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Calculation and analysis of nonlinear dynamic model based on the characteristic line method

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Abstract. To solve the problem that the impact dynamics model is nonlinear and cannot be resolved the analytical solution, the differential equations of the characteristic line compatible relation are constructed, the initial value conditions and the boundary value conditions are determined. The dynamic state of the whole domain is gradually recurred by selecting the appropriate time step and the space step. The theoretical results are consistent with the experimental results, indicating that the method of solving the model is feasible. This method provides an effective theoretical analysis and calculation method for the study of dynamic characteristics of structures under impact.

1. Introduction

The solution of the nonlinear dynamic model is the basis of studying the dynamic characteristics of the structure under the impact environment. Because the impact dynamics model is nonlinear, the analytical solution cannot be found for the nonlinear problems. To solve this problem, the theory and application of nonlinear viscoelastic wave propagation are discussed in the reference [1]. The numerical analysis method is used to simulate the constitutive model constructed in [2-3]. Two-stage finite element modeling and analysis techniques are used in the reference [4], and this method is used to analyze the overall stability and local residual strength of structural frame systems and components. The impact analytical model based on Hertzian elastic contact law is proposed in reference [5], and the analytical method is given. The one-dimensional dynamic model is solved by a precise integration method in reference [6], and the structural dynamic response of missile-borne devices is obtained under a high overload environment. In this paper, the differential equations of the characteristic line compatible relation are constructed, the initial value conditions and the boundary value conditions are determined. The dynamic state of the whole domain is gradually recurred by selecting the appropriate time step and the space step. This method provides an effective theoretical analysis and calculation method for the study of dynamic characteristics of structures under impact.

2. The establishment of the compatible equation based on the characteristic method

Because of the coupling of stress wave and strain rate, the mechanical behavior of viscoelastic material is nonlinear. The propagation of force wave in one-dimensional nonlinear viscoelastic material can be determined by the following three equations, and the dynamic process of the whole viscoelastic material model can be obtained by the following equations [7-10].

(1) Continuous equation

$$\frac{\partial v}{\partial x}=\frac{\partial \varepsilon}{\partial t} \tag{1}$$

(2) Motion equation

$$\rho_0\frac{\partial v}{\partial t}=\frac{\partial \sigma}{\partial x} \tag{2}$$

(3) Constitutive equation

$$\frac{\partial \sigma}{\partial t}+\frac{\sigma}{\theta_2}=(\frac{d\sigma_e}{d\varepsilon}+E_1+E_2)\frac{\partial \varepsilon}{\partial t}+\frac{\sigma_e+E_1\varepsilon}{\theta_2} \tag{3}$$

In equations (1), (2) and (3), $\theta_2=\eta_2/E_2$, $\sigma_e=E_0\varepsilon+\alpha\varepsilon^2+\beta\varepsilon^3$, E_1 , E_2 , E_0 , α , β and θ_2 are the material's property parameter, E_1 , E_2 and E_0 are the elastic modulus, α and β are the nonlinear correlation, θ_2 is the relaxation time, η_2 is viscous parameter, ρ_0 is the material density, v is the particle velocity, σ is the stress , ε is and the strain.

The dynamic process of the whole viscoelastic material model can be described by the above three equations (1), (2) and (3), and the above three equations can be solved by using the characteristic line method.

The concrete practice is as follows:

The above three equations are multiplied by A_0 , B_0 , C_0 , and then added together:

$$[A_0+C_0(\frac{d\sigma_e}{d\varepsilon}+E_1+E_2)]\frac{\partial \varepsilon}{\partial t}+(B_0\rho_0\frac{\partial}{\partial t}-A_0\frac{\partial}{\partial x})v+(C_0\frac{\partial}{\partial t}-B_0\frac{\partial}{\partial x})\sigma+f(\sigma,\varepsilon)=0 \tag{4}$$

In formula (4), $f(\sigma,\varepsilon)$ is a one-item function of σ, ε.

According to the compatibility of the characteristic lines, it can be got as follows:

$$\frac{dx}{dt}=\frac{0}{A_0+C_0(\frac{d\sigma_e}{d\varepsilon}+E_1+E_2)}=\frac{-A_0}{B_0\rho_0}=\frac{-B_0}{C_0} \tag{5}$$

The original partial differential equations can be transformed into ordinary differential equations consisting of the compatible relations of three families of characteristic lines by formula (5).

$$dx = \pm C_V dt \tag{6}$$

$$dv = \pm \frac{1}{\rho_0 C_V} d\sigma \pm \frac{\sigma - \sigma_e - E_1\varepsilon}{\rho_0 C_V \theta_2} dt \tag{7}$$

When $A_0 = B_0$ =0, and $C_0 \neq 0$, then $dx = 0$, then the corresponding third family characteristic lines are:

$$d\varepsilon - \frac{d\sigma}{\frac{d\sigma_e}{d\varepsilon} + E_1 + E_2} - \frac{\sigma - (\sigma_e + E_1\varepsilon)}{\frac{d\sigma_e}{d\varepsilon} + E_1 + E_2} \frac{dt}{\theta_2} = 0 \tag{8}$$

In formula (8), $C_V = \sqrt{\dfrac{\frac{d\sigma_e}{d\varepsilon} + E_1 + E_2}{\rho_0}} = \sqrt{\dfrac{E_0 + E_1 + E_2 + 2\alpha\varepsilon + 3\beta\varepsilon^2}{\rho_0}}$.

From the above, as long as knowing the corresponding material parameters, σ, ε, and v can be solved by using equations(6),(7), and(8) together. Among them, the strength of the material is characterized by σ, the deformation degree of the material is characterized by ε, the particle motion of the material is characterized by v, the velocity of the particle and the compatibility condition of the strain are the embodiment of the conservation of mass. The relation of the stress and velocity is the embodiment of the conservation of momentum. The relationship between the strain and the stress is the embodiment of the constitutive model of the material, and the three parameters are known. The whole impact problem can be solved.

3. Numerical solution of nonlinear dynamic equations

3.1. Characteristic line method for solving nonlinear dynamic equations

Compared with the compatibility relation of linear elastomers, the propagation term of the nonlinear viscoelastic body contains terms related to strain rate and cannot be decoupled. Therefore, equations (6), (7) and (8) do not have analytic solutions at the same time. The only way to solve equations is by numerical means. In this paper, the above equations are solved by the characteristic line method, and the specific solution is shown in Figure 1.

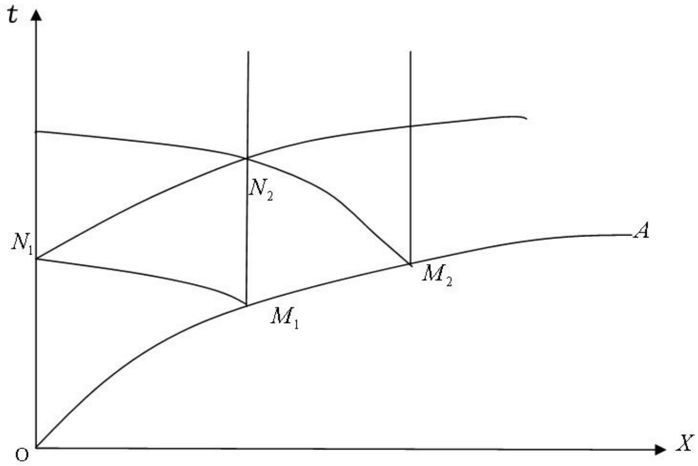

Figure 1. Three characteristic lines of nonlinear dynamic equations.

Figure 1 shows the propagation of stress waves on viscoelastic materials, which can be divided into three regions: the first region is the AOX region, which represents the undisturbed region and the data are all zero; the second region is on the t-axis, which represents the external disturbance of the viscoelastic material model and belongs to the boundary condition; the third one is in the AOX region. It is the AOt region, with the OA line as the firm discontinuous boundary, indicating the disturbance situation in front of and behind the wavefront when the first stress wave propagates, the disturbance area above OA, and the undisturbed area below OA. Thus, in the AOt area, it satisfies:

$$\begin{cases} v(N_2)-v(N_1)=\dfrac{1}{\rho_0 C_V}[\sigma(N_2)-\sigma(N_1)]-[\dfrac{\sigma(N_1)-(\sigma_e+E_1\varepsilon)}{E_0+E_1+E_2+2\alpha\varepsilon+3\beta\varepsilon^2}]\dfrac{[X(N_2)-X(N_1)]}{\theta_2} \\ v(N_2)-v(M_2)=-\dfrac{1}{\rho_0 C_V}[\sigma(N_2)-\sigma(M_2)]+[\dfrac{\sigma(M_2)-(\sigma_e+E_1\varepsilon)}{E_0+E_1+E_2+2\alpha\varepsilon+3\beta\varepsilon^2}]\dfrac{[X(N_2)-X(M_2)]}{\theta_2} \\ \varepsilon(N_2)-\varepsilon(M_1)=\dfrac{\sigma(N_2)-\sigma(M_1)}{\dfrac{d\sigma_e}{d\varepsilon}+E_1+E_2}-\dfrac{\sigma(M_1)-(\sigma_e+E_1\varepsilon)}{\dfrac{d\sigma_e}{d\varepsilon}+E_1+E_2}\dfrac{[t(N_2)-t(M_1)]}{\theta_2} \end{cases} \quad (9)$$

Thus, for any point N_2 in the AOt plane, the above three compatible relations can be obtained simultaneously. However, it is noticed that all the three compatible equations contain functions of ε. ε Changes with time and displacement, and cannot be solved directly. A difference equation which satisfies the CFL condition, i.e., the necessary condition for the convergence of the difference scheme, is constructed. The key idea is to characterize the curve. The solution is simplified to a linear elastic model by linearizing it into several small linear segments, and then the solution of the whole perturbation region is obtained by recursion along three characteristic lines in the entire perturbation region.

The specific measures are as follows:

To obtain the numerical solution of each point in the AOt domain, the points in the computational domain can be divided into: (1) discontinuous points (points on OA); boundary points (points on Ot);

and (3) interior points. The above analysis shows that the breakpoints and boundary points are unique and can be determined.

For any interior point, it is possible to set the internal pointer to A, as shown in figure 2. There must be three classes of characteristic lines passing through this point and all satisfy the compatible equations (6), (7) and (8). As long as the appropriate time and space steps are intercepted, the corresponding difference equations can be constructed along the three characteristic lines. In the area adjacent to the point A, three points B, C, and D having a time step of Δt and a space step of Δx are obtained. Then, the original characteristic line of a curve can be approximated as a straight line, and the complex nonlinear dynamic problem can be simplified as a recursive calculation of several linear problems.

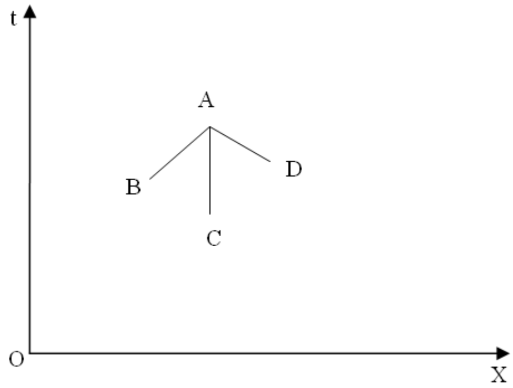

Figure 2. Schematic diagram of internal point calculation.

As long as the initial shock disturbance and loading conditions are known, the numerical solutions of each point in the whole disturbance region can be derived successively by the corresponding differential equations.

Thus, point A satisfies the compatibility equation along three characteristic lines:

Along AB

$$v_A - v_B = \frac{1}{\rho_0 C_B}[\sigma_A - \sigma_B] + F(B)\Delta x \tag{10}$$

Along AD

$$v_A - v_D = -\frac{1}{\rho_0 C_D}[\sigma_A - \sigma_D] + G(D)\Delta x \tag{11}$$

Along AC

$$\varepsilon_A - \varepsilon_C = \frac{\sigma_A - \sigma_C}{E_C} - H(C)\Delta t \tag{12}$$

Because the data of point B, C, and D are all obtained in the previous step, the solution of point A can be calculated by the solution of point B, C and D. By analogy, the solution of point A can be recursively extended to the whole disturbance region until the arrival of the boundary stops, and then the numerical solution of the entire AOt region can be obtained.

3.2. The establishment of difference scheme

When choosing the difference scheme, because the characteristic line is a curve, then select the appropriate space step Δx, and then let $\Delta t = \Delta x / C_V$, the specific feature line family as shown in figure 3. In this way, the AOt in the whole perturbation region is discretized into a lattice, and the solution corresponding to each point is the numerical solution of the entire plane. In particular, it should be pointed out that when solving along the $dx = 0$ characteristic line, the characteristic line $dx = 0$ is a straight line. Then, the nonlinear dynamic equation can be addressed directly by the OA characteristic line, which is consistent with the linear elastic model.

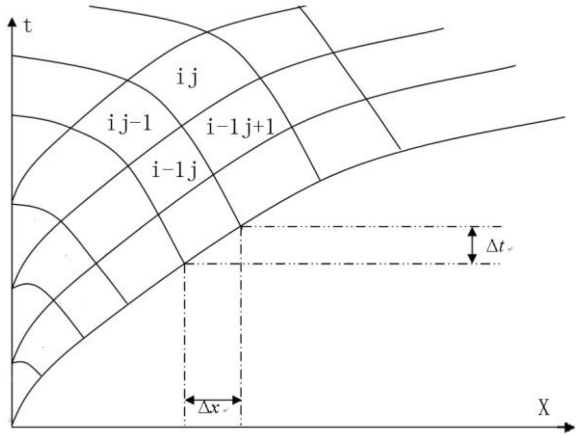

Figure 3. Characteristic lines in calculation area.

According to the physical meaning of the characteristic line, a certain space step and time step can be selected, and the computational domain can be divided into a small area with the characteristic line, as shown in Figure 1. In these regions, all stress states are equal, and the stress states in the adjacent areas satisfy the compatible relationship of stress waves. In this way, with known loading and boundary conditions, as long as the material parameters are known, the stress state of each small area can be calculated recursively from the differential format of the compatible relation. For ij the area, its stress state is ultimately obtained by the logical relation of $ij-1$, $i-1j+1$ and $i-1j$ regions, and the specific equations are as follows:

$$v_{ij} - v_{ij-1} = \frac{1}{\rho C_V}(\sigma_{ij} - \sigma_{ij-1}) - \left(\frac{\sigma_{ij-1} - \sigma_e}{E_0 + E_1 + 2\alpha\varepsilon_{ij-1} + 3\beta\varepsilon_{ij-1}{}^2}\right)\frac{(X_{ij} - X_{ij-1})}{\theta_1} \tag{13}$$

$$v_{ij} - v_{i-1j+1} = -\frac{1}{\rho C_V}(\sigma_{ij} - \sigma_{i-1j+1}) + \left(\frac{\sigma_{ij} - \sigma_e}{E_0 + E_1 + 2\alpha\varepsilon_{i-1j+1} + 3\beta\varepsilon_{i-1j+1}{}^2}\right)\frac{(X_{ij} - X_{i-1j+1})}{\theta_1} \tag{14}$$

$$\varepsilon_{ij}-\varepsilon_{i-1j}=\frac{\sigma_{ij}-\sigma_{i-1j}}{\frac{d\sigma_e}{d\varepsilon}+E_1}+\frac{\sigma_{i-1j}-\sigma_e}{\frac{d\sigma_e}{d\varepsilon}+E_1}\cdot\frac{t_{ij}-t_{i-1j}}{\theta_1} \tag{15}$$

Equations (13), (14) and (15) are the differential equations of nonlinear viscoelastic materials, which are the display formats. All the stress states in the computational domain can be solved, including stress, strain and particle velocity.

4. Simulation calculation and result analysis

Based on the above discrete solution, the stress state of the subregion of the target depends on the stress state of its circumferential region, and the stress state of any subregion will only affect the four adjacent subregions. In such a mechanical property, the continuous equation is discretized by building differential equations (13), (14) and (15). And the numerical calculation program based on Lagrange coordinates is compiled with FORTRAN90. The initial value of the stress state parameters is 0, the boundary condition is set as the free end boundary, and the appropriate time and space step are selected, and stress σ, strain ε, and particle velocity v can be obtained by the recursive calculation.

Combined with the actual situation and material size, the length of the calculation area is set to 77.5mm, the computation time is 50 μs, the space step length is 0.5mm, and the time step is 0.5 μs. The waveforms of the input stress are simplified to rectangular waves and are solved in equations (13), (14) and (15). The comparison between theoretical calculation and experimental results is shown in Table 1.

Table 1. Comparison of output stress experiment and theoretical calculation.

Test number	Input stress /*MPa*	Output stress /*MPa*		Absolute value of error
		Experimental value	Calculated value	
1	165.3	49.9	51.7	3.6%
2	179.5	50.7	52.8	4.1%
3	153.6	48.7	51.1	4.9%
4	149.9	38.8	40.7	4.9%
5	166.1	46.7	48.9	4.7%
6	194.6	65.0	68.1	4.8%

From table 1, it can be seen that the output stress calculated theoretically and the output stress error measured experimentally are within 5 %, indicating that the model solution method is feasible.

5. Conclusion

Based on the propagation characteristics of stress waves, a three-group independent characteristic line equation is constructed. Because the characteristic line equation is nonlinear, the analytical solution cannot be derived. For this reason, the differential equations of the characteristic line compatible relation are constructed, the initial value conditions and the boundary value conditions are determined, and the dynamic state of the entire computational domain is gradually deduced by selecting the appropriate time step length and space step length. The theoretical results are in good agreement with the experimental results, which verify the correctness of the nonlinear dynamic model solving method based on the characteristic line method. This method provides an effective theoretical analysis and calculation method for the study of dynamic characteristics of structures under impact.

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