Presentation of the Exact Technique for Calculation of the Torsional Constant for the T-Section

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Abstract. Both bending and torsional stiffness are the important factors affecting the spatial work of the reinforced concrete engineering structures. While bending stiffness is studied carefully, the torsional stiffness is not sufficiently investigated. This paper studies one of its components, the torsional constant. The accepted in the practice formulae of the strength of materials are appropriate only for thin-walled sections and give a significant error in other cases. That's why the exact technique has been tested on the example of the T-section, which is one of the most common cross-sections applied in engineering (for the reinforced concrete overlaps, bridge superstructures, etc.). The results of the calculation by different methods have been compared. The essential error has been proved.

1. Introduction

The change in values of both bending and torsional stiffness influences significantly the stress redistribution of the spatial structures, as has been proved both theoretically and experimentally [1]. However, the torsional stiffness of reinforced concrete elements is not taken into account in the Building Standards, and its components are not studied sufficiently. Also, in the Building Codes shear modulus of concrete is accepted depending on the Young's modulus, which is not true after passing the zone of elastic deformation.

The torsional stiffness depends on the Kirchhoff modulus and the torsional constant. First, the secant shear modulus of concrete was investigated [1]. The next important task is to determine the torsion constant for the T-section. So, the purpose of this paper is to test the exact technique for computation of the torsional constant for the concrete section, in order to apply it in the subsequent calculations of the torsional stiffness of the reinforced concrete elements when studying stress redistribution in the cross-ribbed systems.

The example of the T-section is considered, as one of the most common cross section, because the overlap is often divided into T-elements for calculation of the overlaps and bridges with consideration of spatial work. The stiffness of the T-elements with normal cracks can be determined by [2]. The torsional constant of the whole (uncracked) element is generally determined as the sum of values of stiffness for the shelf and ribs, which in the case of not thin-walled sections gives a significant error, and the torsional deformations of such a section cannot be considered as the equivalent to the deformations of the thin-walled section. The reason is that the Saint-Venant's principle is exact only for the thin-walled sections. Due to the absence of the exact solution for the case of the not thin-walled elements (T-sections, I-beams, etc.), the torsional stiffness should be found by simulation using the volumetric finite elements and further approximation (e.g., according to [2]).

Therefore, this paper studies the technique for calculation the stiffness of the T-section by the exact formulae of the theory of elasticity.

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2. Desctiption of the applied technique

The formulae of the proposed technique have been derived from the basic equations of the theory of elasticity [3]. The stiffness of the T-section has been determined by the exact method, which considers the T-section element as a prismatic rod of the polygonal section, the sides of which form right angles (figure 1).

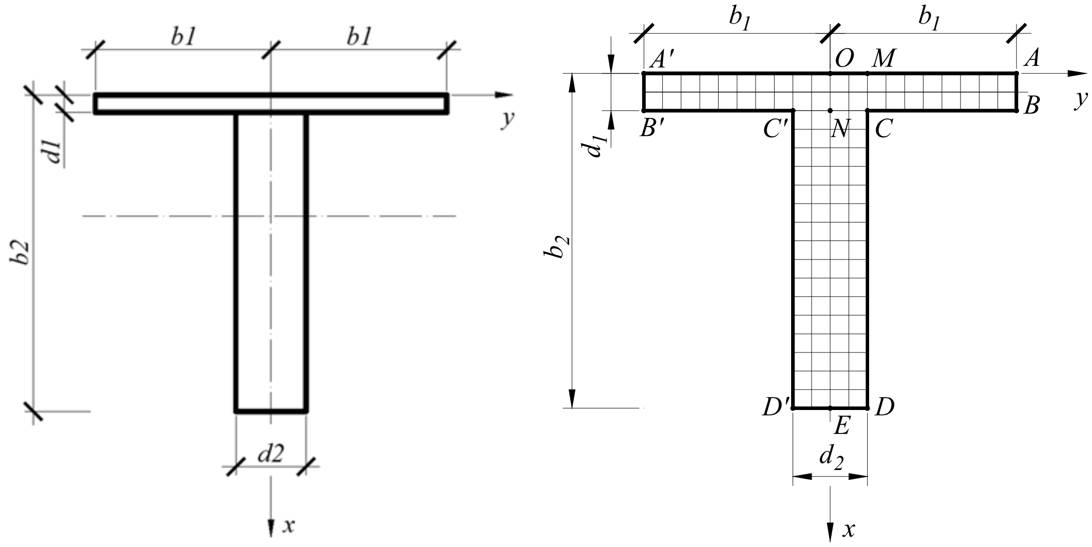

Figure 1. T-section: splitting into elements according to the accepted technique.

Torsional stiffness for T-beam is calculated by the formula derived from the elasticity theory formulae [3]:

$$C = G\left\{\begin{aligned} &\frac{d_2^3 b_2}{3} + \frac{2}{3} d_1^3\left(b_1 - \frac{d_2}{2}\right) - \frac{d_1^3 d_2}{\pi^2}\sum_{k=1,3,\dots}^{\infty}\frac{z_{2k}}{k^3}\left[th\frac{k\pi d_2}{2d_1} + th\frac{k\pi}{2d_1}\left(b_1 - \frac{d_2}{2}\right)\right] + \\ &+\frac{d_2^3 d_1}{\pi^2}\sum_{k=1,3,\dots}^{\infty}\frac{z_k}{k^3} sh\frac{k\pi b_2}{2d_2} sch\frac{k\pi d_1}{2d_2} sch\frac{k\pi(b_2 - d_1)}{2d_2} + \\ &+\frac{64 d_1^4}{\pi^5}\sum_{k=1,3,\dots}^{\infty}\frac{1}{k^5}\left[th\frac{k\pi d_2}{2d_1} - cth\frac{k\pi b_1}{d_1} + \csc h\frac{k\pi b_1}{d_1} sch\frac{k\pi d_2}{2d_1}\right] - \\ &-\frac{32 d_1^4}{\pi^5}\sum_{k=1,3,\dots}^{\infty}\frac{1}{k^5}\left[th\frac{k\pi b_2}{2d_2} + th\frac{k\pi d_1}{2d_1}\right] \end{aligned}\right\} \qquad (1)$$

The expression (1) can be represented as

$$C = C_0\eta(b_1, b_2; d_1, d_2) \qquad (2)$$

where C_0 is the sum of values of stiffness, which form the T-section, η is the influence coefficient for its stiffness.

Notation conventions in formulae (1-2):

b_1, b_2, d_1, d_2 are the geometric dimensions (see figure 1),

z_m are unknown coefficients which are determined from the infinite regular system of linear equations:

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$$z_m = \sum_{n=1}^{\infty} C_{mn} z_n + \gamma_m, \qquad (m = 1, 2, ...) \tag{3}$$

where the components can be defined through the rows:

$$C_{2k-1,2p-1} = C_{2k,2p} = 0$$

$$\begin{cases} C_{2k-1,2p} = \dfrac{2(2k-1)d_1}{\pi d_2} \dfrac{sh\alpha_k d_1 sh\alpha_k (b_2 - d_1) \csc h\alpha_k b_2}{p^2 + (2k-1)^2 (d_1 / d_2)^2} \\ C_{2k,2p-1} = \dfrac{-4kd_2}{\pi d_1} \dfrac{ch\dfrac{k\pi d_2}{2d_1} sh\dfrac{k\pi}{d_1}\left(b_1 - \dfrac{d_2}{2}\right) \csc h\dfrac{k\pi b_1}{d_1}}{(2p-1)^2 + k^2 (d_2 / d_1)^2} \end{cases}$$

$$\begin{cases} \gamma_{2k-1} = \dfrac{4(2k-1)}{d_1 d_2} \dfrac{sh\alpha_k (b_2 - d_1)}{sh\alpha_k b_2} \left[-\dfrac{8d_2^2}{(2k-1)^3 \pi^3} + \dfrac{32 d_1^3}{\pi^4 d_2} sh\alpha_k d_1 \times \sum_{p=1}^{\infty} \dfrac{1 - sh\dfrac{(2p-1)\pi d_2}{2d_1} \csc h\dfrac{(2p-1)\pi b_1}{d_1}}{(2p-1)^2 \left[(2p-1)^2 + (2k-1)^2 (d_1 / d_2)^2 \right]} \right] \\ \gamma_{2k} = \dfrac{8k}{d_1 d_2} \dfrac{sh\dfrac{k\pi}{d_1}\left(b_1 - \dfrac{d_2}{2}\right)}{sh\dfrac{k\pi b_1}{d_1}} \left[\dfrac{4d_1^2}{k^3 \pi^3} \dfrac{1 + (-1)^{k+1}}{ch\dfrac{k\pi b_1}{d_1}} + \dfrac{16 d_2^3}{\pi^4 d_1} ch\dfrac{k\pi b_1}{d_1} th\alpha_k \times \sum_{p=1}^{\infty} \dfrac{1 - sh\alpha_p d_1 \csc h\alpha_p b_2}{(2p-1)^2 \left[(2p-1)^2 + k^2 (d_2 / d_1)^2 \right]} \right] \end{cases} \tag{4}$$

Substituting the found values of the unknown Z_k^+ and Z_k^- in the calculation formula, respectively, the upper and lower limits for the stiffness value C^+ and C^- for the T-section with an arbitrary ratio of the geometric dimensions of the shelves and the wall are obtained. The estimated value of the stiffness is C $C = GI = \dfrac{C^+ + C^-}{2}$, the error is $\delta(\beta) = \dfrac{C^+ + C^-}{C}$. The system (3) is regular. The set of free members $\{\gamma_m\}$ of the system is bounded from above and tends to zero at $m \to \infty$ as $O\left(\dfrac{1}{m}\right)$ which indicates its convergence and allows to determine the unknowns z_m with any accuracy.

3. Comparison of the results with another methods

The comparison for the calculation of torsional stiffness for the T-section by this technique with values obtained by the known formulae of the strength of materials as the sum of the values for rectangles, into which the section can be divided (considering the Saint-Venant's torsional coefficients) and the data of a similar calculation of the thin-walled section by the exact and approximate technique is given below.

The element of the T-section which models the elements of the bridge superstructure has been accepted. The geometric characteristics of the T-section are shown in table 1 and figure 2. Stiffness *C=GI* was calculated by the explained above exact technique and is equal to $G_c I_t = 1125 \times 227329 = 255744712$ kN/cm^2.

Table 1. Geometry of the T-section.

b_1, cm	b_2, cm	d_1, cm	d_2, cm
50	90	5	20

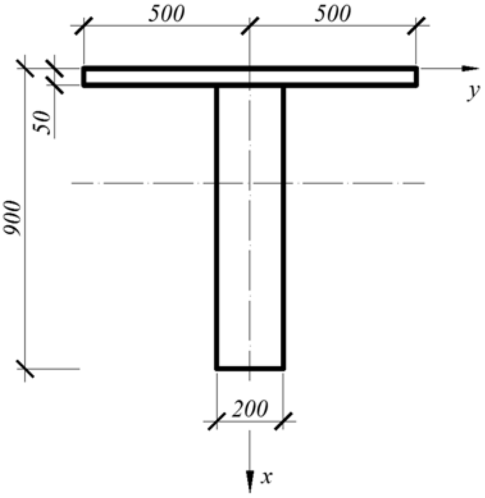

Figure 2. The cross section geometry.

The value of the torsional constant obtained by the formulae of the strength of materials is $I_t = 197111\,\text{cm}^4; G_c I_t = 221749407\ \text{kNcm}^2$ for the concrete of the shear modulus $G_c = 1125\ \text{kN/cm}^2$. The error is $\delta = 14.63\,\%$. When comparing the calculation with the more accurate technique of S.P. Timoshenko $I_t = 197088\, cm^4; G_c I_t = 221725002\ kNcm^2$, and the error is $\delta = 14.63\,\%$ (see table 2).

Table 2. Comparison of the values of the torsional constant calculated by different techniques.

The calculational technique	I_t, cm^4	GI_t, kNcm^2	δ,%
Strength of materials	197111	221749407	14.62
Method of Stephen Timoshenko	197088	221725002	14.63
FME (ANSYS)	217838	245067750	5.64
The exact technique [2]	210367	259721628	

The applied exact technique has been checked by the calculation of the thin-walled profile. Then, the shear stresses are distributed almost uniformly along the thickness of the closed thin-walled section eliminating in the midline of the profile. The torsion constant is determined by assuming that each section works as part of a rectangular cross-section, and its value can be obtained by replacing the integration along the middle line of the profile by approximate summation:

$$J_t = \frac{1}{3}\int_0^L \delta^3(s)\,ds = \frac{1}{3}\sum_{i=1}^{3}\delta_i^3 L_i \tag{5}$$

where L_i, δ_i are the length and the average thickness of the i -the section (figure 3).

In fact, the torsional stiffness is slightly higher than that determined by the formula (5) [4], which is explained by a number of factors for the studied section (the correction coefficient is 1.1-1.2).

If the thickness of the rib of the researched T-section will be decreased by a factor of four, then the thin-walled section will be obtained (see table 3 and figure 3).

Table 3. Geometry of the thin-walled T-section.

b_1, cm	b_2, cm	d_1, cm	d_2, cm
50	90	5	5

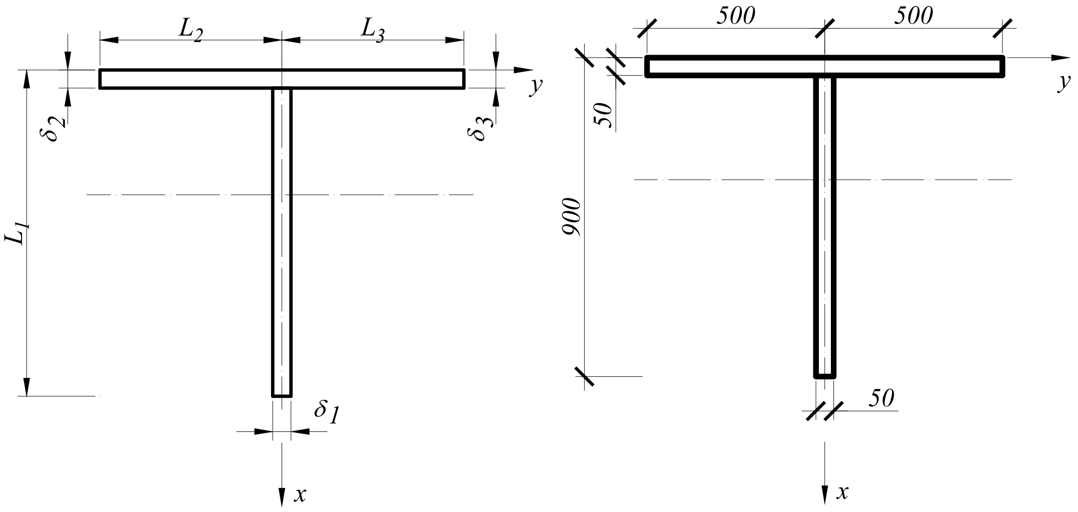

Figure 3. Geometry of the thin-walled T-section.

Value of the torsional constant is $I_t = 7572.56\,cm^4$. The error is δ =*1.848 %* and δ =*1.675 %* when compared to the generally accepted calculation methods (see table 4), which confirms the correctness of the exact technique for calculating the torsional stiffness of the T-section [3] and its effectiveness for different ratio of sizes of the T-section.

Table 4. Comparison of the values of the torsional constant calculated by different techniques.

The calculational technique	I_t, cm^4	GI_t, kNcm^2	δ,%
Strength of materials	7445.83	8662500	1.848
Method for thin-walled sections	7445.73	8671500	1.675
FME (ANSYS)	7963.00	8958375	5.156
The exact technique [2]	7572.56	8573625	

4. Conclusions and prospects of research

The torsional constant of the reinforced concrete T-sections should not be calculated by the traditional approximate technique, because for the nonsymmetrical elements with thick ribs and shelves the Saint-Venant's principle gives a significant error. Being the component of the torsional stiffness, an important factor affecting the spatial work of the engineering structures, the torsional constant should be calculated more precisely. The further studies will show the influence of the refined values of the torsional constant and the secant shear modulus of concrete as the components of the torsional stiffness on the strain redistribution in the elements of the spatial systems.

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