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Analysis of the Strain Energy Release Rate for a Delamination Crack in a Multilayered Beam with Material Non-Linearity

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Abstract. The strain energy release rate for a delamination crack in a multilayered cantilever beam which exhibits material non-linearity is derived by considering the energy balance. The material non-linearity is described by the Ramberg-Osgood stress-strain relation. A cantilever beam made of adhesively bonded lengthwise vertical layers is studied. Each layer has individual widths and material properties. Besides, the delamination is located arbitrary between layers. The strain energy release rate is derived also by differentiating the complementary strain energy with respect to the crack area for verification. It is shown that the solution derived is useful for parametric investigations of delamination in multilayered beams.

1. Introduction

Multilayered materials and structures made of adhesively bonded layers of dissimilar materials have certain advantages such as high strength to weight and stiffness to weight ratios over the homogeneous materials. Therefore, multilayered materials are extensively used in many structural applications where a high performance is required. However, due to the low interlaminar strength, delamination fracture is the predominant failure mode [1-5].

The main goal of the present paper is to develop a strain energy release rate analysis for a delamination crack in a multilayered beam with considering the material non-linearity. A multilayered cantilever beam made by adhesively bonded vertical layers is analyzed. The Ramberg-Osgood stress-strain relation is applied for describing the non-linear mechanical behaviour of the material in contrast to previous works of the author in which delamination in multilayered beams of vertical layers is investigated by using power law stress-strain relations [6, 7].

2. Determination of the strain energy release rate

The delamination crack in the multilayered cantilever beam configuration shown in figure 1 is studied in the present paper. The beam is made of adhesively bonded lengthwise vertical layers of different widths and material properties. The number of layers is arbitrary. It is assumed that in each layer the material exhibits non-linear mechanical behaviour which is treated by the Ramberg-Osgood stress-strain relation. A delamination crack of length, a, is located arbitrary between layers (the widths of the right-hand and left-hand crack arms are b_1 and b_2, respectively. The external loading consists of one moment, M, applied at the free end of right-hand crack arm. Thus, the left hand crack arm is free of stresses. The cross-section of the beam is a rectangle of width, b, and height, h. The beam length is l.

The fracture is investigated in terms of the strain energy release rate, G, by considering the energy balance. By assuming a small increase, δa, of the crack length, the energy balance is written as

$$M\delta\varphi = \frac{\partial U}{\partial a}\delta a + Gh\delta a \qquad (1)$$

where $\delta\varphi$ is the increase of the angle of rotation of the free end of the right-hand crack arm, U is the strain energy. From (1), G is expressed as

$$G = \frac{M}{h}\frac{\partial\varphi}{\partial a} - \frac{1}{h}\frac{\partial U}{\partial a} \qquad (2)$$
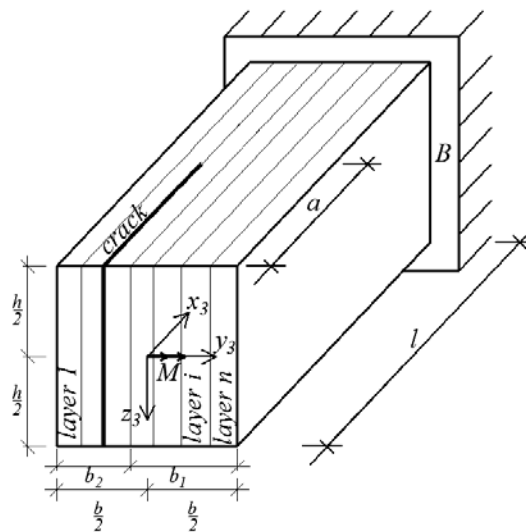

Figure 1. Geometry and loading of the multilayered cantilever beam.

The angle of rotation of the free end of the right-hand crack arm is written as

$$\varphi = a\kappa_1 + (l - a)\kappa_2 \qquad (3)$$

where κ_1 and κ_2 are, respectively, the curvatures of the right-hand crack arm and the un-cracked beam portion, $a \le x_3 \le l$, (figure 1).

The strain energy in the beam is written as

$$U = a\sum_{i=1}^{i=n_R}\iint_{A_i} u_{0_i}\, dA + (l - a)\sum_{i=1}^{i=n}\iint_{A_i} u_{0R_i}\, dA \qquad (4)$$

where n_R and n are, respectively, the numbers of layers in the right-hand crack arm and the un-cracked beam portion, u_{0_i} and u_{0R_i} are the strain energy densities in the i-th layer, respectively, of the right-hand crack arm and the un-cracked beam portion, A_i is the area of the cross-section of the i-th layer.
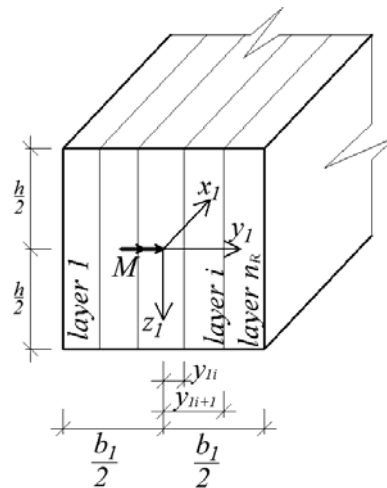
Figure 2. Cross-section of the right-hand crack arm.

The Ramberg-Osgood stress-strain relation for the material in the i-th layer is written as

$$\varepsilon = \frac{\sigma_i}{E_i} + \left(\frac{\sigma_i}{H_i}\right)^{\frac{1}{n_i}} \tag{5}$$

where ε is the lengthwise strain, σ_i is the distribution of the normal stresses in the i-th layer, E_i is the modulus of elasticity in the i-th layer, H_i and n_i are material properties in the same layer.

The distribution of lengthwise strains in the beam cross-section is analyzed by applying the Bernoulli's hypothesis for plane sections since the length to height ratio of the beam under consideration is large. Besides, since the beam is loaded in pure bending, the lengthwise strains are the only non-zero strains. Thus, according to the small strain compatibility equations, the lengthwise strains in the cross-section of the right-hand crack arm are distributed linearly

$$\varepsilon = \kappa_1 z_1 \tag{6}$$

where z_1-axis is shown in figure 2.

The curvature of the right-hand crack arm is determined from the following equation of equilibrium of the cross-section:

$$M = \sum_{i=1}^{i=n_R} \iint_{A_i} \sigma_i z_1 dA \tag{7}$$

Equation (7) can be used also to determine κ_2. For this purpose, n_R, σ_i and z_1 have to replaced, respectively, with n, σ_{R_i} and z_2 where σ_{R_i} is the distribution of the normal stresses in the i-th layer of the un-cracked beam portion, z_2 is the vertical centroidal axis of the cross-section of the un-cracked beam portion.

The strain energy density in the i-th layer of the right-hand crack arm can be obtained by the following formula [8]:

$$u_{0_i} = \frac{\sigma_i^2}{2E_i} + \frac{\sigma_i^{\frac{1+n_i}{n_i}}}{(1+n_i)H_i^{\frac{1}{n_i}}} \tag{8}$$

Formula (8) can be applied also to calculate the strain energy density in the i-th layer of the un-cracked beam portion. For this purpose, σ_i has to be replaced with σ_{R_i}.

By substituting of (3) and (4) in (2), one arrives at

$$G = \frac{M}{h}\left(\kappa_1 - \kappa_2\right) - \frac{1}{h}\sum_{i=1}^{i=n_R}\iint_{A_i} u_{0_i} dA + \frac{1}{h}\sum_{i=1}^{i=n}\iint_{A_i} u_{0R_i} dA \qquad (9)$$

The integration in (9) should be carried-out by using the MatLab program.

In order to verify (9), the strain energy release rate is determined also by using the following formula [6]:

$$G = \frac{dU^*}{hda} \qquad (10)$$

where dU^* is the change of the complementary strain energy, da is an elementary increase of the delamination crack length. The complementary strain energy is obtained by (4). For this purpose, u_{0_i} and u_{0R_i} are replaced, respectively, with $u_{0_i}^*$ and $u_{0R_i}^*$ where $u_{0_i}^*$ and $u_{0R_i}^*$ are the complementary strain energy densities in the i-th layer of the right-hand crack arm and the un-cracked beam portion, respectively. The strain energy release rate obtained by (10) is exact match of the result obtained by (9) which is a verification of the delamination fracture analysis developed in the present paper.

A parametric analysis of delamination fracture is performed by applying the solution to strain energy release rate (9).
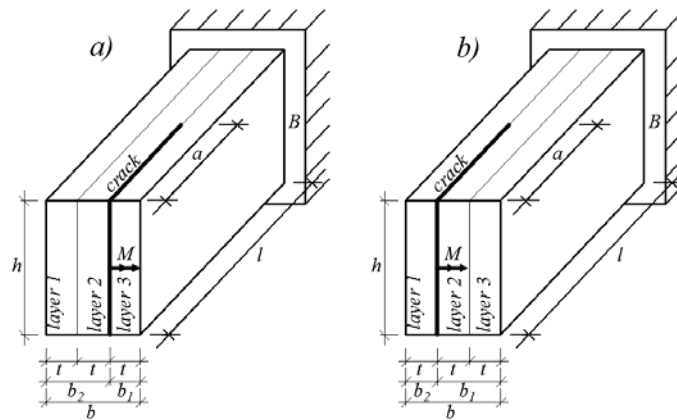

Figure 3. Two three-layered cantilever beam configurations.

The strain energy release rate is presented in non-dimensional form by using the following formula $G_N = G/(E_1 b)$. Two three-layered cantilever beam configurations are considered in order to investigate the influence if the crack location on the strain energy release rate (figure 3).
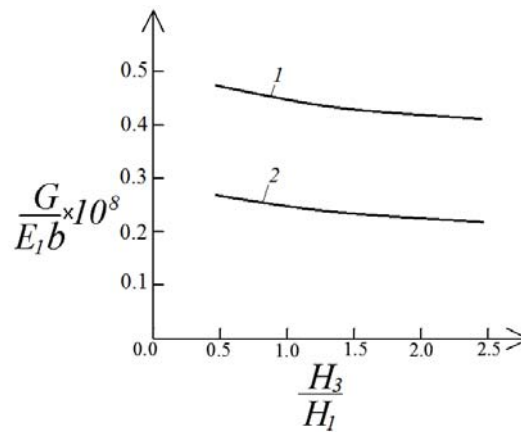
Figure 4. The strain energy release rate plotted against H_3 / H_1 ratio
(curve 1 – for the beam configuration shown in figure 3a,
curve 2 – for the beam configuration shown in figure 3b).

In the configuration shown in figure 3a, a delamination crack is located between layers 2 and 3. A three-layered cantilever beam configuration with a delamination crack between layers 1 and 2 is also analyzed (figure 3b). It is assumed that the width of each layer is $t = 0.005$ m. Besides, it is assumed that $h = 0.020$ m and $M = 40$ Nm. The strain energy release rate in non-dimensional form is presented as a function of H_3 / H_1 in figure 4 at $E_2 / E_1 = 0.6$, $E_3 / E_1 = 0.5$, $H_1 / E_1 = 0.7$, $H_2 / H_1 = 1.2$ and $n_1 = n_2 = n_3 = 0.75$ for the two three-layered beams shown in figure 3. The curves in figure 4 indicate that the strain energy release rate decreases with increasing of H_3 / H_1 ratio. One can observe also in figure 4 that the strain energy release rate is lower when the delamination is between layers 1 and 2. This is due to the fact that the stiffness of the right-hand crack arm is higher when the delamination is between layers 1 and 2 (figure 3a). The strain energy release rate in non-dimensional form is presented as a function of E_3 / E_1 ratio in figure 5 for the beam configuration shown in figure 3a.
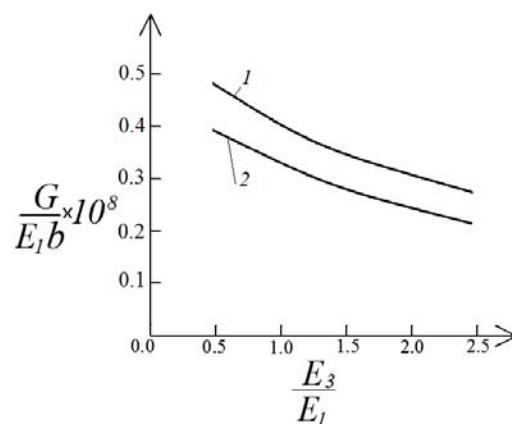

Figure 5. The strain energy release rate plotted against E_3 / E_1 ratio
(curve 1 – at non-linear material behaviour, curve 2 – at linear-elastic behaviour).

It can be observed that the strain energy release rate decreases with increasing of E_3 / E_1 ratio (figure 5). This finding is attributed to the increase of the beam stiffness with increase of E_3 / E_1 ratio.

The strain energy release rate obtained assuming linear-elastic behaviour of the material in each layer is also plotted in figure 5 for comparison with the non-linear solution. It should be noted that the linear-elastic solution to the strain energy release rate is derived by substituting of $H_i \rightarrow \infty$ in (9) because at $H_i \rightarrow \infty$ the non-linear stress-strain relation (5) transforms into the Hooke's law. One can observe that the material non-linearity leads to increase of the strain energy release rate (figure 5). This finding indicates that neglecting the material non-linearity causes underestimation of the strain energy release rate. Therefore, the material non-linearity has to be taken into account in delamination fracture analyses of multilayered structural members and components.

3. Conclusions

Delamination fracture behaviour of a multilayered cantilever beam configuration is analyzed in terms of the strain energy release rate. Non-linear mechanical behaviour of the material is assumed in each layer. The beam is made of adhesively bonded lengthwise vertical layers. The beam is loaded in bending by one external moment applied at the free end of the right-hand crack arm. The Ramberg-Osgood stress-strain relation is applied to describe the non-linear mechanical behaviour of the material. A solution to the strain energy release rate is derived by considering the energy balance. The solution holds for a cantilever beam made of an arbitrary number of lengthwise vertical layers which have individual widths and material properties. Besides, the delamination crack is located arbitrary between layers (thus, the two crack arms have different widths). The strain energy release rate is obtained also by considering the complementary strain energy for verification. The solution derived is very useful for parametric analyses of delamination fracture in multilayered beams exhibiting non-linear mechanical behaviour of the material. It is found that the strain energy release rate decreases with increasing of H_3/H_1 and E_3/E_1 ratios. The investigation reveals also that the strain energy release rate decreases with increasing of the width of the right-hand crack arm cross-section. Concerning the influence of the material non-linearity on the delamination behaviour, the analysis shows that the material non-linearity leads to increase of the strain energy release rate. The results obtained in the present paper can by used in fracture mechanics based structural design of multilayered beams.

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