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On assessment of crack-type defect sizes with using results of modal analysis

A S Chernyatin and I A Razumovsky

Blagonravov Mechanical Engineering Research Institute of the Russian Academy of Sciences, 4 Maly Kharitonyevsky Pereulok, Moscow, 101990, Russia

murza45@gmail.com

Abstract. This paper presents the methodological approach to the assessment of geometrical and localization parameters for crack-type defects (including hidden subsurface flaws) by means mathematical treatment of experimental information represented by the amplitude displacement fields of researchable object registered by optic-interference methods.

1. Introduction

The goal of research, the results of which are presented in this paper, is to assess the capabilities of finding the geometrical parameters for crack-type defect and its localization zone based on the data of experimental analysis of oscillation natural modes and frequencies (simulation analysis).

As of today, numerous experimental methods (as well as appropriate devices) that provide finding capabilities of oscillation modes and frequencies applied to technical systems have been developed, e.g. holographic interferometry, electronic digital speckle interferometry, method of digitized image correlation, digital holography. These methods and devices make it possible to register the displacement field pictures of the examined object that are obtained at any point of time during dynamic process or the oscillations corresponding to definite natural mode of oscillation to a high precision ($\sim 0.1 \mu\text{m}$ /interference band) [1].

It is shown in this paper that the parameters defining defect sizes and location can be determined on the basis of minimizing the complex discrepancy between experimentally registered fields of vibration displacements and their computed values obtained from simulation analysis with FEM invocation at current values of required parameters. Similar approach has been applied successfully in [2,3] with the purpose of estimating the parameters of surface and internal defects in construction components on the basis of mathematical treatment of deformation response fields registered at static loads.

2. Problem definition

The model of a gas turbine blade (Figure 1a) in the form of the profiled plate permanently attached at the bottom (in section $x = 0$) is considered as a subject of inquiry; this plate has the following dimensions: length $L = 0.1$ m; width $W = 0.05$ m; maximal profile thickness $t = 0.01$ m. Plate material (steel): modulus of elasticity $E = 2 \times 10^5$ MPa; Poisson ratio $\mu = 0.3$; density $\rho = 7800$ m³/kg. The blade has a defect in the form of semi-elliptical flat surface crack with length $2a$ and depth b (a , b – axes of semi-ellipse). Location of the crack which is orthogonal to the lower flat surface of the plate is defined by localization parameters x_c, y_c (Figure 1b).



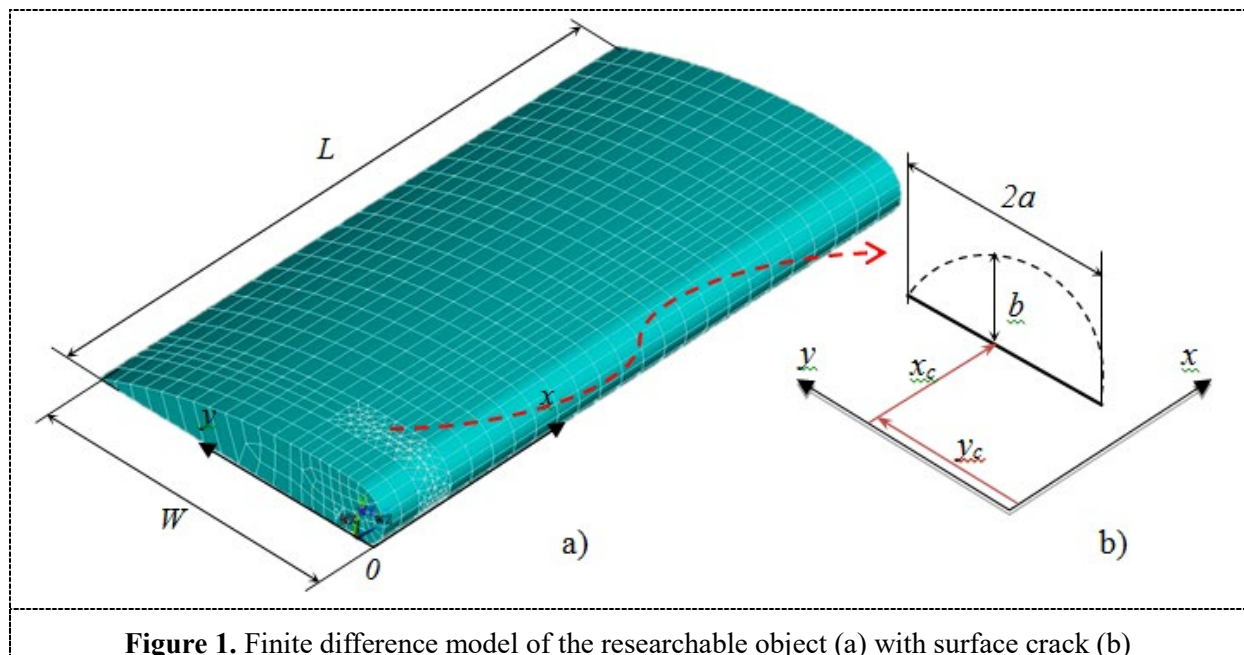


Figure 1. Finite difference model of the researchable object (a) with surface crack (b)

The objective is to assess the capability to determine defect parameters a, b, x_c, y_c (forming vector of unknowns $P = \{a, b, x_c, y_c\}$) through mathematical treatment of amplitude displacement field pictures for plate points $w(x, y)$ (w values at N points form the vector of $e = \{w_j\} j = 1, \dots, N$) corresponding to oscillation modes of the examined object at natural frequencies. It should be pointed out, that modal analysis does not immediately allow obtaining of real values of point oscillation amplitudes (they are defined up to a constant). In view of this several variants for normalizing the displacement fields corresponding to natural modes were developed by authors of this paper.

The finite-element model of the examined object in ANSYS circumference built using 3D elements with quadratic approximation is shown in Figure 1a. The model is divided into two zones: a zone with defect and the basic part. Along the crack front the grid has a crowding, the average size of elements is $(L+W)/100$. The finite-element model is parametric, and a special APDL macro was developed for its generation and subsequent calculation of natural frequencies and modes.

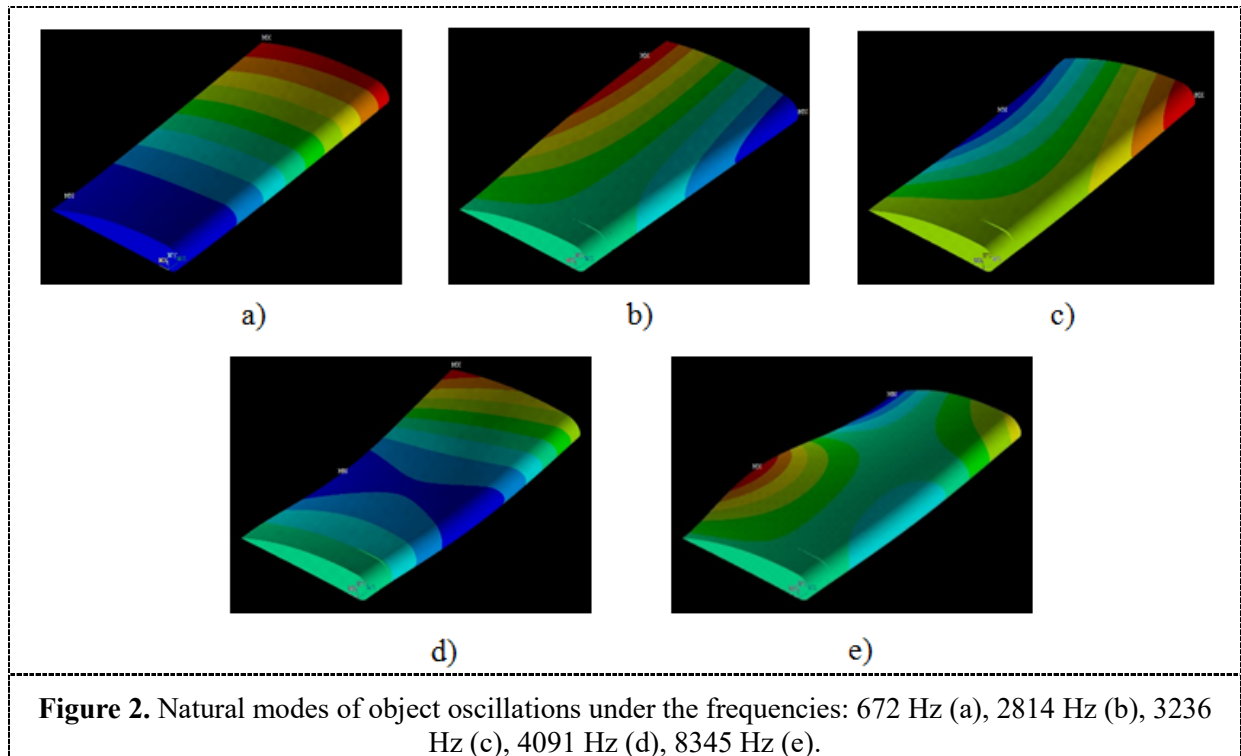
3. Calculation procedure

The proposed approach for determining $P = \{a, b, x_c, y_c\}$ vector on the basis of processing of displacement fields recorded experimentally consists of following stages:

- Handling the problem of modal analysis for the examined object – with the purpose of generating the so-called “response database” [2].
- Handling the problem of determining the required parameters of defect based on mathematical treatment of the interference images pertaining to the oscillation natural modes of the object as a multiparameter nonlinear optimization problem [2,3].
- Assessment of the effect of experimental data inaccuracies on the precision of determining the parameters characterizing the defect sizes and location.

The modal analysis carried out with the purpose of determining the natural frequencies and modes of oscillations based on solving of the eigenvalue problem is the “point of departure” for more complicated inquiries on dynamic system responses, which, in particular, can be used for defect identification. For the purpose of determining natural frequencies and modes of oscillations, the preconditioned conjugate gradient (PCG) method coupled with Lanczos algorithm was applied. This method is preferable to the direct methods of solving equations with sparse matrices (as it requires far less resources) and possesses a superlinear rate of convergence – even in the problems of high dimensionality and with ill condition sine qua non.

Figure 2 represents the calculation oscillation modes corresponding to the first five frequencies for the blade with the defect with parameters of $P_0^* = \{10; -5; 10; 5\}$ mm.



For the purpose of solving the problem of mode analysis for the object in question, a series of problems of finding the amplitude displacement for surface points of the examined object $w(x,y)$ under variation of vector P components is solved for the “provisioning” of the response database. This procedure is repeated for each natural mode. Based on the hyper-spline surfaces obtained in such a manner for each j -th component of e vector the approximation of the $e_i = F(P)$ form is set forth.

It should be noted, that the normalized module of the relative gradient g_w is used in the calculation, rather than the w displacement in the points of the examined object; this allows to circumvent the problem of uncertain scaling for displacement fields.

In the next step the vector P components are determined according to the displacement field $w^*(x,y)$ available from experiments and presented through the e^* vector of g_w gradient values in actual points of measurements on the basis of minimizing the objective function I , which reproduces the integrated divergence between e^* and e , which, in turn, is calculated by dint of well-formed response database. It should be noted that within the confines of this work the numerical experiment is performed when the w^* displacements are determined by the numerical finite element simulation. The defect with P_0^* parameters (see above) was adopted as the definable one, and the measurement points in which w^* value is calculated are located in the nodes of a rectangular mesh corresponding to the undamaged (inverse) surface of the blade (see Figure 1a). This way, the defect is “undetected” in relation to the surface of measurements. Mean-square deviation was adopted as an objective function. Taking into account the fact that optic-interference methods allow to obtain a practically unlimited volume of experimental information, the number of N points of displacement “measuring” was imputed as $17 \times 17 = 289$.

For the purpose of examining the effect of the errors of the w values measuring on the precision of determining the P defect parameters, a computed procedure of e^* vector “noise masking” was conducted by introducing random departures out of $\pm \Delta e$ range to each vector component with subsequent use of the provided approach of P vector determination. This procedure was repeated multiple times in order to obtain statistical parameters.

4. Results

The results of the numerical experiment for determining the defect parameters in the form of mathematical expectation M and dispersion D for P vector components which are classified as valid (i.e. corresponding to P_0^*) values are presented in table 1.

Table 1. Results of defect parameters determination.

Calculation conditions		Δe	x_c		y_c		a		b	
$N_0 N_0$	Oscillation mode		M	D	M	D	M	D	M	D
1	1	$\pm 15\%$	Not determined, exact values were given				0,131	0,231	1,124	0,240
2	1, 2						0,927	0,082	1,0314	0,048
3	1, 2, 3						0,964	0,007	1,023	0,002
4	1, 2, 3, 4						0,995	0,003	1,002	0,002
5	1, 2, 3		1,000	0,000	1,016	0,005	0,952	0,007	1,037	0,006
6	1, 2, 3, 4, 5		1,000	0,000	1,000	0,006	0,996	0,005	1,002	0,003

The following conclusions can be made on the basis of the results obtained.

- Under exceptionally wide adopted scatter band of experiment uncertainties the usage of registration results for the oscillation mode corresponding to one of natural frequencies does not assess the values of defect parameters. Herewith it is obvious that, unlike the sizes of hidden defect, the zone of its localization can be assessed with a precision eligible for practice based on visual examination of the oscillation mode.

- In the case of oscillation modes obtained for several natural frequencies, the proposed approach allows to determine both the localization and the geometric defect parameters reliably and precisely. One can expect that the precision of problem solving with the use of displacement fields registered at the поверхности выхода дефекта, will be higher, compared to the provided example, when the defect emerges to a surface that does not allow to register the displacement fields.

Acknowledgments

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