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Crack propagation simulation for ductile material under fretting fatigue loads using cohesive zone models

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Abstract. The object of this study is fretting fatigue cracks subjected to cyclic loading both normal and tangential. The cycle cohesive zone model (CCZM) and extended finite element method (XFEM) are used to describe the fretting behaviour. The fretting fatigue damage is described with a damage evolution equation. Both the extended finite element method as well as cohesive zone models have been implemented into the commercial FEM software ABAQUS, via the user interfaces UEL and UMAT. The CCZM parameters are obtained from a material fatigue test result and fretting fatigue experiment.

1. Introduction

Fretting fatigue caused by multi-axis cycle loads, is a typical failure phenomenon. It is a challenge to find proper experimental methods and theoretical models to describe the fretting fatigue process under multi-axis loading. Mutoh's [1] studies show that fretting fatigue cracks usually occur at the edge of contact.

As an effective fracture mechanics analysis method, the cohesion zone model has been widely used in the simulation of crack initiation and propagation processes. When it is combined with the traditional finite element method, it is necessary to arrange the cohesive region on the crack propagation path so that the cohesive region has predefined coincidences with the crack propagation path. However, for mixed mode cracks such as fretting fatigue cracks, the direction of crack propagation is one of the research objects, which limits the application of the cohesion model in engineering fracture problems. The extended finite element method (XFEM) can effectively avoid the above limitations [2]. When a certain rule is reached, the unit is defined as cohesive zone. The study found that in the process crack growth cohesion zone model parameters have little effect on the crack growth direction, whereas fracture criterion plays a major role.

Under cyclic loading, a change of contact area results in the dynamic change of the maximum principal stress of the contact edge and the position of the contact edge. Therefore, the determination of crack initiation position is difficult. For the composite crack in the contact area, the different crack initiation positions will cause a change of local stress state, which will affect the simulation of the fatigue crack propagation direction and the calculation of fatigue crack growth rate. In this paper, Xu's composite mode crack initiation criterion is used to accumulate the damage at each point in the cohesive zone, so as to judge the crack initiation position. The method is used to determine the fretting fatigue crack initiation position under cyclic loading in both normal and tangential directions.



2. Cohesive zone model (CZM)

2.1. Cohesive zone model

The cohesive zone model was proposed by Needleman based on strip-yield model [3]. The main purpose was to eliminate the stress singularity at the crack tip. Needleman simplified the interface failure as material separation in the cohesive zone, and proposed the potential energy as a function of the surface force-displacement characteristic curve (cohesive constitutive relation), and established the theoretical framework of the cohesive zone from initial separation to complete fracture process.

The theory of cohesion model is based on the assumption that there is a no thickness narrow strip ahead of the crack tip. This area is defined as a cohesive zone, as shown in Figure 1. In the cohesive zone, the original material constitutive relation is replaced by a cohesive constitutive law, which relates the cohesive traction to the separation displacement of the cohesive surface. Crack growth occurs when the separation at the tail of the cohesive zone (physical crack tip) reaches a critical value at which the cohesive traction vanishes. The cohesive zone model method does not involve the crack tip stress singularity in classical fracture mechanics. The material failure is controlled by displacement and stress equivalents, which is consistent with the general strength of material theory.

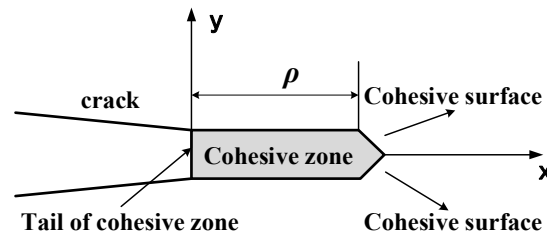


Figure 1. Cohesive zone at crack tip.

The damage of materials is described by the cohesion relation between displacement (δ_n, δ_t) and traction (T_n, T_t) under two-dimensional mixed mode loading. The cohesive zone model form is not fixed. There are a variety of cohesive zone models used to simulate the damage of materials.

At the initial stage of material damage, the traction in the cohesive zone increases with the increase of the opening displacement. This can be regarded as the hardening process of ductile materials. When the traction reaches the threshold value, the traction will decrease with an increase of the opening displacement.

For composite cracks, both normal and tangential opening displacements coexist and affect the propagation of cracks. Needleman uses a potential energy function to describe the relationship between surface forces and normal and tangential displacements.

$$\Phi(\delta_n, \delta_t) = \frac{9}{16} \sigma_{\max,0} \delta \left\{ 1 - \left[1 + z \left(\frac{u_n}{\delta} \right) - \frac{1}{2} \alpha z^2 \left(\frac{u_t}{\delta} \right)^2 \right] \exp \left[-z \left(\frac{u_n}{\delta} \right) \right] \right\} \quad (1)$$

The relationship between normal traction T_n and open displacement u_n , as well as the relationship between shear traction T_t and open displacement u_t can be obtained by derivatives of the above potential energy functions.

$$T_n = \partial \Phi / \partial u_n = \sigma_{\max,0} e^{\left[z \left(\frac{u_n}{\delta} \right) - \frac{1}{2} \alpha z^2 \left(\frac{u_t}{\delta} \right)^2 \right]} \exp \left[-z \left(\frac{u_n}{\delta} \right) \right] \quad (2)$$

$$T_t = \partial \Phi / \partial u_t = \sigma_{\max,0} e^{\left[\alpha z \left(\frac{u_t}{\delta} \right) \right]} \exp \left[-z \left(\frac{u_n}{\delta} \right) \right] \quad (3)$$

From equation (2) and equation (3), cohesion stiffness matrix T can be expressed as:

$$\begin{bmatrix} \dot{T}_t \\ \dot{T}_n \end{bmatrix} = T \begin{bmatrix} \dot{u}_n \\ \dot{u}_t \end{bmatrix}, \quad T = \begin{bmatrix} \frac{\partial T_n}{\partial u_n} & \frac{\partial T_n}{\partial u_t} \\ \frac{\partial T_t}{\partial u_n} & \frac{\partial T_t}{\partial u_t} \end{bmatrix} \quad (4)$$

2.2. Cyclic cohesion zone model for ductile materials

Combined with a damage evolution equation and a cohesion zone fracture criterion, a cyclic cohesion zone model (CCZM) is developed to calculate damage accumulation and simulate fatigue crack propagation. In the CCZM, the damage evolution equation is more decisive than the cohesion zone fracture criterion. In the process of fatigue damage, the strength of the material is determined by the fracture strength as well as cyclic cumulative fatigue strength. In the cyclic cohesion zone model (CCZM), both of above damage evolution equations are integrated to describe the fatigue crack evolution process [4].

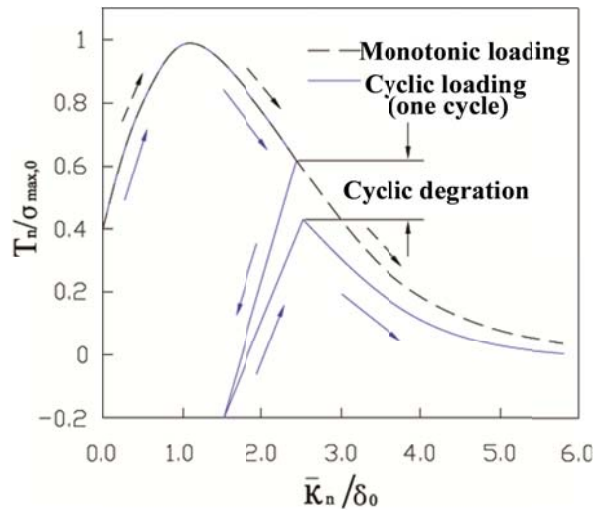


Figure 2. Cohesive zone model of ductile material.

The rupture of material under monotonic loading can be characterized by the conventional cohesive law, while the fatigue failure process under cyclic loading is described by accumulation of a linear unloading and reloading cumulative process. This mode of fatigue crack propagation is mainly mode I crack propagation, the normal traction mainly determines the propagation rate of a fatigue crack, and the effect of shear stress on damage can be neglected. The experimental results of crack propagation in cracked specimens show the monotonicity and the cyclic degradation process of the cohesive zone model for ductile material as shown in Figure 2. The fatigue failure of materials is mainly caused by damage accumulation under cyclic loading, which is determined by the damage accumulation equation. Therefore, the relationship between traction and opening displacement in the cohesion zone model under cycle loading can be expressed as follows:

$$T_n = \frac{\phi_n}{\delta_0} \exp\left(-\frac{\kappa_n}{\delta_0} - \frac{\delta_t^2}{\delta_0^2}\right), \quad T_t = \mu \frac{\delta_t}{\delta_0} \quad (5)$$

where

$$\kappa_n = \bar{\kappa}_n + \kappa_0 \quad (6)$$

μ is the shear stiffness, δ_t is the tangential opening displacement, δ_0 is the characteristic cohesive length, $\bar{\kappa}_n$ is the normal separation in the XFEM, and the parameter κ_0 stands for a model parameter

to consider the threshold value of cohesive traction. As $\kappa_0 = 0$, the cohesive energy ϕ_n generated by normal displacement can be calculated as follow:

$$\phi_n = e\sigma_{\max}\delta_0 \quad (7)$$

Where the Euler's number $e = \exp(1)$, and σ_{\max} is the cohesive normal strength of material. In the crack simulation under monotonic loading, the parameters of σ_{\max} and δ_0 is fitted to the experimental curve. Under cyclic loading, failure may occur even if the load level is obviously lower than the strength of the material, which indicates σ_{\max} decreases with the cyclic loading history.

3. Evolution equations for damage accumulation

The cyclic cohesion model combines the cyclic damage accumulation rule with the conventional cohesion rule to describe the fatigue damage accumulation process of materials. The accumulation of fatigue damage can be described by the damage variable D_c , which is determined by the fatigue damage evolution equation for the cyclic loading process. The fatigue damage evolution equation proposed by Xu [5] is as follows:

$$\dot{D}_c = \frac{|\dot{\kappa}_n|}{d_\Sigma} \left(\frac{T_n}{\sigma_{\max,0}} + 1 \right)^m H(\sigma_{eq} - f_0) \quad (8)$$

$\sigma_{\max,0}$ is the initial cohesive strength of the material, d_Σ is the accumulated cohesive length which scales the increment of material separation $\dot{\kappa}_n$, and m is the additional parameter in evolution equation. H represents the Heaviside function which signifies that nucleation of damage once the equivalent principal stress σ_{eq} , ahead of the cohesive zone tip is greater than the material fatigue limit f_0 .

In the process of fatigue damage failure of brittle materials, it is assumed that the separation in cohesive zone can be return to zero during unloading process. However, it is different from the process of damage for ductile materials, due to the potential local yielding of material. According to the suggestion in reference [6], it is assumed that the stiffness of the material will follow the following expressions during unloading and reloading.

$$k_n = \sigma_{\max,0} (1 - D_c) e / \delta_0 \quad (9)$$

This assumption leads to the existence of residual displacement. Furthermore, the material damage is caused in both unloading and reloading processes, except for the penetration, i.e. $\bar{\kappa}_n < 0$.

4. Criteria of crack growth

4.1. Non-local maximum principal stress criterion at crack tip

The non-local maximum principal stress criterion (modified maximum principal stress at the crack tip) is selected to control the crack propagation direction. In order to reduce the influence of element size on numerical stability, the average stress is usually used as the fracture criterion.

The non-local maximum principal stress criterion is mainly used to calculate the crack initiation life and crack propagation rate in the cohesive zone. The weighted average of n_{gp} Gaussian integral points in a circular region with a crack tip as the center is taken as the nonlocal stress. The weighting function can be expressed as [7]:

$$\hat{w}(r) = \frac{1}{l\sqrt{2\pi}} \exp\left(-\frac{r^2}{4l^2}\right), \quad w(r) = \frac{\hat{w}(r)}{\sum_{i=1}^{n_{gp}} \hat{w}_i A_i} \quad (10)$$

Non local stress is expressed as:

$$\tilde{\sigma}(x) = \sum_{i=1}^{n_{sp}} \sigma_i w_i A_i \quad (11)$$

r is the distance from Gauss point to the tip of cohesive zone, and l is the decreasing speed of weight function \hat{w} as r increasing. The nonlocal stress tensor is obtained by the sum of the local stress σ_i at Gauss point i and the product of weighting function w_i and the area A_i .

The nonlocal maximum principal stress at the tip of the cohesive region will be used to compare with the fatigue limit f_0 of the material. The failure process of cyclic cohesive models for quasi brittle materials and ductile materials is different. For brittle materials, when the non-local maximum principal stress exceeds the fatigue limit f_0 , the σ_{\max} begins to decrease, which is considered to be the incubation period of the cohesive region. The additional degree of freedom of the element is not activated, and the displacement of the element is still in a continuous state; once the traction force exceeds the strength σ_{\max} of the material, the additional degree of freedom of the element is activated, and the tip of the cohesive zone extends to the element, and the crack enters the initiation stage in the element. For ductile materials, the incubation period of damage accumulation for quasi-brittle materials will not be considered. When the maximum principal stress exceeds the fatigue limit f_0 , the tip of the cohesion zone will be extended to the element immediately.

4.2. Comprehensive stress criterion for crack initiation and propagation direction

The results of fretting fatigue experiments show that the direction of initiation of fretting fatigue cracks on the contact surface depends on the shear action of the material, and when the crack extends to the interior of the material, the direction of propagation of the crack depends on the tensile action of the material.

For plane stress problems, the maximum nonlocal principal stress (NMPS) direction θ_n can be obtained by nonlocal stress calculation. According to the criterion, the direction of the crack is perpendicular to the direction of the non-local maximum principal stress.

$$\tan(2\theta_n) = \frac{-2\tilde{\tau}_{xy}}{\tilde{\sigma}_x - \tilde{\sigma}_y} \quad (12)$$

At the same time, the direction of the largest non-local shear stress (NMSS) can be obtained through the following expressions:

$$\tan(2\theta_t) = \frac{\tilde{\sigma}_x - \tilde{\sigma}_y}{2\tilde{\tau}_x} \quad (13)$$

In order to judge the direction of crack initiation by using the maximum shear stress during the crack initiation stage, a comprehensive stress criterion is introduced to judge the direction of crack propagation by using the maximum principal stress during the crack propagation stage. The criteria can be expressed as [8]:

$$\theta = \begin{cases} \theta_t & \text{or } \theta_t + \pi/2 \\ \theta_n & + \pi/2 \end{cases} \quad \begin{cases} \text{if } \frac{T_n - \sigma_0}{\sigma_0} < \frac{|T_t| - \tau_0}{\tau_0} & \text{and } |T_t| > \tau_0, \\ \text{if } \frac{T_n - \sigma_0}{\sigma_0} > \frac{|T_t| - \tau_0}{\tau_0} & \text{and } T_n > \sigma_0 \end{cases} \quad (14)$$

σ_0 and τ_0 are cohesive zone thresholds under tensile and shear failure respectively. The first one represents the direction of shear failure and the second represents the direction of tensile failure. When the principal stress or shear stress is greater than the corresponding threshold value, the cohesion region expands along this direction in the XFEM element.

5. Fretting fatigue crack simulation

It is assumed that the material for fretting fatigue is ZL702A high strength aluminium alloy. The aluminium alloy is a typical ductile material. Therefore, Xu-Needlman model of ductile material is used as the cohesion model in the numerical calculation of the fretting fatigue crack initiation and propagation. The fatigue evolution equation is based on Xu's fatigue damage evolution model suitable for ductile materials.

Using the cyclic cohesion model to predict the fatigue crack, the cohesion parameters and damage parameters must be determined. These parameters include cohesive length δ_0 , cohesive threshold T_0 , initial cohesive strength $\sigma_{\max,0}$, fatigue limit f_0 of cohesion region, and cumulative cohesive length d_Σ . Through calculation and analysis, the cyclic cohesion and damage parameters of ZL702A under multi-axial cyclic loading are obtained, which is shown in Table 1

Table 1. Cohesive parameters and damage parameters of ZL702A aluminium alloy.

σ_b (MPa)	δ_0 (mm)	T_0 (MPa)	$\sigma_{\max,0}$ (MPa)	f_0 (MPa)	d_Σ
260~300	0.0022	137.9	882.6	3.38%	215

Figure 3 shows a local fretting fatigue damage calculation model for contact between the body and the main bearing cover. The model size is 6.24mm×6.24mm, and the contact edge fillet size of the main bearing cover is 1mm. A cohesion zone is set at the contact edge between punch and specimen, which is shown in Figure 4.

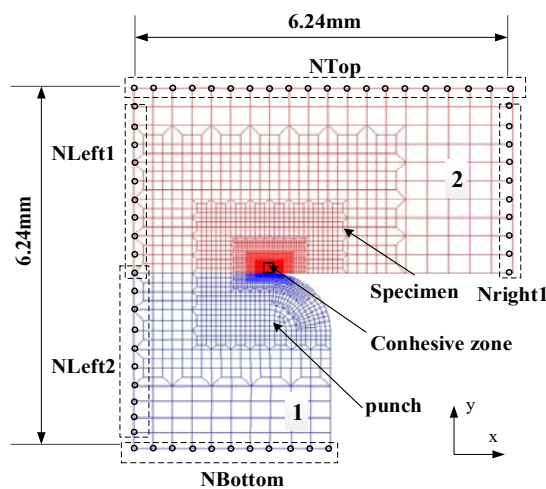


Figure 3. Calculation model of fretting fatigue.

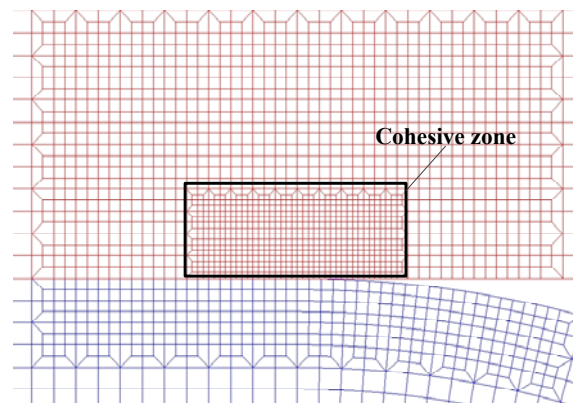


Figure 4. Cohesive zone of the model.

The size of cohesion judgment zone is 0.4 mm×0.16 mm, the mesh size is 0.01×0.01 mm, and there are 606 units in cohesive zone. The whole calculation model has 4503 units. A contact surface is set between the punch and the specimen. The contact algorithm adopts the Lagrange multiplier method, and the friction coefficient between the contact surfaces is 0.6.

The ABAQUS calculation input file is generated according to the pre-defined cohesive force model cycle number. Subsequently, the self-defined subroutine of the extended finite element method combined with the cohesion model and the input file of ABAQUS calculation are submitted to the ABAQUS solver, which outputs the resulting information through calculation. Since the post-processing program of ABAQUS CAE cannot display user-defined units, it is necessary to post-process the results through PYTHON script to generate visual result files.

Sinusoidal loads were applied to the model along the Y-axis and z-axis. The above load phase difference was 90 degrees, and the load peak value was 6500 N. As shown in Figure 5, after 1.5×10^5 cycles, it can be seen that the crack originates at the stress concentration position at the edge of the contact zone, and the crack initiation angle is about 45 degrees. The result is basically consistent with

the actual crack initiation angle in Figure 6, which shows that the fretting fatigue crack initiation point and propagation path in the contact area of the standard specimen under multi-axial loading. The crack propagates about 0.1 mm along 45 degrees and then perpendicular to the contact surface. The final crack length is 0.14 mm.

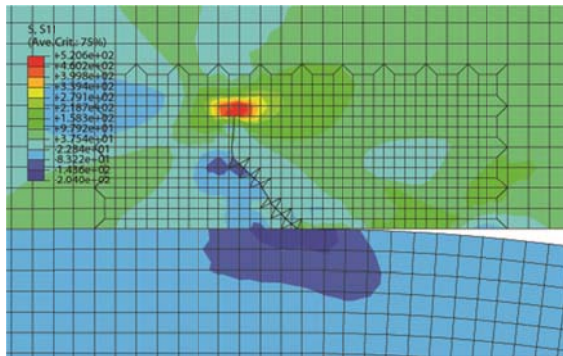


Figure 5. Simulation results of local fretting fatigue cracks under multi-axial cyclic loading.

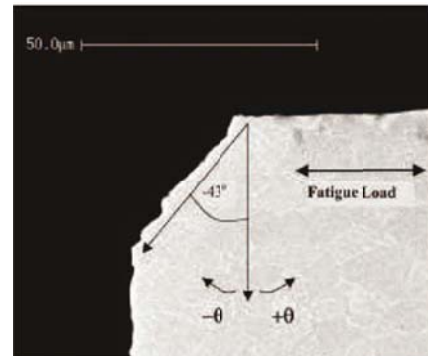


Figure 6. Actual fretting crack initiation angle.

6. Conclusions

The cyclic cohesion model and the extended finite element method are suitable for predicting the crack initiation and propagation in a fretting fatigue contact region. The relationship between fretting fatigue load and local fretting fatigue crack initiation and propagation can be obtained by this method. The calculated results are consistent with the actual crack characteristics.

Acknowledgments

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