3. The resonance case

The resonance case may be of particular interest. It occurs when the characteristic root of (8) is equal to frequency in the right part of the equation under consideration. It may be observed that $-i\omega_k$ is the characteristic root of (8), consequently if one of the boundary functions $w_1(t)$ and $w_0(t)$ has the same frequency there is a resonance in the system. For definiteness we shall consider a case when $w_1(t) = \cos(\omega_1 t)$ and $w_0(t) = 0$. Set the parameters of the problem are the same as done above. It

gives the expression of $T_k(t)$ and $W(x,t) = Y_k(x)T_k(t)$. The shape of the real part $W(x,t)$ for $x = 0.5$ is shown in figure 2.

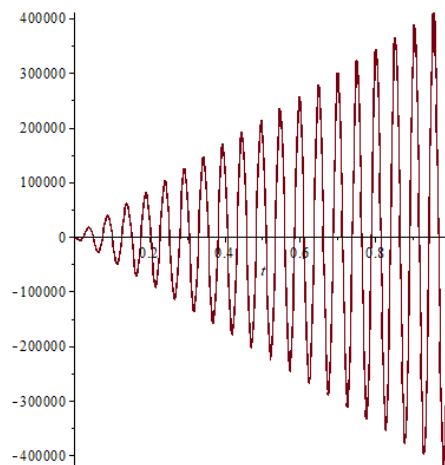


Figure 2. Shape of the real part of $W(x,t)$.

From the graphic, it is gathered that the deflection increases linearly with the time, hence the resonance occurs in the system.

4. General case of plate bending

The governing partial differential equation for the free vibration of a plate in a moving system can be written as

$$\frac{\partial^4 w}{\partial \xi^4} + 2 \frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + \frac{\partial^4 w}{\partial \eta^4} = -\frac{\mu}{D} \frac{\partial^2 w}{\partial t^2}. \quad (9)$$

As shown above, equation (9) in the fixed coordinate system takes the form

$$\frac{\partial^4 w}{\partial x^4} - r^2 \frac{\partial^2 w}{\partial x^2} + 2q^2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + q^4 \frac{\partial^4 w}{\partial y^4} + p^2 \left(\frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial^2 w}{\partial x \partial t} + \frac{\partial^2 w}{\partial t^2} \right) = 0.$$

Where $p^2 = \frac{12(1-\nu^2)mV^2L^2}{Eh^3}$, $r^2 = \frac{12(1-\nu^2)T_0}{LEh^3}$, $q^2 = \frac{L^2}{a^2}$, a is the band width. Assume that the plate is fixed along two opposite edges parallel to the x axis and that it is simply supported along other two edges. In this case, the boundary conditions along x and y are

$$x = 0, x = 1 : w = 0, \frac{\partial w}{\partial x} = 0, y = 0, y = 1 : w = 0, \frac{\partial^2 w}{\partial y^2} = 0.$$

Then the solution of the motion equation can be taken in the form

$$w(x, y, t) = \sum_n \sin(\pi n y) Y(x) e^{-i\omega_n t}.$$

Solving this problem for certain parameters in the same way, the expression of the deflection can be find in terms of double series:

$$w(x, y, t) = \sum_n \sin(\pi n y) \left(\sum_m Y_{nm}(x) e^{-i\omega_m t} \right).$$

The first natural frequencies are of great importance in practice. Thus, this is crucial to find the nature of their dependence on parameters of the problem. For certain values of parameters the numerical results are obtained and presented here in the graphical form in Fig 3 to 4. In Fig. 3 the

dimensional ω_1 is plotted against the dimensional L and in Fig. 4 the dimensional ω_1 is plotted against the dimensional V .

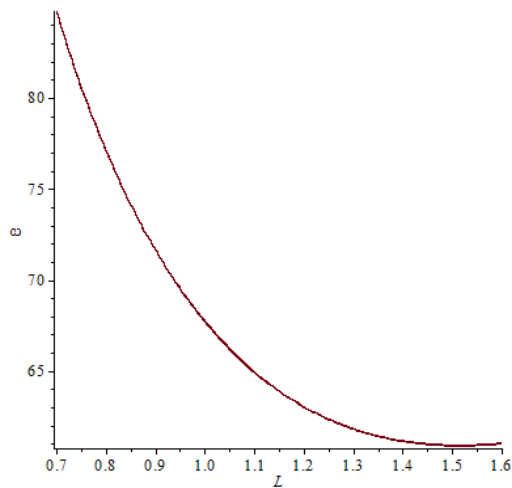


Figure 3. Shape of $\omega_1(L)$

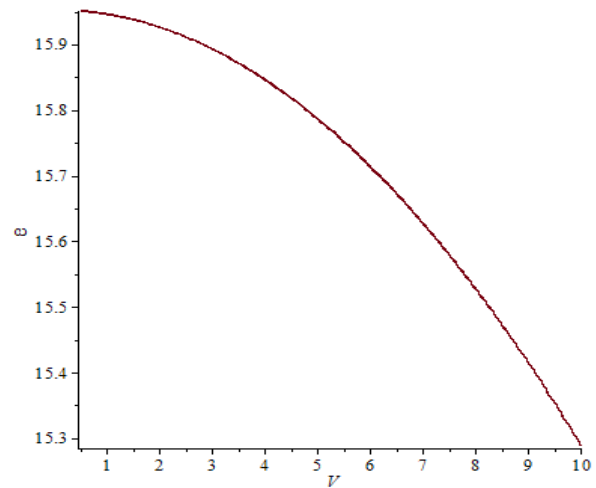


Figure 4. Shape of $\omega_1(V)$

5. Conclusions

The natural frequencies are also resonance frequencies. Thus their investigating can help to avoid resonance occurrence and structural failures. It was found in this paper that the first natural frequency remains static for growing V and T_0 . The first value of natural frequency ω_1 decreases with growth of the distance between rolls and band width and increases with growth of the plate thickness.

References

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- [2] Atamuratov A ZH 2013 Investigating methods of solving mathematical physics problems using an example of vibrations of a rectangular membrane *J. Young scientist* **10** 1-5