IOP Conference Series: Materials Science and Engineering

PAPER • OPEN ACCESS

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To cite this article: V V Dodonov 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **474** 012009

IOP Conf. Series: Materials Science and Engineering 474 (2019) 012009 doi:10.1088/1757-899X/474/1/012009

Dynamical Casimir effect meets material science

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Abstract.

The aim of this study is to analyze attempts to observe the so called Dynamical Casimir effect in cavities, changing their optical lengths by means of fast time variations of material properties (dielectric permeability or conductivity) of thin slabs attached to the cavity walls. The emphasis is made on the case of semiconductor slabs excited by short laser pulses. Considering the evolution of the classical electromagnetic field in this case, an approximate analytical solution for an infinite set of coupled ordinary differential equations for the mode amplitudes is derived under certain simplifying assumptions. According to this solution, an amplification of the initial field can be made, provided the induced dielectric permeability can become negative with a large absolute value. Evaluations of the feasibility of such a scenario are given.

1. Introduction

In 1970, Moore [1] showed that motions of ideal boundaries of a one-dimensional cavity can result in a generation of quanta of the electromagnetic field from the initial vacuum quantum state. Since that time, such a remarkable effect (called after [2,3] as *Dynamical Casimir Effect* – DCE) attracted attention of many researchers. Reviews of numerous publications on this subject can be found in [4, 5]. One of the main theoretical results obtained in the middle of 1990s was a simple formula for the mean number of quanta that could be generated during time t inside an ideal three-dimensional non-degenerate cavity whose walls perform a periodic motion with the *double* eigenfrequency $2\omega_0$ of the fundamental cavity mode (the parametric amplification of the vacuum fluctuations) [6] $\langle n\rangle(t) = \sinh^2(\epsilon\kappa\omega_0 t)$. Here ϵ is the maximal relative displacement of the boundary with respect to the wavelength $\lambda = 2\pi c/\omega_0$ and $\kappa < 1$ is a numerical coefficient which depends on the cavity geometry. For example, $\kappa = [\lambda/(2L_0)]^3$ for a rectangular cavity, where L_0 is the average distance between vibrating walls.

Scholars seeked for a possible experimental verification of the DCE since the early 1990s. One of the main obstacles is the extreme smallness of parameter ϵ that could be achieved for real cavities with moving walls. Indeed, consider cavities with dimensions of the order of a few centimeters, possessing the fundamental eigenfrequencies $\omega_0/2\pi$ of the order of a few GHz. An idea of [6] was not to move the wall as a whole (since it seems practically impossible at high frequencies [4]), but to excite oscillations of the *surface* of the cavity wall. Note that the amplitude a of a standing acoustic wave at frequency $\omega_w = 2\omega_0$ is related to the relative deformation amplitude δ as $a = v_s\delta/\omega_w$, where v_s is the sound velocity. Since the maximal deformation of usual materials cannot exceed $\delta_{max} \sim 10^{-2}$, the maximal possible velocity of the boundary is $v_{max} \sim \delta_{max} v_s \sim 50$ m/s for $v_s \sim 5$ km/s. Therefore, the maximal relative displacement $\epsilon = a/\lambda$ equals $\epsilon_{max} \sim v_s\delta_{max}/(4\pi c) \sim 10^{-8}$. Using this value, one could get $\sinh^2(5) \sim 5000$ photons in the initially empty ideal cavity with $\omega_0/(2\pi) = 3\,\mathrm{GHz}$ and $\kappa = 1/2$ after $t \sim 50\,\mathrm{ms}$. The required cavity quality factor Q is about 10^8. If the value of δ_{max} could be increased by one order of magnitude (and this is the material science problem), then the necessary time and the cavity quality factor could be diminished by the same one order of magnitude. The excitation of the surface oscillations with the maximal amplitude at such a high frequency is a challenge. Probably, this can be achieved with the aid of the so-called 'film bulk acoustic resonators' (FBARs) [7], as was proposed in connection with the DCE in [8, 9].

In fact, the fundamental mechanism of the DCE is the parametric amplification of initial vacuum fluctuations due to temporal changes of the cavity eigenfrequencies. But such changes can be achieved not only by means of changing the cavity geometry, but also by changing the material properties of the cavity. Hence the idea of *simulating* the DCE and other quantum effects was suggested by Yablonovitch about three decades ago [2]. Namely, he proposed to use a medium with a rapidly decreasing in time refractive index ('plasma window'). Similar ideas were put forward later in [10, 11]. A possibility to use *quantum circuits* instead of cavities to simulate the DCE was discussed in [12–16], and it was realized in [17, 18]. The dielectric permeability can be changed in nonlinear optical materials illuminated by strong laser pulses. This way of modelling effects predicted by the quantum field theory in a laboratory was considered by several authors, started from [19–21] (see [22] for a short review). However, no real experiments were performed till now due to the smallness of the nonlinear optical parameters. This is one of challenges for the material science: to find (construct) materials with high nonlinear optical coefficient $\chi^{(3)}$ in the microwave domain.

On the other hand, fast big variations of electric properties can be achieved in semiconductors illuminated by laser pulses, as was pointed out by Yablonovitch [2]. Following this idea, attempts to observe the analogue of 'true' DCE in *microwave cavities* were performed [23, 24]. In that experiment, a semiconductor slab was periodically illuminated by chains of short (a few picosecond) laser pulses, so that its conductivity changed from almost zero to almost metallic values. This resulted in periodical changes of the fundamental mode frequency of the order of 10^{-3} (or bigger). However, in spite of many efforts, the amplification of the microwave field was not observed. The main difficulty consists in the inevitable losses inside the semiconductor slab with a finite electric conductivity. Theoretical models accounting for these losses at the quantum level were elaborated in [4, 25]. However, it seems that some important factors were not taken into account there (although experimental results showed a good agreement with some theoretical predictions). In order to find a possible drawback, the preliminary analysis of the problem in the *classical* regime was performed in [26]. The main question was as follows: is it possible in principle to amplify the classical EM field by means of strong temporal changes of the electrical properties of a thin conducting slab inside the cavity? What kinds of materials should be used? Some answers (generalizing results of [26]) are given in the next sections.

2. The classical EM field dynamics: approximate solutions for a thin slab with time-dependent parameters

Firstly, consider the Maxwell equations for the EM field inside an ideal cavity, filled in with a stationary, isotropic, non-dispersive and non-magnetic dielectric material:

$$c\,\mathrm{rot}\mathbf{B} = \partial\mathbf{D}/\partial t, \qquad c\,\mathrm{rot}\mathbf{E} = -\partial\mathbf{B}/\partial t, \qquad \mathbf{D}(\mathbf{r},t) = \varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r},t), \quad \varepsilon(\mathbf{r}) \geq 1. \tag{1}$$

These equations admit the complete set of solutions in the form $\mathbf{E}(\mathbf{r},t) = \mathbf{E_n}(\mathbf{r})\exp(-i\omega_\mathbf{n}t)$, $\mathbf{B}(\mathbf{r},t) = -i\mathbf{B_n}(\mathbf{r})\exp(-i\omega_\mathbf{n}t)$, where real time independent functions $\mathbf{E_n}(\mathbf{r})$ and $\mathbf{B_n}(\mathbf{r})$ satisfy the equations (where the 'vector' index $\mathbf{n}$ combines three integers (l, m, n) and $k_\mathbf{n} = \omega_\mathbf{n}/c$)

$$\mathrm{rot}\mathbf{B_n} = \varepsilon(\mathbf{r})k_\mathbf{n}\mathbf{E_n}, \quad \mathrm{rot}\mathbf{E_n} = k_\mathbf{n}\mathbf{B_n}. \quad \int \varepsilon(\mathbf{r})\mathbf{E_n}\mathbf{E_m}dV = \int \mathbf{B_n}\mathbf{B_m}dV = 8\pi\delta_{\mathbf{nm}}. \tag{2}$$

In the general case, $\mathbf{E}(\mathbf{r},t)=\sum_{\mathbf{n}}\mathbf{E}_{\mathbf{n}}(\mathbf{r})f_{\mathbf{n}}(t)$ and $\mathbf{B}(\mathbf{r},t)=\sum_{\mathbf{n}}\mathbf{B}_{\mathbf{n}}(\mathbf{r})g_{\mathbf{n}}(t)$, the total energy can be written as

$$W=\int dV\left(\varepsilon\mathbf{E}^2+\mathbf{B}^2\right)/(8\pi)=\sum_{\mathbf{n}}\left(f_{\mathbf{n}}^2+g_{\mathbf{n}}^2\right). \tag{3}$$

For the conducting and nonstationary medium, described by means of functions $\sigma(\mathbf{r},t)$ and $\varepsilon(\mathbf{r})+\delta\varepsilon(\mathbf{r},t)$, the first equation in (1) is replaced with $c\,\mathrm{rot}\mathbf{B}=4\pi\sigma\mathbf{E}+\partial\left[(\varepsilon+\delta\varepsilon)\mathbf{E}\right]/\partial t$. This form implies the neglect of temporal dispersion, which seems reasonable for microwave fields. Using (2), one can write $\sum_{\mathbf{n}}\mathbf{E}_{\mathbf{n}}\left[\varepsilon\left(\dot{f}_{\mathbf{n}}-g_{\mathbf{n}}\omega_{\mathbf{n}}\right)+\delta\varepsilon\dot{f}_{\mathbf{n}}+f_{\mathbf{n}}\left(4\pi\sigma+\partial\delta\varepsilon/\partial t\right)\right]=0$. Then, following the standard procedure with account of (2), one can replace the partial differential equations with the ordinary ones for the time-dependent amplitudes

$$\dot{g}_{\mathbf{m}}=-\omega_{\mathbf{m}}f_{\mathbf{m}},\qquad \dot{f}_{\mathbf{m}}=\omega_{\mathbf{m}}g_{\mathbf{m}}-\sum_{\mathbf{n}}\alpha_{\mathbf{mn}}f_{\mathbf{n}}-\frac{d}{dt}\sum_{\mathbf{n}}\beta_{\mathbf{mn}}f_{\mathbf{n}}, \tag{4}$$

$$\alpha_{\mathbf{mn}}(t)=\int dV\sigma(\mathbf{r},t)\mathbf{E}_{\mathbf{m}}(\mathbf{r})\mathbf{E}_{\mathbf{n}}(\mathbf{r})/2,\quad \beta_{\mathbf{mn}}(t)=\int dV\delta\varepsilon(\mathbf{r},t)\mathbf{E}_{\mathbf{m}}(\mathbf{r})\mathbf{E}_{\mathbf{n}}(\mathbf{r})/(8\pi). \tag{5}$$

These equations can be significantly simplified for rectangular cavities with $0<z<L$, $|x|<L_x/2$, $|y|<L_y/2$, provided functions $\varepsilon(\mathbf{r})$, $\sigma(\mathbf{r},t)$ and $\delta\varepsilon(\mathbf{r},t)$ depend on the single longitudinal variable z only. Suppose that $\varepsilon=const>1$ for $0<z<L_s<L$ and $\varepsilon\equiv 1$ for $L_s<z<L$. Then, the consequence of equations (2) is the Helmholtz equation $\Delta\mathbf{E}_{\mathbf{n}}+\mathbf{k}_{\mathbf{n}}^2\varepsilon(z)\mathbf{E}_{\mathbf{n}}=0$. It is known [25, 27, 28] that the TM configuration of the electromagnetic field is much more advantageous than the TE one for the field excitation due to the DCE. In this case,

$$B_z\equiv 0,\quad E_z=\Phi(\mathbf{r}_\perp)\psi(z),\quad E_x=\mathbf{k}_\perp^{-2}\frac{\partial\Phi}{\partial x}\frac{d\psi}{dz},\quad E_y=\mathbf{k}_\perp^{-2}\frac{\partial\Phi}{\partial y}\frac{d\psi}{dz}.$$

$$\Delta_\perp\Phi+\mathbf{k}_\perp^2\Phi=0,\qquad \psi''+\left[\mathbf{k}_n^2\varepsilon(z)-k_\perp^2\right]\psi=0.$$

Since the tangential components of the electric field must disappear near all surfaces of the ideal cavity, the boundary conditions for function $\psi(z)$ are $\psi'\equiv d\psi/dz=0$ at $z=0$ and $z=L$, whereas $\Phi\left(\mathbf{r}_\perp\right)=\cos\left(k_l x\right)\cos\left(k_m y\right)$, $k_l=(1+2l)\pi/L_x$, $k_m=(1+2m)\pi/L_y$. As a consequence, modes with different indexes l and m are independent. We assume that $l=m=0$. Then, equations (4) can be written in the vector form

$$\dot{\mathbf{g}}=-\Omega\mathbf{f},\qquad \dot{\mathbf{f}}=\Omega\mathbf{g}-\mathcal{A}\mathbf{f}-d(\mathcal{B}\mathbf{f})/dt, \tag{6}$$

where $\mathbf{f}\equiv(f_0,f_1,f_2,\ldots)$, $\mathbf{g}\equiv(g_0,g_1,g_2,\ldots)$, $f_m\equiv f_{00m}$, $g_m\equiv g_{00m}$. The diagonal matrix Ω has elements $\omega_m\equiv\omega_{00m}$. The elements of symmetrical matrices $\mathcal{A}(t)$ and $\mathcal{B}(t)$ are as follows:

$$a_{mn}(t)=\int_0^{L_s}dz\sigma(z,t)\chi_{mn}(z),\quad b_{mn}(t)=\int_0^{L_s}dz\delta\varepsilon(z,t)\chi_{mn}(z)/(4\pi), \tag{7}$$

$$\chi_{mn}(z)=(L_xL_y/8)\left[\psi_m(z)\psi_n(z)+\mathbf{k}_\perp^{-2}\psi_m'(z)\psi_n'(z)\right]. \tag{8}$$

An immediate consequence of (6) is the equation $dW/dt=-2\left[\mathbf{f}\mathcal{A}\mathbf{f}+\mathbf{f}d(\mathcal{B}\mathbf{f})/dt\right]$, where $W=\mathbf{f}^2+\mathbf{g}^2$. Since $\sigma(z,t)\geq 0$, matrix $\mathcal{A}$ is *non-negative*: $\mathbf{f}\mathcal{A}\mathbf{f}\geq 0$ for any vector $\mathbf{f}$. Therefore, $dW/dt\leq 0$ if $\sigma>0$ and $\mathcal{B}=0$. This means that the total energy W always decreases if $\delta\varepsilon=0$.

Since matrices Ω, $\mathcal{A}$, and $\mathcal{B}$ do not commute, equations (6) can be solved only numerically in the general case. However, approximate analytical solutions can be found under certain assumptions for thin slabs with $L_s\ll L$. For the totally empty cavity, one has

$$\psi_n(z)=N_n\cos\left(\pi nz/L\right),\quad N_n^{-2}=L_xL_yL\left(1+\mu n^2\right)\left(1+\delta_{n0}\right)/(64\pi), \tag{9}$$

$$\omega_n^2 = \omega_0^2\left(1+\mu n^2\right), \quad \omega_0 = c\pi/(L\sqrt{\mu}), \quad \mu = L_x^2 L_y^2\left[L^2\left(L_x^2+L_y^2\right)\right]^{-1}. \tag{10}$$

If $L_s \ll L$, then the electric field outside the slab remains practically the same as in the stationary case, as well as the eigenfrequencies and normalization constants. But E_z component of the electric field inside the slab is smaller: $\psi_n(z < L_s) \approx N_n \cos\left(\zeta_n \pi n z/L\right)/\varepsilon$, $\zeta_n \approx \sqrt{\varepsilon + (\varepsilon-1)/(\mu n^2)}$. Therefore, the ratio of the second term (containing the product of derivatives) to the first one in the function $\chi_{mn}(z)$ (8) inside the slab equals approximately $\mu(\pi m n \varepsilon L_s/L)^2$, so that this term can be neglected for not very big values of mode numbers m and n and the dielectric constant ε. Using one more approximation $\cos\left(\zeta_n \pi n z/L\right) \approx 1$, one can simplify matrices $\mathcal{A}(t)$ and $\mathcal{B}(t)$ as follows:

$$\mathcal{A}(t) = a(t)\Psi, \quad \mathcal{B}(t) = b(t)\Psi, \quad a(t) = \int_0^{L_s} \sigma(z,t)dz/L, \quad b(t) = \int_0^{L_s} \delta\varepsilon(z,t)dz/(4\pi L). \tag{11}$$

Here Ψ is the symmetric matrix with the factorized elements $\Psi_{mn} = \Psi_m \Psi_n$, $\Psi_0 = \sqrt{4\pi}/\varepsilon$, $\Psi_{n\neq 0} = \left[\varepsilon^2\left(1+\mu n^2\right)/(8\pi)\right]^{-1/2}$. Moreover, $\Psi^n = \rho^{n-1}\Psi$ for $n = 1, 2, 3, \ldots$, where [26]

$$\rho = \mathrm{Tr}(\Psi) = \sum_{k=0}^{\infty} \Psi_k^2 = \left(4\pi/\varepsilon^2\right)\left[1 + 2\sum_{n=1}^{\infty}\left(1+\mu n^2\right)^{-1}\right] = 4\pi^2\left(\varepsilon^2\sqrt{\mu}\right)^{-1}\coth\left(\pi/\sqrt{\mu}\right) \tag{12}$$

Consequently, any function of matrix Ψ is proportional to Ψ itself. In view of this finding, let us make one more approximation, assuming that matrix Ω is proportional to the unity matrix I: $\Omega = \varpi I$. Of course, this assumption is rather artificial, but it enables us to solve equations (6) analytically:

$$\mathbf{g}(t) = \left[u_g(t)I + v_g(t)\Psi\right]\mathbf{g}(0) + \left[z_g(t)I + w_g(t)\Psi\right]\mathbf{f}(0), \tag{13}$$

$$\mathbf{f}(t) = \left[u_f(t)I + v_f(t)\Psi\right]\mathbf{g}(0) + \left[z_f(t)I + w_f(t)\Psi\right]\mathbf{f}(0). \tag{14}$$

The time-dependent scalar functions in (13) and (14) must obey the following set of linear ordinary differential equations and initial conditions:

$$\dot{u}_f = \varpi u_g, \quad \dot{u}_g = -\varpi u_f, \qquad \dot{z}_f = \varpi z_g, \quad \dot{z}_g = -\varpi z_f, \tag{15}$$

$$\dot{v}_f = \varpi v_g - a(t)\left(u_f + \rho v_f\right) - \frac{d}{dt}\left[b(t)\left(u_f + \rho v_f\right)\right], \qquad \dot{v}_g = -\varpi v_f, \tag{16}$$

$$\dot{w}_f = \varpi w_g - a(t)\left(z_f + \rho w_f\right) - \frac{d}{dt}\left[b(t)\left(z_f + \rho w_f\right)\right], \qquad \dot{w}_g = -\varpi w_f, \tag{17}$$

$$u_g(0) = z_f(0) = 1, \qquad u_f(0) = z_g(0) = v_f(0) = v_g(0) = w_f(0) = w_g(0) = 0. \tag{18}$$

Solutions to (15) are obvious: $u_g(t) = z_f(t) = \cos(\varpi t)$, $u_f(t) = -z_g(t) = \sin(\varpi t)$. Equations (16) and (17) are more complicated, because their coefficients can be arbitrary functions of time satisfying the conditions $a(0) = b(0) = 0$. To simplify the problem, let us suppose that functions $a(t)$ and $b(t)$ grow from zero to some constant values a and b during a very short time interval $\delta \to 0$, then remain at constant levels until the instant $t_* - \delta$, and finally return very quickly to the initial zero values at $t = t_*$. Then (16) and (17) turn into linear inhomogeneous equations with constant coefficients a and b for $\delta < t < t_* - \delta$. To find the new initial conditions at $t = \delta$ we integrate equations (16) and (17) from 0 to δ, assuming that all functions remain bounded during this short interval, and then take the limit $\delta \to 0$. Thus we arrive at the conditions at $t = 0+$: $v_f(0+) = v_g(0+) = w_g(0+) = 0$, $w_f(0+) = -b/(1+b\rho)$. Using the standard scheme, one can find solutions for any instant of time in the interval $\delta < t < t_* - \delta$. To find the values at $t = t_*$, one has to integrate equations (16) and (17) from $t_* - \delta$ to t_* and again take the limit

$\delta \to 0$. Then $v_g(t_*) = v_g(t_* - \delta)$, $w_g(t_*) = w_g(t_* - \delta)$, whereas $v_f(t_*) = v_f(t_* - \delta)(1 + b\rho) + bu_f(t_*)$ and $w_f(t_*) = w_f(t_* - \delta)(1 + b\rho) + bz_f(t_*)$. The final results are

$$v_f(t_*) = (r\rho)^{-1} \left[2\varpi\beta e^{-\lambda t_*} \sinh(\kappa t_*) - r \sin(\varpi t_*)\right] = -w_g(t_*),$$

$$v_g(t_*) = (r\rho)^{-1} \left\{r \left[e^{-\lambda t_*} \cosh(\kappa t_*) - \cos(\varpi t_*\right] + a\rho e^{-\lambda t_*} \sinh(\kappa t_*)\right\},$$

$$w_f(t_*) = (r\rho)^{-1} \left\{r \left[e^{-\lambda t_*} \cosh(\kappa t_*) - \cos(\varpi t_*\right] - a\rho e^{-\lambda t_*} \sinh(\kappa t_*)\right\},$$

where

$$\lambda = a\rho/(2\beta), \quad \kappa = r/(2\beta), \quad r = \sqrt{(a\rho)^2 - 4\varpi^2\beta}, \quad \beta = 1 + b\rho. \tag{19}$$

The total energy variation equals

$$\Delta W \equiv \mathbf{f}^2(t_*) + \mathbf{g}^2(t_*) - \mathbf{f}^2(0) - \mathbf{g}^2(0) = h_{ff}Y_f^2 + h_{gg}Y_g^2 + 2h_{fg}Y_fY_g, \tag{20}$$

$$Y_f = \sum_{n=0}^{\infty} \Psi_n f_n(0), \quad Y_g = \sum_{n=0}^{\infty} \Psi_n g_n(0), \tag{21}$$

$$\rho h_{ff} = e^{-2\lambda t_*} \left\{[a\rho \sinh(\kappa t_*) - r \cosh(\kappa t_*)]^2 + 4\varpi^2\beta^2 \sinh^2(\kappa t_*)\right\}/r^2 - 1 \tag{22}$$

$$\rho h_{gg} = e^{-2\lambda t_*} \left\{[a\rho \sinh(\kappa t_*) + r \cosh(\kappa t_*)]^2 + 4\varpi^2\beta^2 \sinh^2(\kappa t_*)\right\}/r^2 - 1 \tag{23}$$

$$h_{fg} = -4a\varpi\beta e^{-2\lambda t_*} \sinh^2(\kappa t_*)/r^2. \tag{24}$$

3. Discussion and conclusions

Formulas (20)-(24) show that the possibility of the EM field amplification depends crucially on the sign of coefficient $\beta = 1 + b\rho$. Indeed, the energy variation in the short time limit ($|\kappa|t_* \ll 1$ and $|\lambda|t_* \ll 1$) equals $\Delta W = -2at_*Y_f^2/\beta + \mathcal{O}(t_*^2)$. Consequently, for short pulses of variations of material properties, the total energy always decreases if $\beta > 0$, but it can increase if $\beta < 0$. A similar behavior of ΔW can be observed for long time as well. If $\beta > 0$, then $\lambda > 0$. If κ is real, then $\kappa < \lambda$, so that $h_{ff} = h_{gg} = -1/\rho$ and $h_{fg} = 0$ for $(\lambda - \kappa)t_* \gg 1$. The same asymptotical values of these coefficients arise for $\lambda t_* \gg 1$ if κ is an imaginary number. Therefore, the total energy always decreases for long excitation pulses if $\beta > 0$. On the contrary, if $\beta < 0$, then κ is real and $\lambda < 0$, so that the total energy can grow as $\exp\left[(|\lambda| + |\kappa|)\, t_*\right]$ for $|\kappa|t_* \gg 1$ due to some kind of parametric instability.

The fundamental conclusion is that the possibility of the DCE simulation in cavities by means of temporal variations of conductivity inside a thin slab depends on the sign and maximal absolute value of the accompanying dielectric permeability variation $\delta\varepsilon$. If $\delta\varepsilon > 0$, then $b(t) > 0$, so that no field amplification can be expected. However $\delta\varepsilon$ can be negative, e.g., according to the Drude model: $\delta\varepsilon(\mathbf{r}, t) = -4\pi n(\mathbf{r}, t)e^2/\left[m_{ef}\left(\omega^2 + \gamma^2\right)\right]$. Here $n(\mathbf{r}, t)$ is the concentration of free carriers created by the laser pulse, e the electron charge, m_{ef} the effective mass, and γ is the collision frequency. Therefore, the intriguing question is: whether condition $|b|_{max}\rho > 1$ can be fulfilled in realistic situations? For realistic values $\mu < 1$ one can replace the coth function in the right-hand side of (12) by unity. In particular, for the cubical cavity, $\mu = 1/2$ and $\rho \approx 56/\varepsilon^2$. Since we consider the non-dispersive case, we have to assume that $\varpi \ll \gamma$. Then, using equations (10)-(12), one can arrive at the requirement $\gamma^2 < 8\pi^2 n_{max}e^2 L_s/(\lambda_0 m_{ef}\varepsilon^2)$, where n_{max} is the maximal concentration of free carriers inside the slab and $\lambda_0 = 2\pi c/\omega_0$ is the wave length corresponding to the fundamental cavity eigenfrequency. Certainly, n_{max} cannot be higher than the electron concentration in good metals $n_{met} \sim 10^{23}\,\mathrm{cm}^{-3}$. Using this value together with the free electron mass for m_{ef}, $\varepsilon \sim 10$, and taking $L_s/\lambda_0 \sim 10^{-3}$, one obtains the upper limit for the collision frequency $\gamma < 10^{14}\,\mathrm{s}^{-1}$. For $n_{max} \sim 10^{19}\,\mathrm{cm}^{-3}$ (as for inferior metals) the limitation is $\gamma < 10^{12}\,\mathrm{s}^{-1}$. In view of (11), the requirement $|b|_{max}\rho > 1$ means that it is necessary to achieve negative values of $\delta\varepsilon$ of the order of -10^4 (under the same assumptions). Variations of the dielectric function in the photo-excited GaAs to the negative values of the order of -10 were reported in [29] for the visible light (in a good agreement with the Drude approximation). For the microwave frequencies, one can expect bigger absolute values of these negative changes, but this seems to be a task for future experiments.

Note that coefficient b can be written as $b = \chi\mathcal{N}_s/L$, where $\mathcal{N}_s$ is the total number of carriers created by the laser pulse per unit area of the semiconductor surface and χ is the dielectric susceptibility per one carrier ($\delta\varepsilon = 4\pi\chi n$). In turn, $\mathcal{N}_s = \xi\Phi/E_g$, where Φ is the fluence of laser pulse, E_g the energy gap of the semiconductor and ξ the efficiency of free carriers generation. In addition we have $L\sqrt{\mu} \approx L_\perp$ - the transverse cavity dimension. Therefore, the condition $|b_{max}|\rho > 1$ results in the following requirement for the fluence: $\Phi > \varepsilon^2 L_\perp E_g/\left(4\pi^2\chi f\right)$. For $L_\perp \sim 10\,\mathrm{cm}$, $E_g \sim 2\,\mathrm{eV}$, $f = 1$, $\gamma = 10^{12}\,\mathrm{s}^{-1}$ and other parameters used above, one obtains $\Phi > 400\,\mathrm{J/m}^2$. This value is close to the threshold $1\,\mathrm{kJ/m}^2$ of the permanent damage for the GaAs material [29]. Note the strongly negative role of the factor $\varepsilon^2 \sim 100$ for the semiconductors like GaAs. Consequently, materials with smaller ε would be much better. Concluding, the dream to observe analogs of the DCE in cavities with periodical variations of the effective length is still alive, although the main challenges are nowadays in the realm of the material science.

Acknowledgments
A partial support of the Brazilian funding agency CNPq is acknowledged.

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