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To cite this article: Jun-Min Bai *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **473** 012049

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Markov Modeling for the Availability of Firearms

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Abstract. In order to give a reasonable forecast of firearms combat readiness, analysis of gun working method is the stochastic process of which time and status are both discrete, to simulate it by Markov process, and to deduce the calculation model of the steady state availability of firearms. Through the calculation model, as long as the known of gun state before start to work, we can just determine the probability of which gun is in the normal state at each firing time, this model can provide an important theoretical basis for maintenance test of the gun.

1. Introduction

The availability of firearm refers to the probability of being useful whenever it is needed, which is the criterion for firearm's performance [1] and it is the comprehensive reflection of reliability, maintainability and supportability for firearm products, meanwhile, it is one of the most important parameters of firearm system effectiveness [2]. For now, the existing model of availability is usually based on the maintenance time which is mainly applied to some large electronic equipment, such as radar and communications satellite. However, the performance for the firearm should be measured by the number of firing ammunition in its life cycle, but not described by working time or other time parameter, which means the model of time based cannot be used. In this article, the time span of firearm is regarded as a stochastic process which switches between normal and failure state, markov process is used to build model of availability, in this way, we can get a reasonable forecast of firearm combat readiness, meanwhile, we can find out the weakness in the process of using and maintenance so that we can improve it in time, the efficiency of firearm will be greatly improved.

2. Availability Analysis of Firearms

The work process of firearms is a discrete random process both in time and position, that means it may be normal or not at any time, in this way, the probability in the normal state is used to describe whether it is available or not. The concept of availability in firearms is the probability that it is normal at any moment during shooting. For any random shooting time t , state of firearms is:

$$X(t) = \begin{cases} 0 & (\text{normal at time } t) \\ 1 & (\text{fault at time } t) \end{cases} \quad (1)$$

The transient availability at time t is $A(t) = p\{X(t)=0\}$, which is only related to whether the firearm is normal or not at time t , but not related to whether the fault is happened or repaired before t , higher availability will naturally lead to higher reliability. Even if the reliability is low, but there is something wrong with the quick fix, availability will remain relatively high. Firearms are often in a state of continuous work under the condition of real war, the transient availability $A(t)$ is unreliable, so the steady-state availability is referred [3].

If there is a limit:



$$\lim_{t \rightarrow \infty} A(t) = A \quad (2)$$

A is called the steady-state availability ($0 \leq A \leq 1$), which means the proportion of firearm is normal during long-time shooting. Firearm belongs to repairable unit [3], for the reasons that it can keep working and continue to work when repaired again and again until it is dead.

3. Gun Availability Model based on Markov Process

Markov process [4] is a random process like this, if state of a process at time t_i is known, then process in the possibility of various state is completely determined at any time t_j after t_i , which will not be affected by state before. Which means the probability of what state the system is after time t can be confirmed by the state before [5]. We can generalize this feature to firearms so that markov process can be used to describe the availability of firearms.

Typical firearms' failure includes bullet/shell stuck or unloaded etc, and these types of failures on the total number of 80%-90% [6], by analysing the principle of fault produced we can know that the restoration belongs to basic repair which means similar failure still happens after being repaired, there is no relationship with when it is in fault last time. The probability of a component fault for firearms is really rare, and the situation of a component broken twice in the same product has never happened, fully repair refers to some components being replaced, but other component still has a continuous failure rate. Model of availability can be built based on assumptions as follows:

- 1) The life and repair of the unit obeys index distribution, failure rate λ and repair rate μ are both constant;
- 2) Each time of the maintenance aims at specified unit, other components will not be affected.
- 3) For the situation of many times failures, failure is not affected by the fault before and it will not affect failures behind, the generation of failure only related to the component's reliability, which means failure is independent in the aspect of time [7].

The working process of firearms includes two kinds of state normal/ fault, it is able to shoot in the state of normal but not in fault. Markov process of firearms can be described as follows:

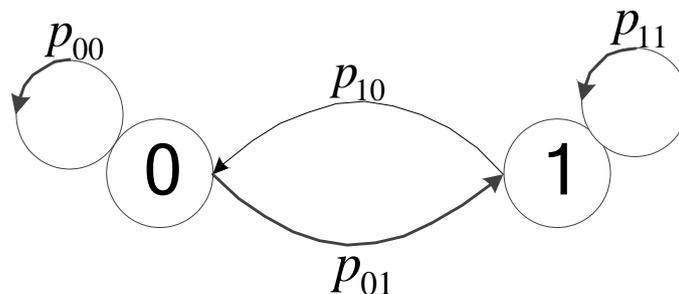


Figure 1. Markov process of firearms.

State "0" and "1" mean normal/fault respectively. $X(t)$ represents the state of firearms at time t -th shooting, it is a homogeneous markov chain and its state space is $E = \{0,1\}$. So that the state of firearms in life test is: $0 \rightarrow 0, 0 \rightarrow 1, 1 \rightarrow 0$ and $1 \rightarrow 1$ represent C_{00}, C_{01}, C_{10} and C_{11} respectively, state transition probabilities $p_{00}, p_{01}, p_{10}, p_{11}$ can be described as below:

$$\begin{cases} p_{00} = p \left\{ X(t+1)=0 | X(t)=0 \right\} \approx C_{00}/(C_{00} + C_{01}) = 1 - \lambda \\ p_{01} = p \left\{ X(t+1)=1 | X(t)=0 \right\} \approx C_{01}/(C_{00} + C_{01}) = \lambda \\ p_{10} = p \left\{ X(t+1)=0 | X(t)=1 \right\} \approx C_{10}/(C_{10} + C_{11}) = \mu \\ p_{11} = p \left\{ X(t+1)=1 | X(t)=1 \right\} \approx C_{11}/(C_{10} + C_{11}) = 1 - \mu \end{cases} \quad (3)$$

State transition matrix is:

$$p = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} = \begin{bmatrix} 1-\lambda & \lambda \\ \mu & 1-\mu \end{bmatrix} \quad (4)$$

We can define:

$$p(t) = \begin{bmatrix} p_0(t) \\ p_1(t) \end{bmatrix} = \begin{bmatrix} p\{X(t)=0\} \\ p\{X(t)=1\} \end{bmatrix} \quad (5)$$

The initial work state of firearms is:

$$p(0) = \begin{bmatrix} p_0(0) \\ p_1(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (6)$$

Markov chain's one-dimensional distribution is determined by the initial distribution $p(0)$ and t times of transition probability matrix at any moment $t \in T$ (T is the life of firearms), t times of transition probability matrix is equal to t -th power of one time transition probability matrix.

$$p(t) = p^t p(0) \quad (7)$$

Characteristic equation of matrix p is:

$$|kE - p| = \begin{vmatrix} k - (1-\lambda) & -\lambda \\ -\mu & k - (1-\mu) \end{vmatrix} = [k - (1-\lambda)][k - (1-\mu)] - \lambda\mu = 0 \quad (8)$$

E is second order identity matrix, eigenvalues are $k_1 = 1, k_2 = 1 - \lambda - \mu$, eigenvectors can be solved like this.

When $k_1 = 1$, $\varphi_1 = [m_1 \ n_1]^T$ is defined as eigenvectors, homogeneous linear equations below can be solved.

$$[k_1 E - p]\varphi_1 = \begin{bmatrix} \lambda & -\lambda \\ -\mu & \mu \end{bmatrix} \begin{bmatrix} m_1 \\ n_1 \end{bmatrix} = 0 \quad (9)$$

So that $\varphi_1 = [1 \ 1]^T$.

When $k_2 = 1 - \lambda - \mu$, $\varphi_2 = [m_2 \ n_2]^T$ is defined as eigenvectors, homogeneous linear equations below can be solved.

$$[k_2 E - p]\varphi_2 = \begin{bmatrix} -\mu & -\lambda \\ -\mu & -\lambda \end{bmatrix} \begin{bmatrix} m_2 \\ n_2 \end{bmatrix} = 0 \quad (10)$$

So that $\varphi_2 = [-\lambda \ \mu]^T$.

$$\varphi = [\varphi_1, \varphi_2] = \begin{bmatrix} 1 & -\lambda \\ 1 & \mu \end{bmatrix}; \varphi^{-1} = \frac{1}{\lambda + \mu} \begin{bmatrix} \mu & \lambda \\ -1 & 1 \end{bmatrix}; \phi = \begin{bmatrix} 1 & 0 \\ 0 & 1 - \lambda - \mu \end{bmatrix} \quad (11)$$

$$p = \varphi \phi \varphi^{-1}; p(t) = p^t = \varphi \phi^t \varphi^{-1} = \frac{1}{\lambda + \mu} \begin{bmatrix} \mu & \lambda \\ \mu & \lambda \end{bmatrix} + \frac{(1 - \lambda - \mu)^t}{\lambda + \mu} \begin{bmatrix} \lambda & -\lambda \\ -\mu & \mu \end{bmatrix} \quad (12)$$

We know that $0 < \lambda, \mu < 1$, so $(1 - \lambda - \mu)^t \rightarrow 0$ when $t \rightarrow \infty$.

$$\lim_{t \rightarrow \infty} p^t = \frac{1}{\lambda + \mu} \begin{bmatrix} \mu & \lambda \\ \mu & \lambda \end{bmatrix} \tag{13}$$

Matrix limit here means the limit of each matrix element. The result shows that probability of process being state "0" or "1" is $\mu/(\lambda + \mu)$ or $\lambda/(\lambda + \mu)$ after a long time, so we can conclude the model for steady availability is:

$$p(t) = \lim_{t \rightarrow T} p^t = \frac{\mu}{\lambda + \mu} (t \in T) \tag{14}$$

4. Instance Analysis

The complete life test data of two 14.5mm anti-aircraft guns is collected in this paper, the cumulative use of bullets $6000 \times 2 = 12000$, and failure statistics is shown in table 1 & 2.

Table 1. Failure statistics.

NO. \ Name	Ammunition resisted	Unloaded	Ammunition jammed	Ammunition dropped	Stop shooting	Pin breakage	Total
1#	5	2	1	—	1	—	9
2#	6	1	—	2	—	1	10

Table 2. Failure statistics.

NO. \ Time	1	2	3	4	5	6	7	8	9	10
1#	1226	1774	2322	2572	2622	2822	4604	5295	5495	—
2#	507	2504	2554	2654	2854	3004	3254	4596	5425	5520

By observing the data above we could know that failures such as ammunition resisted/jammed/dropped or unloaded account for more than 90% of the total, and relevant repair belongs to basic repair. The main reason is size/strength of components or ammunition belt and the mouth size of bullet box are dimensionally out of tolerance, some of the failures are blamed for inappropriate use and maintenance. Continuous failure rate will not be affected in the case of a normal life consumption, situation of state transition can be arranged in table 3 according to the table 2.

Table 3. State transfer statistics.

NO. \ State Transfer	C_{00}	C_{01}	C_{10}	C_{11}
1#	5982	9	9	0
2#	5980	10	10	0

1-th / t-th times of transition probability matrix can be calculated as follows according to formula (4) and table 3.

$$p_1 = \begin{bmatrix} 0.9985 & 0.0015 \\ 1.0000 & 0.0000 \end{bmatrix}, p_2 = \begin{bmatrix} 0.9983 & 0.0017 \\ 1.0000 & 0.0000 \end{bmatrix}$$

$$p_1^t = \begin{bmatrix} 0.9985 & 0.0015 \\ 0.9985 & 0.0015 \end{bmatrix}, p_2^t = \begin{bmatrix} 0.9983 & 0.0017 \\ 0.9983 & 0.0017 \end{bmatrix}$$

Steady availability for 1# and 2# are $A_1 = 0.9985$, $A_2 = 0.9983$ respectively, and availability curves are as follows:

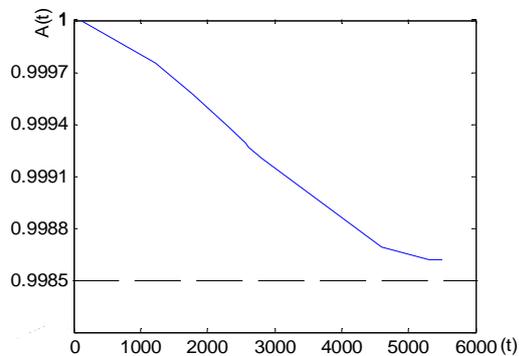


Figure 2. Availability curve for 1#.

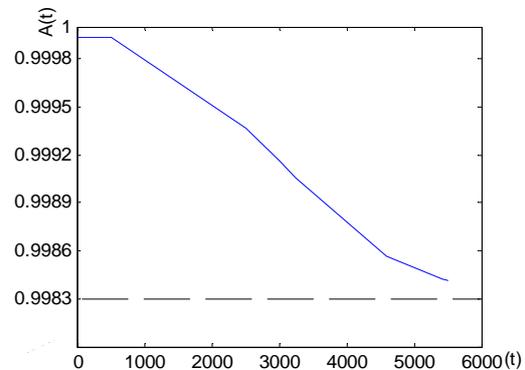


Figure 3. Availability curve for 2#.

5. Conclusion

A conclusion of work time and state are random for firearms is got in this paper, which is based on the mechanism and characteristics of the failure, firearms are defined as products with single unit and repairable; Subsequently, Markov process is used to build the state transition probability matrix for firearms; Model of availability or steady availability is deduced, which is used to calculate firearm's availability in the condition of initial state is known; Finally, the calculation model is verified in the example of 14.5mm anti-aircraft guns. This paper has some reference value for reasonable prediction of weapons and equipment reliability.

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