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The behavior of laced built-up columns made of aluminum alloy

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Abstract. The second-order analysis is used to examine the laced built-up member with two extruded I-sections under combined compression and bending. The member bottom end is fixed, and the upper one is free in the case of in-plane buckling. The member is restrained against out-of-plane buckling at both ends. The global initial sway imperfection is taken into account instead of the local bow imperfection.

1. Introduction

Eurocodes EN 1993 [1] and EN 1999 [2] provide the guidance limited to the design of uniform built-up member under compression with hinged ends laterally supported only. This way of calculation follows the concept published in [3] based on equivalent bending and shear stiffness. Eurocodes [1, 2] use the local bow imperfection $e_0 = L / 500$. The numerical examples of built-up steel members were solved according to the former Czechoslovak standard ČSN 731401 in [4], according to the former German code in [5, 6] and according to Eurocode [1] in publications[7, 8]. However, the calculations in [6] are not complete, while those in [7] contain significant errors. This paper investigates a more general case described in the abstract. The numerical example of the built-up member made of aluminum alloy is only part of the large parametrical study of the battened and laced built-up members. In the parametrical study, the analysis was used based on the Rubin analytical solutions and formulas [4]. All numerical results were verified by the independent program IQ 100 [9], which revealed zero deviations.

2. Analyzed laced built-up column

The analysis of the second order with imperfection is used in calculations. The geometrical equivalent global initial sway imperfection is taken according to [1, 2]. The column is made of EN-AW 6061-T6 aluminum alloy, of buckling class A. The material possesses the yield strength f_0 = 240 MPa and elastic modulus modulus E = 70 GPa. The material safety margin γ_{M1} = 1.1 is used, while the design values of the external forces and moments applied to the column top (figure 1) are

$$F_{Ed} = 430\,kN, \quad H_{Ed} = 11\,kN, \quad M_{Ed,e} = 175\,kNm \tag{1}$$

The height of the column equals to L = 7.2 m. The boundary conditions are defined by the buckling lengths: $L_{cr,y} = L$ and $L_{cr,z} = 2L$ (table 6.8 in [2]). The distance between the posts of the lacing is a = 1.2 m. The initial sway imperfection is determined according to clause 5.3.2 [1, 2]:

$$\phi = \phi_0\, \alpha_h\, \alpha_m = \frac{1}{200} 0.745\ 1.0 = 3.727\ 10^{-3}, \quad \phi = \frac{1}{268.328} = 3.727\ 10^{-3} \tag{2}$$

The total design value of the horizontal force at the top of the column after the replacement of the initial imperfections by the equivalent horizontal forces:

$$H_{Ed,tot} = H_{Ed} + \phi N_{Ed} = 11kN + 1.603kN = 12.603\,kN \tag{3}$$

The properties of the cross-section class 3 extruded I section (Class 3 cross-section: flanges 120 x 12 mm, section height 240 mm, web thickness 9 mm, radius 16 mm), which creates a chord of the built member are as follows:

$$A_{ch} = 50.44\ cm^2,\ I_{ch,y} = 4742\,cm^4,\ I_{ch,z} = 348.5\,cm^4,\ W_{ch,el,y} = 395.2\ cm^3,\ W_{ch,el,z} = 58.09\ cm^3 \tag{4}$$

The distance between the centroids of the chords is h_0 = 40 cm. The properties of the built-up member (two I sections):

$$A = 100.88\ cm^2, \quad I_y = 9484\,cm^4, \quad I_z = 162099\ cm^4, \quad i_y = 9.7\ cm, \quad i_z = 40.1\ cm \tag{5}$$

$$\mu = 0, \qquad I_{eff} = 2\left[\mu I_{ch,z} + A_{ch}(h_0/2)^2\right] = 1614\ 10^6 mm^4, \quad I_{eff} = 161402\ cm^4 \tag{6}$$

The two chords (I-sections) of the member are connected by the lacing consisting of diagonals L 40 x 4 mm and posts L 40 x 4 mm, Class 4 cross-section.

The discrete structure of the built-up column smeared to a continuum has the bending stiffness EI_{eff} = 112 981 kNm^2 and shear stiffness S_v = 45 721.5 kN.

$$n = 2,\ A_d = 3.08\,cm^2,\ A_v = 3.08\ cm^2,\ d = \sqrt{a^2 + h_0^2} = 1.442\,m,\ S_v = \frac{nEA_d a h_0}{d^3\left[1 + \dfrac{A_d h_0^3}{A_v d^3}\right]} = 9429.9\,kN \tag{7}$$

The parameter of the member ε used in the analysis of the 2nd order takes into account the influence of the shear deformations through the parameter γ

$$\gamma = \frac{1}{1 - \dfrac{N_{Ed}}{S_v}} = \frac{1}{1 - \dfrac{430kN}{9429.9\,kN}} = 1.048, \qquad \varepsilon = L\sqrt{\frac{\gamma N_{Ed}}{EI_{eff}}} = 7.2\,m\sqrt{\frac{1.048\ \ 430\,kN}{112\,981\,kNm^2}} = 0.455 \tag{8}$$

3. Internal forces of the smeared continuum

The internal forces N_{Ed} (constant), V_{Ed} (non-linear) and M_{Ed} (non-linear) are calculated according to the theory of the 2nd order with the sway initial imperfection and the influence of the shear deformations (S_v = 9429.9 kN, γ = 1.048, ε = 0.455, ξ_H = 1.0). Their distributions are drawn in figure 1 with solid lines and their values are written in bold. The internal forces were calculated via Eqs. [5], which are valid for any point of horizontal force H_{Ed} application x_H from the interval (0 m $\leq x_H \leq L$) or ($0 \leq \xi_H = x_H / L \leq$ 1.0) and for any point of bending moment $M_{Ed,e}$ application x_M from the interval (0 m $\leq x_M \leq L$) or ($0 \leq \xi_M = x_M / L \leq 1.0$)

The bending moment due to horizontal force $H_{Ed,tot}$ at the top of the column:

$$M_{Ed}^{H}(\xi) = \left\{\frac{\gamma \sin[(1-\xi_H)\varepsilon]\sin(\xi\varepsilon)}{\varepsilon \sin(\varepsilon)} - \frac{\gamma \sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)}\,\frac{\sin(\varepsilon) - \sin[(1-\xi_H)\varepsilon]}{\varepsilon \cos(\varepsilon)}\right\} L H_{Ed,tot}, \quad \text{for } \xi \leq \xi_H \tag{9}$$

$$M_{Ed}^{H}(\xi) = \left\{\frac{\gamma \sin(\xi_H\varepsilon)\sin[(1-\xi)\varepsilon]}{\varepsilon \sin(\varepsilon)} - \frac{\gamma \sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)}\,\frac{\sin(\varepsilon) - \sin[(1-\xi_H)\varepsilon]}{\varepsilon \cos(\varepsilon)}\right\} L H_{Ed,tot}, \quad \text{for } \xi > \xi_H \tag{10}$$

The bending moment due to external bending moment $M_{Ed,e}$ at the top of the column:

$$M_{Ed}^{M}(\xi) = \left\{-\frac{\cos[(1-\xi_M)\varepsilon]\sin(\xi\varepsilon)}{\sin(\varepsilon)} - \frac{\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)}\,\frac{\cos[(1-\xi_M)\varepsilon]}{\cos(\varepsilon)}\right\} M_{Ed,e}, \qquad \text{for } \xi \leq \xi_M \tag{11}$$

$$M_{Ed}^{M}(\xi)=\left\{\frac{\cos[(\xi_M\varepsilon)]\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)}-\frac{\sin[(1-\xi)\varepsilon]}{\sin(\varepsilon)}\,\frac{\cos[(1-\xi_M)\varepsilon]}{\cos(\varepsilon)}\right\}M_{Ed,e}\,, \qquad \text{for } \xi > \xi_M \quad (12)$$

The total bending moment and shear force are as follows:

$$M_{Ed}(\xi)=M_{Ed}^{H}(\xi)+M_{Ed}^{M}(\xi),\quad V_{Ed}(\xi)=[\mathrm{d}M_{Ed}(\xi)\,/\,\mathrm{d}\xi]\,/\,L \qquad (13)$$

The distributions of the internal forces N_{Ed} (constant), V_{Ed} (non-linear) and M_{Ed} (non-linear) calculated according to the theory of the 2nd order with the sway initial imperfection but without the influence of the shear deformations ($S_v = \infty$ kN, $\gamma = 1.0$, $\varepsilon = 0.444$, $\xi_H = 1.0$, $\xi_M = 1.0$) are written in figure 1 in italic.

The distributions of the internal forces N_{Ed}, V_{Ed} (constant) and M_{Ed} (non-linear) calculated according to the theory of the 1st order with the sway initial imperfection ($\varepsilon = 0$, $\xi_H = 1.0$, $\xi_M = 1.0$) are drawn in figure 1 with dotted lines. Their values are given in the brackets. If these values are multiplied by the ratio 11/12.603, one obtains the results valid for the case without imperfection.

Figure 1 also shows the values of the horizontal deflections at the top of the built-up column.

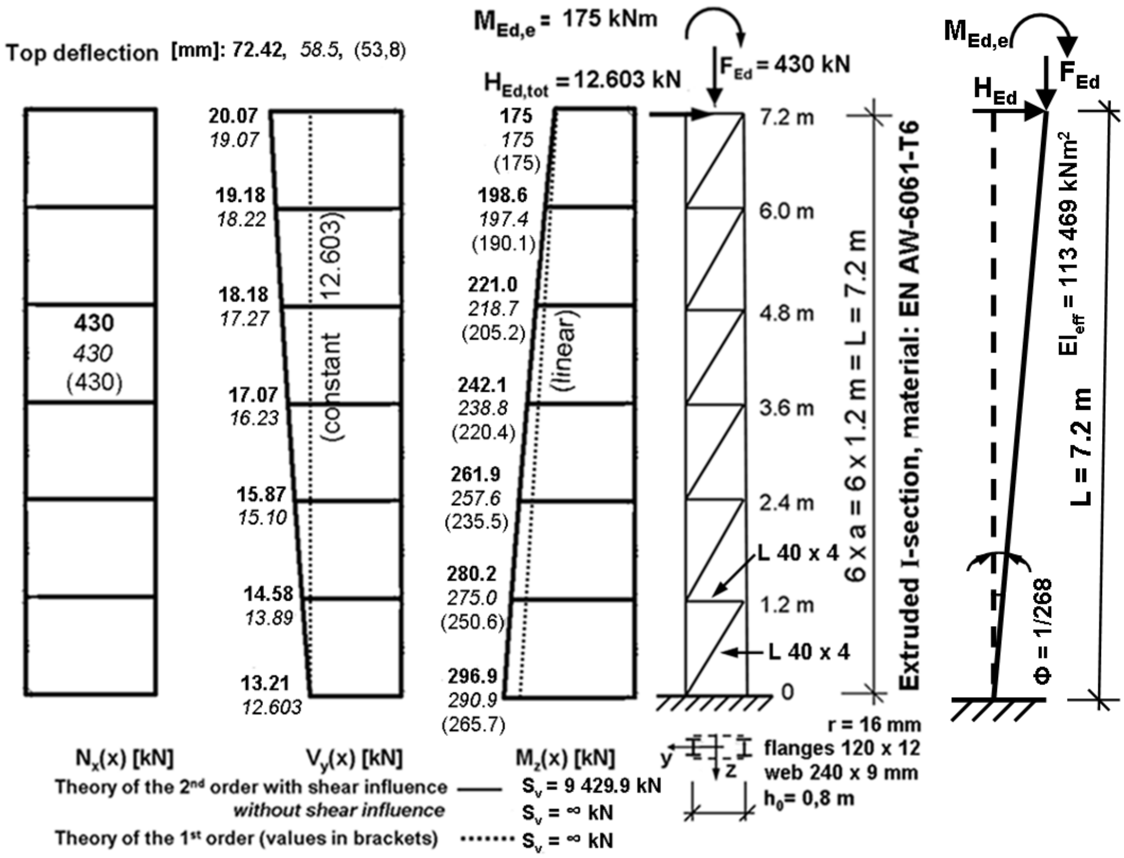

Figure 1. Column geometry, actions and distributions of the internal forces $N_x(x)$, $V_y(x)$, $M_z(x)$.

4. Internal forces of the components of the built-up column

Internal forces in the chords, diagonals, and posts may be calculated from the internal forces of the continuum with smeared stiffness (figure 2a). This approximate calculation according to [1, 2] was performed twice with bending stiffness: (i) $EI_z = 162099$ cm^4 ($\mu = 1$), (ii) $EI_{eff} = 161400$ cm^4 ($\mu = 0$). The differences are negligible. The values in figure 2a are compared with the internal forces in the chords, diagonals, and posts calculated more exactly on the calculation model represented by the frame structure (figure 2b). The frame structure was calculated by the second order theory twice: (i) all compression and tension forces were taken into account. These values are more exact and are given in figure 2b in brackets, (ii) only compression forces were taken into account. The differences between (i) and (ii) values are from the practical point of view negligible. The comparison of the values in figure 2a and 2b confirmed that approximate Eurocode way of internal forces calculation on the model with smeared bending and shear stiffnesses is possible to use in the practical design of laced built-up columns.

4.1. Verification of the in-plane buckling of the column

The axial forces in the chords $N_{ch,Ed}$ used for the verification of the in-buckling resistance of the chord determined at the fixed bottom end

$$N_{ch,c,Ed} = \frac{N_{Ed}}{2} + \frac{M_{Ed}(x=0)}{W_{eff}} A_{ch} = 586.26\,kN \quad \text{(right chord in compression)} \tag{14}$$

$$N_{ch,c,Ed} = \frac{N_{Ed}}{2} - \frac{M_{Ed}(x=0)}{W_{eff}} A_{ch} = -156.26\,kN \quad \text{(left chord in tension)} \tag{15}$$

The in-plane buckling resistance of the chord in compression at the bottom fixed end (figure 2a)

$$N_{cr,ch,z} = \frac{\pi^2 EI_{ch,z}}{a^2} = 1672.2\,kN,\ \lambda_{ch,z} = 45.6,\ \bar{\lambda}_{ch,z} = 0.851,\ \ \alpha = 0.2,\ \bar{\lambda}_0 = 0.1 \tag{16}$$

$$\phi_{ch,z} = 0.937, \quad \chi_{ch,z} = 0.752, \quad N_{ch,b,z,Rd} = \chi_{ch,z} A_{ch} \frac{f_0}{\gamma_{M1}} = 827.65\,kN \tag{17}$$

Verification equation of the in-plane buckling resistance of the chord in compression:

$$U_{ch,b,z} = \frac{N_{ch,c,Ed}}{N_{ch,b,z,Rd}} = \frac{586.26}{827.65} = 0.708 < 1.0 \tag{18}$$

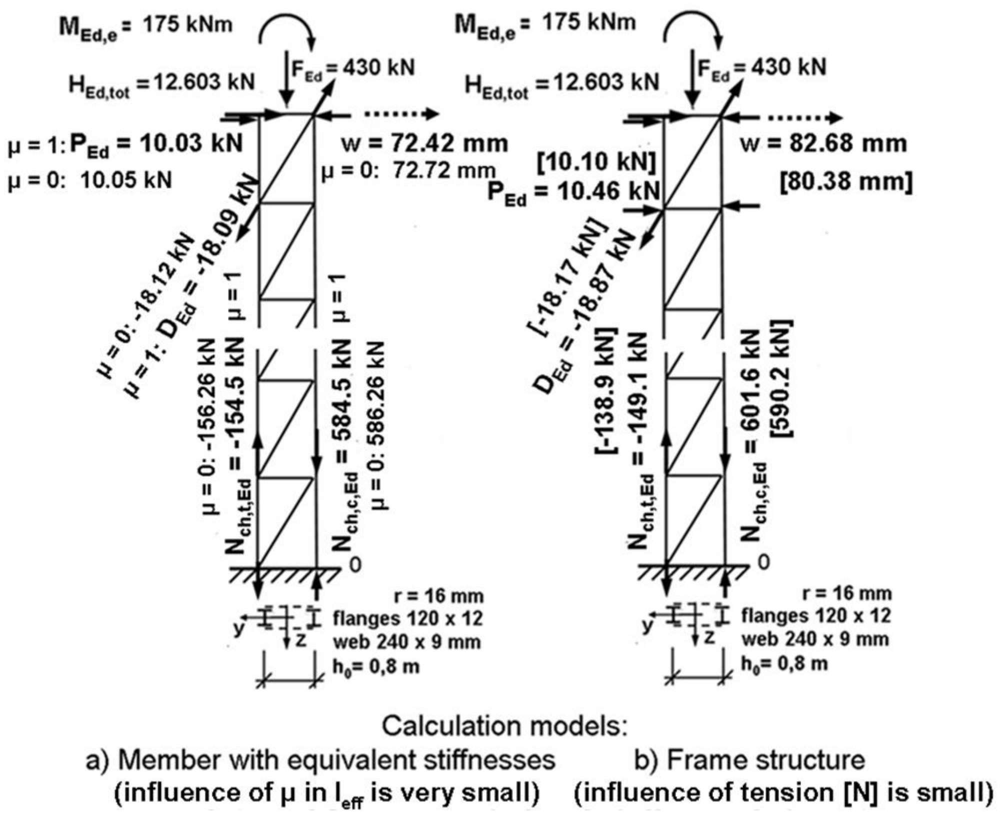

Figure 2. Comparisons of axial forces per 1 chord, 1 diagonal and 1 post calculated by the model

of: (i) continuum with smeared stiffnesses, according to [1, 2] and (ii) discrete frame structures

4.2. Verification of the out-of-plane buckling of the column

$$N_{cr,y} = \frac{\pi^2 EI_y}{L_{cr,y}^2} = 1264\,kN,\ \lambda_y = 74.3,\ \bar{\lambda}_y = 1.384,\ \ \alpha = 0.2,\ \bar{\lambda}_0 = 0.1 \tag{19}$$

$$\phi_y = 1.586, \quad \chi_y = 0.424, \quad N_{b,y,Rd} = \chi_y A \frac{f_0}{\gamma_{M1}} = 932.2\,kN \tag{20}$$

Utility grade due to the axial force only is

$$U_{b,y} = \frac{N_{Ed}}{N_{b,y,Rd}} = \frac{430}{932.2} = 0.461 \tag{21}$$

Utility grade due to bending moment only. The bending moment $M_{z,Ed.H,Me}$ = 235.565 kNm in the section x = 3.6 m due to the horizontal force H_{Ed} = 22 kN and $M_{Ed,e}$ = 175 kNm is taken into account:

$$M_{z,Ed,Htot,Me}(x = 3.6\,m) = 242.236\,kNm, \quad M_{z,Ed,H,Me}(x = 3.6\,m) = 235.565\,kNm \tag{22}$$

$$W_{eff} = \frac{I_{eff}}{0.5h_0} = 4035\,cm^3,\ \ N_{ch,M} = \frac{M_{z,Ed,H,Me}}{W_{eff}} A_{ch} = 294.46\,kN,\ \ U_M = \frac{\gamma_{M1} N_{ch,M}}{A_{ch} f_0} = 0.268 \tag{23}$$

Verification of the out-of-plane buckling resistance of the column is done in the middle of the column height. Utility grade due to the axial force and the bending moment is

$$U_y = U_{b,y} + U_M = 0.461 + 0.268 = 0.729 < 1.0 \tag{24}$$

4.3. Verification of the cross-section resistance of the diagonal in tension

The diagonals L 40 x 4 are in tension. The maximal axial force is at the top of the column (figure 2a)

$$D_{Ed} = \frac{V_y(x = 7.2m)}{n} \frac{d}{h_0} = \frac{20.10}{2} \frac{1.442}{0.8} = 18.12\,kN \tag{25}$$

Utility grade

$$U_{D,t} = \frac{\gamma_{M1} D_{Ed}}{A_d f_0} = 0.27 < 1.0 \tag{26}$$

If diagonals would be in the compression

$$\lambda_v = \frac{d}{i_d} = \frac{1442}{7.8} = 184.9 \approx \lambda_{\lim} = 180,\ \chi_d = 0.13,\ N_{b,d,Rd} = \chi_v A_{eff} \frac{f_0}{\gamma_{M1}} = 7.989\,kN \tag{27}$$

$$U_{d,c} = \frac{D_{Ed}}{N_{b,d,Rd}} = \frac{18.12}{7.989} = 2.27 > 1.0 \tag{28}$$

4.4. Verification of the buckling resistance of the post in compression

The maximal axial compression force in the post L 40 x 4 (figure 2a)

$$P_{Ed} = \frac{V_y(x = 7.2m)}{n} = \frac{20.10}{2} = 10.05\,kN \tag{29}$$

The buckling length and the slenderness about the weakest axis

$$L_{cr,v} = h_0 = 800\,mm, \quad \lambda_v = \frac{h_0}{i_v} = \frac{800}{78} = 102.56 \tag{30}$$

The post L 40 x 4 is the class 4 cross-section

$$\beta_v = \frac{b - t - r}{t} = \frac{40 - 4 - 6}{4} = 7.5 > 6\,\varepsilon_m = 6(250MPa/\,f_0)^{0.5} = 6.214 \ \rightarrow \ \text{Class 4} \tag{31}$$

Reduction factor for the local buckling

$$\rho_c = C_1 \frac{\varepsilon_m}{\beta_v} - C_2 \left(\frac{\varepsilon_m}{\beta_v} \right)^2 = 10 \frac{1.021}{7.5} - 24 \left(\frac{1.021}{7.5} \right)^2 = 0.916 \quad (32)$$

Effective area

$$A_{v,eff} = \rho_c A_v = 0.916\ 3.08 = 2.822 cm^2 \quad (33)$$

Critical force of the flexural buckling

$$N_{cr,v} = \frac{\pi^2 EI_v}{L_{cr,v}^2} = \frac{\pi^2\ 70GPa 1.86\, cm^4}{(80\, cm)^2} = 20.078\, kN \quad (34)$$

Relative slenderness and effective relative slenderness (formula (BB.1) in [1])

$$\bar{\lambda}_v = \sqrt{\frac{A_{v,eff}\, f_0}{N_{cr,v}}} = 1.837\,, \qquad \bar{\lambda}_{v,eff} = 0.35 + 0.7 \bar{\lambda}_v = 1.636 \quad (35)$$

$$\Phi_v = 0.5 \left[1 + 0.2 \left(\bar{\lambda}_{v,eff} - 0.1 \right) + \bar{\lambda}_{v,eff}^2 \right] = 1.991\,, \qquad \chi_v = \frac{1}{\Phi_v + \sqrt{\Phi_v^2 - \bar{\lambda}_{v,eff}^2}} = 0.32 \quad (36)$$

The design flexural buckling resistance of the post and utility grade are

$$N_{b,v,Rd} = \chi_v A_{eff} \frac{f_0}{\gamma_{M1}} = 19.692\, kN\,, \qquad \frac{P_{Ed}}{N_{b,v,Rd}} = \frac{10.05}{19.692} = 0.51 < 1.0 \quad (37)$$

5. Conclusion

It was confirmed that the general procedure proposed by the authors is possible to use for in-plane buckling verification of the laced built-up members with any boundary conditions under any combination of actions. Eurocodes [1, 2] give no guidance for such special cases.

The out-of-plane flexural buckling resistance verification of the built-up column took into account also influence of the bending moment due to in-plane actions. This was not solved in [3-9].

Acknowledgment

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