

PAPER • OPEN ACCESS

## Fracture Analysis for Torsion Problems of a Deep Sea Spar Platform Main Body

To cite this article: Nan Liu *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **473** 012007

View the [article online](#) for updates and enhancements.

# Fracture Analysis for Torsion Problems of a Deep Sea Spar Platform Main Body

Nan Liu<sup>1</sup>, Jisen Liu<sup>2</sup>, Wei Liu<sup>3</sup>

<sup>1</sup> School of transportation civil engineering, Shandong Jiaotong University, China

<sup>2</sup> Branch of water administration supervision, Jinan, China

<sup>3</sup> School of transportation civil engineering, Shandong Jiaotong University, China

**Abstract.** Attention should be paid to variations of crack effects on the stability of offshore structure during building and using. The body of the deep-water Spar platform is the important support of the whole structure, once produced edge cracks in certain parts, the overall structure of the platform would be severely affected. By using boundary integral equations which applied to the edge cracks of the cylinder, this paper discussed singularity of the crack tip; then used the boundary element numerical calculation method to divide the crack boundary into many units, used different interpolation functions to calculate the stress intensity factor of crack tip; Finally, calculated torsional fracture of spar platform with edge cracks under different wind loads, got the allowed maximum crack length of Spar's body with cracks under different wind scale. It was concluded that used the boundary element calculation method was practical in ocean engineering.

## 1. Introduction

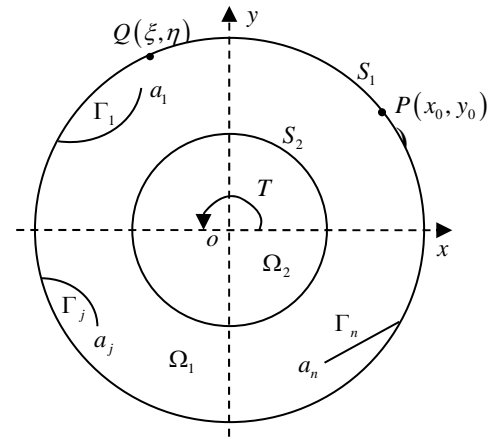
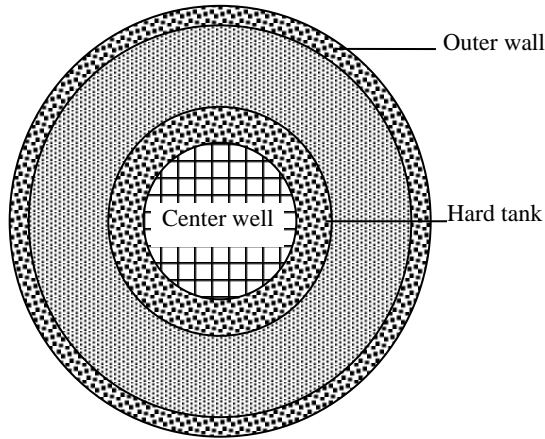
Deep-sea Spar platform has been served in the Marine environment for a long time, various complex environmental loads (such as earthquakes, etc.) may cause the torsional damage of the platform, which may be one of the most important factors in the whole structure damage [1], and the wind load may cause the upper structures of the platform to have a torsion effect on its main part, the main reason is that the wind surface of the whole platform is asymmetric [2]. This article analyzed torsional fracture of the main structure of Spar platform with edge cracks, used the boundary element numerical method to divide the boundary into many units, calculated the stress intensity factor at the crack tip by using different interpolation functions, calculated torsional rigidity of a cylinder with a straight edge crack by programming, and calculated the maximum allowable crack length of the cracked body under different wind conditions at last, which proves that boundary element method is practical in marine engineering.

## 2. Basic structure of the Spar platform

Main body plays an important role in the whole spar structure. Figure 1 shows a cross-section of the main part of Truss Spar platform which has been put into use at present. From top to bottom, it is divided into hard cabin, middle section and soft cabin. As we can see, the body of the Spar platform is not a simple cylinder structure, but a structure made by many different materials from the inside out. Its internal also contains a lot of facilities and equipment, and its structural type is quite complex, so it's very difficult to analysis directly. We should simplify the main structure appropriately during torsional failure analysis under various marine environmental loads [3]. Each component of the main body as if it is made of one material, as shown in figure 2, the section is a circular structure, the boundary between the outermost and the outer layer is denoted as  $S_1$ , the boundary between the inner layer and hollow part is denoted as  $S_2$ , thus the circular structure is divided into two parts: the outside is denoted as  $\Omega_1$ ,



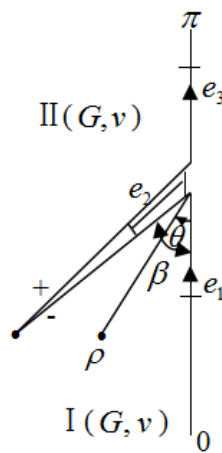
the edge contains curve crack  $\Gamma_k (k = 1, 2, \dots, n)$ , and the inside is denoted as  $\Omega_2$ , which without cracks. The whole main structure is subjected to torsion, the torque is  $T$ , shear modulus of materials and poisson ratio are  $G$  and  $\nu$  respectively.



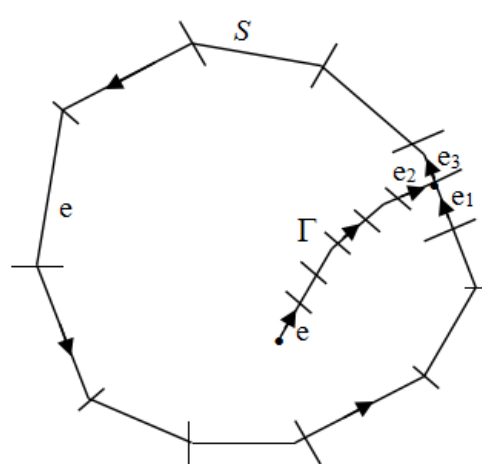
**Figure. 1** Major structures of spar platform. **Figure. 2** Section geometry of a cracked cylinder.

### 3. Crack tip singularity for torsion of a cylinder with edge crack

In this section, the singularity of crack tip is considered. Take the local area with cracks on the boundary as an example, as shown in figure 3.



**Figure 3.** Areas and units division.



**Figure 4.** Division on boundary and crack.

(1) For area I ( $0 < \theta < \beta$ ),  $(G, \nu)$ :

$$w_1 = \rho^{\lambda_1} A_1 \cos(\lambda_1 \theta)$$

$$\tau_{\theta 1} = -G \lambda_1 \rho^{\lambda_1 - 1} A_1 \sin(\lambda_1 \theta)$$

$$\gamma_{\rho 1} = \lambda_1 \rho^{\lambda_1 - 1} A_1 \cos(\lambda_1 \theta)$$

Where,  $\lambda_1 = \pi / \beta$  ( $A_1 \neq 0$ ,  $\lambda_1 > 1$ ).

(2) For area II ( $\beta < \theta < \pi$ ),  $(G, \nu)$ :

$$w_2 = \rho^{\lambda_2} C_2 \cos[\lambda_2 (\theta - \beta)]$$

$$\tau_{\theta 2} = -G\lambda_2 \rho^{\lambda_2-1} C_2 \sin[\lambda_2(\theta - \beta)]$$

$$\gamma_{\rho 2} = \lambda_2 \rho^{\lambda_2-1} C_2 \cos[\lambda_2(\theta - \beta)]$$

Where,  $\lambda_2 = \pi / (\pi - \beta)$  ( $A_2, B_2 \neq 0, 0 < \beta < \frac{\pi}{2}$ ,  $\lambda_2 > 1$ ),  $C_2 = \frac{A_2}{\cos(\lambda_2 \beta)}$ .

We should note that when  $\beta = \frac{\pi}{2}$ ,  $\lambda_1 = \lambda_2 = 2$ , this will cause the interpolation function on the different units repeated. In order to make the following interpolation function meaningful, when  $\beta = \frac{\pi}{2}$ , we choose  $\lambda_1 = \lambda_2 = 4$ , this will correspond to the characteristic solution  $k = 2$ .

#### 4. Boundary element numerical calculation

##### 4.1. Selection of interpolation functions

The resulting boundary integral equation satisfies the condition of displacement singleness, but it is very difficult to solve [4]. Therefore, other methods are considered instead of direct solution. Much practice proves that the boundary element method is a feasible numerical method. The boundary and cracks are divided into M plus N linear units, as shown in figure 3 and 4, that is  $S + \Gamma = \sum_e L_e$ ,  $S = \bigcup_{j=1}^M L_j$ ,

$T = \bigcup_{j=1+M}^{M+N} L_j$ , each linear unit  $L_e$  is mapped into the interval  $-1 \leq t \leq 1$ , the point  $Q$  on the unit  $L_e$  is denoted as

$$Q = Q_1^e N_1(t) + Q_2^e N_2(t), (|t| < 1, Q \in L_e) \quad (1)$$

Where,  $Q_1^e$  and  $Q_2^e$  are the two terminal nodes of unit  $e$ , take shape functions as follows:

$$N_1(t) = \frac{1}{2}(1-t), \quad N_2(t) = \frac{1}{2}(1+t)$$

The interpolation function on the boundary  $S$  and crack  $\Gamma$  is defined as follows. The function of the interpolation function is to close the linear equations after the boundary integral equation is discretized and to satisfy the displacement single value condition. First, the linear interpolation function of the boundary  $S$  is defined as follow:

$$F(Q) = F_1^e N_1(t) + F_2^e N_2(t) \quad (|t| \leq 1, Q \in L_e \subset S) \quad (2)$$

The unit on the crack  $\Gamma$  can be divided into general linear unit and crack tip unit, where the general linear unit does not contain crack tip, and its interpolation function is

$$F(Q) = F_1^e N_1(t) + F_2^e N_2(t) \quad (|t| \leq 1, Q \in L_e \subset \Gamma) \quad (3)$$

If the initial node ( $t = -1$ ) of the unit is a crack tip point,  $F(Q)$  is denoted as

$$F(Q) = \frac{F_1^e}{\sqrt{N_2(t)}} + (F_2^e - F_1^e) N_2(t) \quad (|t| \leq 1, Q \in L_e \subset \Gamma) \quad (4)$$

The unit  $e_2$  which is adjacent to unit  $e_1$  on the boundary  $S$ , as shown in figure 4, whose interpolation function is

$$F(Q) = (F_1^{e_1} - F_2^{e_1})N_1(t) + \frac{F_2^{e_1}}{[N_1(t)]^{1-\lambda_1}} \quad (|t| \leq 1, Q \in L_{e_1} \subset S) \quad (5)$$

The unit  $e_2$  which is adjacent to unit  $e_3$  on the boundary  $S$ , the unknown function  $F(Q)$  is denoted as

$$F(Q) = \frac{F_1^{e_3}}{[N_2(t)]^{1-\lambda_2}} + (F_2^{e_3} - F_1^{e_3})N_2(t) \quad (|t| \leq 1, Q \in L_{e_3} \subset S) \quad (6)$$

For the unit  $e_2$  where the crack  $\Gamma$  intersects with the boundary, the following interpolation function is introduced [5]

$$F(Q) = \frac{F_{2c}^{e_2}}{[N_1(t)]^{1-\lambda_2}} + (F_{1c}^{e_2} - F_{2c}^{e_2} - Q_{2c}^{e_2})[N_1(t)] + \frac{Q_{2c}^{e_2}}{[N_1(t)]^{1-\lambda_1}} \quad (|t| \leq 1, Q \in L_{e_2} \subset \Gamma) \quad (7)$$

Thus, on the unit  $e_1$ ,

$$F(Q) = \frac{\partial \varphi(Q)}{\partial s(Q)} = - \frac{\partial \varphi(Q)}{\partial \rho} \bigg|_{\theta=0} = - \frac{1}{\alpha} \frac{\partial w_1(\rho, 0)}{\partial \rho} = - \frac{1}{\alpha} \lambda_1 A_1 \rho^{\lambda_1-1}$$

$$F(Q) = (F_1^{e_1} - F_2^{e_1})N_1(t) + \frac{F_2^{e_1}}{[N_1(t)]^{1-\lambda_1}}, \quad \rho = l_{e_1} N_1(t)$$

Therefore,

$$\lim_{\rho \rightarrow 0} [(\rho)^{1-\lambda_1} F(Q)] = - \frac{1}{\alpha} \lambda_1 A_1 = (l_{e_1})^{1-\lambda_1} F_2^{e_1}$$

Consequently,

$$F_2^{e_1} = - \frac{1}{\alpha} \lambda_1 A_1 (l_{e_1})^{\lambda_1-1} \quad (8)$$

In the same way, on unit  $e_2$ ,

$$F_{2c}^{e_2} = - \frac{1}{\alpha} \lambda_2 C_2 (l_{e_2})^{\lambda_2-1} \quad (9)$$

$$Q_{2c}^{e_2} = - \frac{1}{\alpha} \lambda_1 A_1 (l_{e_2})^{\lambda_1-1} \quad (10)$$

On unit  $e_3$ ,

$$F_1^{e_3} = - \frac{1}{\alpha} \lambda_2 C_2 (l_{e_3})^{\lambda_2-1} \quad (11)$$

Then,

$$F_{2c}^{e_2} = (l_{e_2})^{\lambda_2-1} (l_{e_3})^{1-\lambda_2} F_1^{e_3} \quad (12)$$

$$Q_{2c}^{e_2} = (l_{e_1})^{1-\lambda_1} (l_{e_2})^{\lambda_1-1} F_2^{e_1} \quad (13)$$

#### 4.2. Boundary element numerical calculation

The following singular integral formula can be obtained [2]:

$$\int_{-1}^1 F(Q(t)) \frac{dt}{t} = F_2^e - F_1^e, \text{ (for liner unit on the boundary } S, L_e \subset S);$$

$$\int_{-1}^1 F(Q(t)) \frac{dt}{t} = F_2^e - F_1^e, \text{ (for common unit on a crack that does not contain a crack tip, } L_e \subset \Gamma);$$

$$\int_{-1}^1 F(Q(t)) \frac{dt}{t} = F_1^e \sqrt{2} \ln(3 - 2\sqrt{2}) + (F_2^e - F_1^e), \text{ ( for the initial node unit, } L_e \subset \Gamma).$$

For unit  $e_1, e_2, e_3$ , When  $\beta$  takes any value,  $\lambda_1$  and  $\lambda_2$  are not natural numbers, so the Newton's binomial theorem no longer holds, here we only calculate in the case of  $\beta = \pi/2$ , then,  $\lambda_1 = \lambda_2 = 4$ , so

$$\int_{-1}^1 F(Q(t)) \frac{dt}{t} = -F_1^{e_1} + \frac{1}{6} F_2^{e_1}, \text{ (for unit } e_1 \text{ on boundary } S, L_e \subset S);$$

$$\int_{-1}^1 F(Q(t)) \frac{dt}{t} = -F_{1c}^{e_2} + \frac{1}{6} F_{2c}^{e_2} + \frac{1}{6} Q_{2c}^{e_2}, \text{ (for unit } e_2 \text{ where crack intersects with the boundary, } L_e \subset \Gamma);$$

$$\int_{-1}^1 F(Q(t)) \frac{dt}{t} = -\frac{1}{6} F_1^{e_3} + F_2^{e_3}, \text{ (for unit } e_3 \text{ on boundary } S, L_e \subset S).$$

The algebraic equations above can solve out the unknown variables  $\{F_j\}$ ,  $Q_{2c}^{e_2}$  and  $\lambda_i$  of the functions  $F(Q)$ , then the stress intensity factor can be calculated as follow:

$$K_{III}(a_j) = -\frac{G\alpha}{2} \sqrt{2\pi l_e} F_1^e \quad (14)$$

### 5. An example of torsion analysis of Spar platform under wind load

In this section, the torsional fracture of the main body in the deep sea Spar platform with linear edge crack under different wind loads is calculated, as shown in figure 5. The main part of the platform is regarded as a simplified cylinder composed of high strength concrete materials containing linear edge cracks[6]. Building facilities on the deck of the platform are 16 to 26m above sea level, and the distance between the main center and the point of acting force of wind load is  $b = 16m$ , and  $A = 200m^2$ .

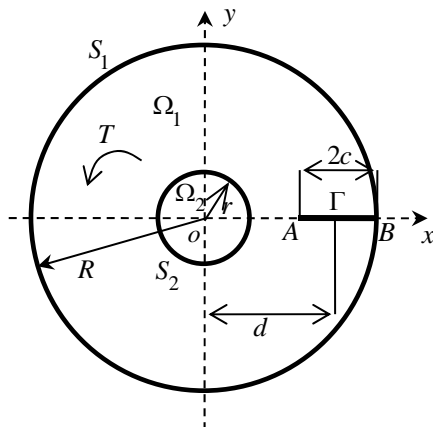


Figure 5. Torsion of a spar with a straight crack.

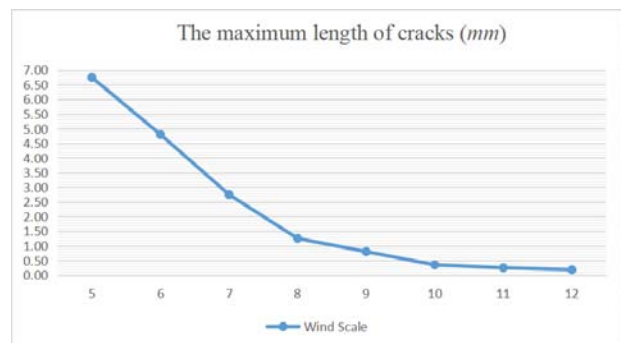


Figure 6 The maximum length of cracks of main body

The main parameters involved in the calculation are as follows:

$$k = 1.5, \quad k_z = 1.1, \quad \kappa = 0.613 \cdot N \cdot s^2 / m^4, \quad K_{IC} = 1.105 MN / m^{3/2}, \quad [\tau] = 2.22 Mpa; \\ R = 16.155 m, \quad r / R = 0.4, \quad c / R = 0.3, \quad d / R = 0.7$$

By using the present method and a FORTRAN program [7-10], the numerical results are obtained as

$$D^* = 0.996652, \quad K_{III}^*(A) = -0.597362, \quad K_{III}^*(B) = 0$$

$$\text{Where, } D^* = D / \left( \frac{\pi}{2} G R^4 \right), \quad K_{III}^* = K_{III} / (G \alpha R \sqrt{\pi c})$$

From the strain energy density factor criterion (Li, 1998), we can obtain:

$$K_{IIIc} = \sqrt{1 - 2\nu} K_{Ic} = 0.816 K_{Ic} = 0.90168 (MN / m^{3/2}), \quad (\nu = 0.167)$$

According to the fracture criterion, the allowable maximum crack length of the Spar platform containing the main part of the linear edge crack under different wind loads can be calculated, as shown in figure 6. It is known that when there is fresh breeze, the maximum allowable crack size is 6.7577mm, but when hurricane is coming, the maximum is 0.1805mm. With the increase of wind load, the maximum crack length permitted of the main body with cracks in the platform decreases. This shows that in marine engineering, if we want to avoid the fracture failure of the structure caused by torsion, we must pay attention to the existence of cracks. The security of the platform can be guaranteed only by minimizing the length of cracks in the platform structure, which is also applicable to the design and use of various platforms on the sea [11].

## 6. Conclusions

The following main conclusions are obtained by analysis:

(1) Under the action of wind load and other factors, cracks in the structure cannot be completely avoided, and cracks are the most important reason for torsional fracture damage in the most critical supporting part of Spar platform.

(2) Analysis of Torsion fracture is very necessary for the safety of platform structure, so it should be carried out throughout the design, construction and using of ocean platform.

(3) The computing method of boundary element numerical is practical in Marine engineering. At the same time, if the structural fracture damage caused by torsion is to be avoided as much as possible, we must pay attention to the cracks, only by minimizing the crack length of the platform structure can we keep its safety, which is also applicable to the design and using of various platforms on the sea.

## Acknowledgments

This work was supported by the PhD research start-up and College fund of Shandong Jiaotong University.

## References

- [1] Zhang M Z 2001 Earthquake Resistant Engineering **3** p 11-14
- [2] Lu Z Z 2005 Fracture Analysis for Torsion Problem of Isotropic or Anisotropic Bar with Curvilinear Cracks Master's Dissertation of Ocean University of China chapter **4** pp 41-50
- [3] Pan T W 2010 Static and Dynamic Analysis for the Structure of Gravity Platform in Shallow Sea Doctoral Dissertation of Ocean University of China chapter **5** pp 61-85
- [4] Guo Z H 1973 Journal of Peking University **1** p 43
- [5] Guo Z H 1979 J. of Mech. **3** p 298
- [6] Guo Z H 1980 J. of Mech. **2** p 208
- [7] Paker A P 1981 E.&F.N. Spon Ltd p 36-38
- [8] Li Q F 1998 Fracture Mechanics and Its Application in Engineering (Harbin: Harbin engineering university press) p 61-65

- [9] Zhang X E 2000 Sichuan Arch. **20** 4 p 51
- [10] Lu Z Z, Wang Y B and Chen L Z 2007 Eng. Mech. **24** 11 p 41-46
- [11] Liu N 2013 Fracture Analysis of Offshore Platform Structures Doctoral Dissertation of Ocean University of China chapter **6** pp 76–102