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Nonlinearities Evaluation in the Aircraft Parameter Identification

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Abstract. The authors consider the algorithmic and methodological support for estimating nonlinear dependencies when performing the parameter identification of the aircraft mathematic models using the flight test data. Examples of the considered methods practical application to identifying the nonlinear parameters of several modern aircrafts are presented.

1. Introduction

The work refers to the problem of algorithmic and methodological support for aircraft parameter identification from the flight tests data. Currently in this area a significant progress is achieved. Thus, the basic approaches and the main stages of the identification technology are described in [1-3], the identification algorithms and examples of practical use in [4-7]. However, the problem of improving identification methods still remains relevant. The identifying the aerodynamic coefficients of an aircraft is a complex task, the solution of which should apply a wide range of techniques: decomposing a general task into a number of simpler ones, assessing the reliability of results by comparing data from different sources, using reliable results from related disciplines. The paper discusses the use of these techniques for solving one of the most difficult tasks of processing flight experiment data, which include the identification of nonlinear aerodynamic dependencies. At the same time, much attention is paid to the form of presentation of results, since in the task of identification, taking into account the characteristics of the operator and convenience of perception of the results significantly increase the interpretation.

2. Mathematical model of spatial motion

In the problem of identifying the aircraft aerodynamic coefficients, the most reliable result is the spatial motion equations, obtained directly from the basic laws of mechanics. These differential equations form the following non-linear system, where it is assumed that the aircraft has a symmetry plane[8]:

$$\frac{d\alpha}{dt} = \omega_z - \frac{1}{\cos \beta} \left[\left(\frac{a_x}{V} - \omega_y \sin \beta \right) \sin \alpha + \left(\frac{a_y}{V} + \omega_x \sin \beta \right) \cos \alpha \right],$$

$$\frac{d\beta}{dt} = \frac{a_z}{V} \cos \beta - \left(\frac{a_x}{V} \sin \beta - \omega_y \right) \cos \alpha + \left(\frac{a_y}{V} \sin \beta + \omega_x \right) \sin \alpha,$$



$$\begin{aligned}
\frac{dV}{dt} &= a_x \cos \alpha \cos \beta - a_y \sin \alpha \cos \beta + a_z \sin \beta, \\
\frac{d\omega_z}{dt} &= \frac{I_x - I_y}{I_z} \omega_x \omega_y + q \frac{S b_A}{I_z} m_z - \frac{k_d \omega_y}{I_z} - \frac{(P_{right} + P_{left}) y_d}{I_z} + \frac{I_{xy} (\omega_x^2 - \omega_y^2)}{I_z} \\
\frac{d\omega_y}{dt} &= \frac{I_x (I_z - I_x) - I_{xy}^2}{I_x I_y - I_{xy}^2} \omega_x \omega_z + \frac{k_d \omega_z I_x}{I_x I_y - I_{xy}^2} + \frac{(P_p - P_l) z_d I_x}{I_x I_y - I_{xy}^2} - \\
&\quad - \frac{I_{xy} (I_z - I_x - I_y)}{I_x I_y - I_{xy}^2} \omega_y \omega_z + \frac{I_x q S l m_y + I_{xy} q S l m_x}{I_x I_y - I_{xy}^2}, \\
\frac{d\omega_x}{dt} &= \frac{I_y (I_y - I_z) + I_{xy}^2}{I_x I_y - I_{xy}^2} \omega_y \omega_z - \frac{I_{xy} (I_y + I_x - I_z)}{I_x I_y - I_{xy}^2} \omega_x \omega_z + \\
&\quad + \frac{I_y q S l m_x + I_{xy} q S l m_y}{I_x I_y - I_{xy}^2} + \frac{I_{xy} (k_d \omega_z + (P_{right} - P_{left}) z_d)}{I_x I_y - I_{xy}^2}, \\
\frac{d\vartheta}{dt} &= \omega_y \sin \gamma + \omega_z \cos \gamma, \\
\frac{d\gamma}{dt} &= \omega_x - \tan \vartheta (\omega_y \cos \gamma - \omega_z \sin \gamma), \\
\frac{d\psi}{dt} &= \frac{1}{\cos \vartheta} (\omega_y \cos \gamma - \omega_z \sin \gamma),
\end{aligned} \tag{1}$$

$$\frac{dH}{dt} = V [\cos \alpha \cos \beta \sin \vartheta - \sin \alpha \cos \beta \cos \vartheta \cos \gamma - \sin \beta \cos \vartheta \sin \gamma]$$

acceleration along the body-fixed axes

$$\begin{aligned}
a_x &= \frac{qS(-c_x + c_p)}{m} - g \sin \nu, \\
a_y &= \frac{qSc_y}{m} + g \cos \nu \sin \gamma, \\
a_z &= \frac{qSc_z}{m} + g \cos \nu \sin \gamma
\end{aligned} \tag{2}$$

overloads along the body-fixed axes:

$$n_x = \frac{qS(-c_x + c_p)}{mg}, n_y = \frac{qSc_y}{mg}, n_z = \frac{qSc_z}{mg}, \tag{3}$$

where α, β – angles of attack and sideslip, rad; $\omega_x, \omega_y, \omega_z$ – angle rates about body-fixed axes frame, rad/s; ν, γ, ψ – angles of pitch, roll, yaw, rad; V – true air speed, m/s; H – flight altitude, m; m_x, m_y, m_z – coefficients of aerodynamic torques; c_x, c_y, c_z – coefficients of aerodynamic force in body-fixed coordinates system; I_x, I_y, I_z, I_{xy} – momentum of inertia about axes of body-fixed frame, kg·m²; m – aircraft's weight, kg; l, b_A – wing span and length of mean aerodynamic chord, m; S – wing surface area, m²; $q = \rho_H V^2 / 2$ – dynamic air pressure, Pa; ρ_H – air density at the altitude of flight, kg/m³; $c_p = P/qS$ – thrust coefficient; P_{right}, P_{left} – thrust of right and left engines, N; k_d – kinematic momentum of engine rotors, kg·m²/s; y_d, z_d – engine coordinates in body-fixed axes frame, m.

The system of differential equations (1), and algebraic relations (2) and (3), enables to construct a model to check the consistency of onboard measurements [1, 6], as well as models to identify longitudinal and lateral movement [1-3].

3. Identification of aerodynamic characteristics

Identification of aerodynamic characteristics should be carried out separately for the longitudinal and lateral channels. The corresponding models can be easily obtained from the general model (1) with regard to formulas (2) and (3) in the same way as in [1, 6] the model is obtained for checking the flight data. To identify the lift coefficients and the pitch moment, it's enough to take the equations for the angle of attack, the angular velocity of pitch as the model of the object, and in the model of observations to use the angle of attack, angular velocity and normal overload. To identify the lateral movement coefficients, the model for the object must include equations for the sideslip angle and angular velocities of the roll and yaw. The observation vector contains the measured values of the same signals, that is, full-component observations, as well as lateral overload. When identifying a longitudinal channel, the variables of lateral movement are replaced by measurements [5], and vice versa.

Aerodynamic characteristics are usually non-linear. The generally accepted approach is to use the data partitioning [1], that is to sort the data, for example, by the angle of attack: $\alpha \in [\alpha_k, \alpha_{k+1}]$, $k=1, 2, \dots, N_k-1$, where $[\alpha_k, \alpha_{k+1}]$ is the interval of the values of the angle of attack 2 ... 4 degrees wide; N_k - the number of sections splitting the full range of variation of the angle of attack. It is assumed that in this area the object is linear. Then, for example, the longitudinal channel coefficients may be presented

$$c_{ye} = c_{y0} + c_y^\alpha \alpha + c_y^{\delta_B} \delta_B,$$

$$m_z = m_{z0} + m_z^\alpha \alpha + m_z^{\delta_B} \delta_B + m_z^{\bar{\omega}_z} \cdot \frac{b_A}{V} \omega_z + m_z^{\ddot{\alpha}} \cdot \frac{b_A}{V} \frac{d\alpha}{dt}$$

However, the linearity assumption is not always true. Therefore, to approximate non-linear dependencies, polynomials or splines are used, the coefficients of which are included into the vector of identifiable parameters. The orders of polynomials or splines in this case are usually chosen not higher than the third. For example, the following approximations are used to estimate the dependence of the pitch moment coefficient on the angle of attack:

- polynomials of the second or third order

$$m_z(\alpha) = m_{z0} + m_z^\alpha \alpha + m_z^{\alpha^2} \alpha^2, \quad m_z(\alpha) = m_{z0} + m_z^\alpha \alpha + m_z^{\alpha^2} \alpha^2 + m_z^{\alpha^3} \alpha^3$$

- 1-order splines

$$m_z(\alpha) = m_z(\alpha_k)(1-t) + m_z(\alpha_{k+1})t,$$

$$\text{where } \alpha \in [\alpha_k, \alpha_{k+1}], k=1, 2, \dots, N_k-1,$$

$$t = \frac{\alpha - \alpha_k}{h_k}, \quad h_k = \alpha_{k+1} - \alpha_k$$

- 3-order Hermitian splines, differing from classical cubic splines in that they do not require solving equations for nodal points

$$m_z(\alpha) = \phi_1(t)m_z(\alpha_k) + \phi_2(t)m_z(\alpha_{k+1}) + \phi_3(t)h_k \frac{dm_z(\alpha_k)}{d\alpha_k}(\alpha_k) + \phi_4(t)h_k \frac{dm_z(\alpha_{k+1})}{d\alpha_{k+1}}(\alpha_k),$$

$$\text{where } \phi_1(t) = (1-t)^2(1+2t), \phi_2(t) = t^2(3-2t), \phi_3(t) = t(1-t)^2, \phi_4(t) = -t^2(t-1)$$

and the variables are defined above.

The polynomials or splines coefficients are included into the vector of estimated parameters. To identify this vector of unknown parameters, as in the previous case, the maximum likelihood method is used [1-3]. The effectiveness of this approach strongly depends on whether the order of the polynomial or the location of the spline nodes corresponds to an unknown nonlinearity.

4. Graphic representation of nonlinearities

Identification is performed in the dialogue mode, therefore it is very important to take into account the peculiarities of the human operator perception. To improve the presentation of the results in the longitudinal channel, the following approach is proposed. From (1) - (3) it follows that estimates of

the lift coefficient in a velocity coordinate system and the pitch moment coefficient for discrete points in time can be calculated as follows:

$$c_{ye}(t_i) = \frac{(n_y(t_i) \cos \alpha(t_i) + n_x(t_i) \sin \alpha(t_i)) mg - P \sin(\alpha(t_i) + \phi_d)}{qS}, \quad (4)$$

$$m_z(t_i) = \left(\frac{J_z}{qS b_A} \right) \left(\frac{d\omega_z(t_i)}{dt} - \frac{J_x - J_y}{J_z} \omega_x(t_i) \omega_y(t_i) + \frac{(k_{\phi} \omega_y(t_i) + P y_d)}{J_z} \right)$$

where ϕ_d is the installation angle of the engines, and the other symbols are explained in the description of the model (1) - (3). When using the formulae (4) the pitch angular velocity derivative we find numerically [1,6], we set the thrust according to the a priori characteristics of the engine.

Consider the widespread special case when nonlinear coefficient models have the form:

$$c_{ye} = c_{ye}(\alpha) + c_{ye}^{\delta_B} \delta_B, \quad m_z = m_z(\alpha) + m_z^{\delta_B} \delta_B + m_z^{\bar{\omega}_z} \frac{b_A}{V} \omega_z + m_z^{\bar{\alpha}} \frac{b_A}{V} \frac{d\alpha}{dt} \quad (5)$$

The expression for the moment coefficient we write in the form

$$m_z = m_z(\alpha) + m_z^{\delta_B} \delta_B + (m_z^{\bar{\omega}_z} + m_z^{\bar{\alpha}}) \frac{b_A}{V} \omega_z, \quad (6)$$

where the complex of rotational derivatives is represented as one parameter [6,8].

Now we will define an approximation structure for nonlinearities (5) - (6), which in general is unknown at this stage, and perform parametric identification. Experience shows that the accuracy of estimates of linear coefficients is usually high while the nonlinearities parameters estimates are approximate. Then, using the obtained estimates of the linear coefficients and formulas (4), one can present the estimates of nonlinear dependences in an explicit form as a function of time:

$$c_{ye}(\alpha(t_i)) = c_{ye}(t_i) - \tilde{c}_{ye}^{\delta_B} \delta_B(t_i), \quad (7)$$

$$m_z(\alpha(t_i)) = m_z(t_i) - \tilde{m}_z^{\delta_B} \delta_B(t_i) - (\tilde{m}_z^{\bar{\omega}_z} + \tilde{m}_z^{\bar{\alpha}}) \cdot \frac{b_A}{V} \omega_z(t_i) \quad (8)$$

Next, graphs of these estimates are plotted in the function of the angle of attack, which graphically displays nonlinearity. This enables to determine the structure of nonlinearity, that is, the order of the polynomial or the location of the spline nodes.

5. Examples of non-linear dependencies identification using flight test data

The results of applying the above approach to flight tests data are presented in Figure 1. The operational range angles attack was divided into intervals (data partitioning) in which the aircraft control stick symmetrical deflections were performed so that the increments of the angle of attack were $\pm (2 \dots 3)$ degrees. In figure 1, the estimates, calculated by formula (8), are shown as points, which merge into a “cloud” due to the large number of samples. It is clearly seen that at angles of attack above 11 degrees non-linear dependencies take place. Maneuvers at all 5 intervals were performed with overlapping, which allows to confirm the consistency of the results.

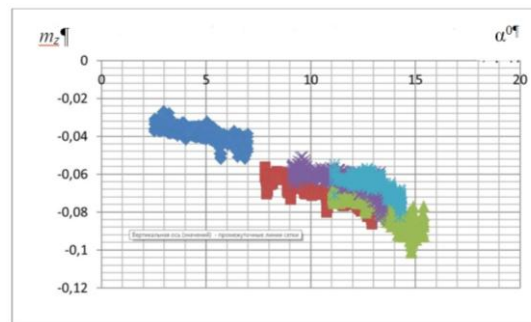


Figure 1. The results of identification of dependency when splitting into intervals.

In figure 2, for another aircraft, identification estimates of the non-linearity are shown as 5 “clouds” in comparison with the estimates obtained from wind tunnel tests (shown by large dots). In general, the degree of compliance is high, despite some differences at angles of attack of more than 14 degrees.

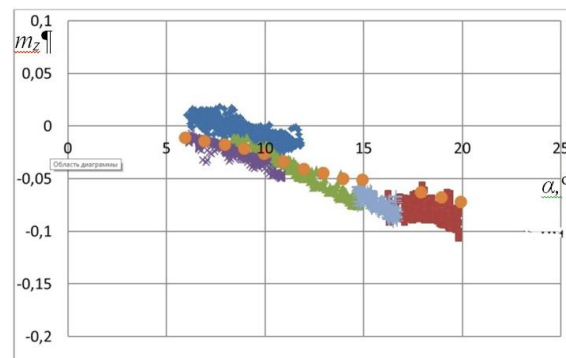


Figure 2. Comparison of the pitch moment identification estimates and the wind tunnel data.

In figure 3, the identification estimates of the lift coefficient from 5 overlapping flight intervals coincide very closely with the wind tunnel data.

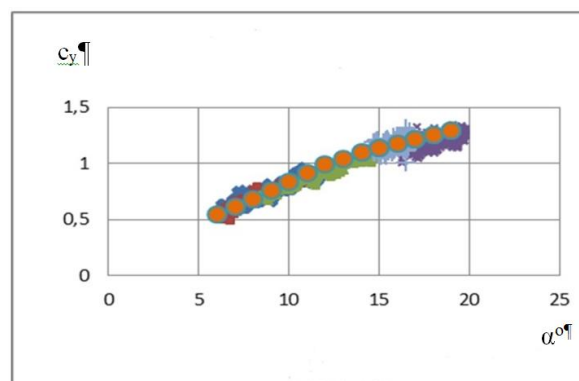


Figure 3. Comparison of the lift force identification estimates and the wind tunnel data.

Consider an example that characterizes the importance of the data presentation in the practical implementation of the aerodynamic parameters identification. In figure 4 for six flight segments, estimates of the lift coefficient by the maximum likelihood method are shown (six colored lines), in which 2-order polynomials were used to describe nonlinearity. The same figure shows the dependence taken from the wind tunnel data (indicated by the blue line). Initially, when analyzing the identification results for 6 areas, estimates of the coefficients of polynomials turn out to be significantly different. Then it is decided to display the polynomials on the same graph so that the identification estimates are displayed only for those values of the angle of attack that took place in the flight test interval. From figure 4 it is clear that this methodical technique allows us to conclude that there is a high degree of consistency between the estimates for different areas and, therefore, that the bank does not comply with the identification estimates obtained in the flight experiment. Indeed, the graphs of polynomial estimates have different slopes and displacements (that is, different coefficients of polynomials), but in general they are close to each other.

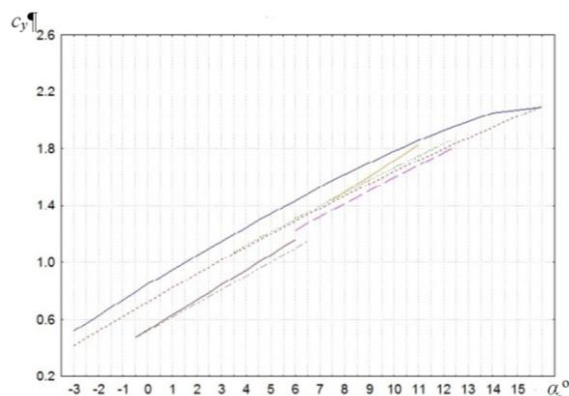


Figure 4. Comparison of polynomial lift coefficient estimates with and the wind tunnel data.

6. Conclusion

The article presents algorithmic and methodological approaches to the processing of flight data when identifying aircraft non-linear aerodynamic coefficients according to flight tests in the operational range of angles of attack. Examples of practical application of the considered methods and algorithms for identifying the aerodynamic coefficients of modern aircraft are presented. The value of the form of presenting the identification results for their correct interpretation is shown.

The presented algorithmic and methodical results make it possible to correct the mathematical models of aircraft according to flight test data.

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