Multi-Scale Full Waveform Inversion based on Curvelet Transform

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Abstract. Full waveform inversion (FWI) is a rapidly developed inversion method in recent years, and the resolution is much higher than traditional inversion methods, such as tomography and migration velocity analysis. However, FWI is much dependent on starting model and low frequency information, and easy to fall into local minima because of cycle skipping issue. We proposed Curvelet transform multi-scale strategy to improve the effect of bad starting model by selecting different scale information of observed data during inversion. Test of frequency domain FWI on Marmousi model shows that the result of inversion has a better stability and convergence with Curvelet transform multi-scale method than traditional methods.

1. Introduction

FWI is an effective tool to obtain accurate velocity models with complex geological structures. Compared with conventional velocity model building method (eq. tomography, migration velocity analysis), the FWI could get a higher resolution result by taking full advantage of kinematics and kinetics information of the prestack seismic records [1, 2].

Successful application of FWI requires a good starting model to ensure that the modeled waveforms are kinematically less than half a cycle away from the recorded data. If one fails to provide a proper starting model, the gradient updates may fall down to the wrong direction and the cycle skipping could happen [3].

In this paper we proposed a new multi-scale FWI method combining with multi-scale characteristic of Curvelet transform. FWI goes from low frequency to high frequency to update the velocity. And at the different stages of FWI, select different scales of observed seismic data involved in the inversion according to the use of frequency. Thus simulated data can be better matched with observed data, and reduce the dependence on initial model of FWI.

2. Theory

2.1. Full wave form Inversion Theory

FWI algorithm is an iterative process, and every iteration should carry out at least two times forward modeling (one for the incident wavefield and another for the back-propagated residual wavefield). And the cost of the inversion is highly depended on the forward modeling. In this paper, we choose the finite difference method for solving two dimensional frequency-domain acoustic wave equation:

$$\frac{\omega^2}{\kappa(x,z)} p(x,z,\omega) + \frac{\partial}{\partial x}\left(\frac{1}{\rho(x,z)}\frac{\partial p(x,z,\omega)}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\rho(x,z)}\frac{\partial p(x,z,\omega)}{\partial z}\right) = -s(x,z,\omega), \tag{1}$$

where $p(x,z,\omega)$, $\kappa(x,z)$, and $s(x,z,\omega)$ denote the wavefield, bulk modulus, and source term, respectively, ω is angle frequency and $\rho(x,z)$ denotes the density.

In the inversion problem, the objective function $\mathrm{C}(m)$ is expressed as the difference between the real seismic data d^{obs} and the simulated seismic data d^{cal} (Brossier, et al., 2010):

$$\mathrm{C}(m)=\frac{1}{2}[d^{obs}-d^{cal}]^T[d^{obs}-d^{cal}]=\frac{1}{2}\left\|d^{obs}-d^{cal}\right\|_2^2, \quad (2)$$

where 'T' denotes conjugate transpose; $\|\|_2^2$ denotes $l2$ norm, and m is geological model, such as subsurface velocity model.

The process of local optimization is to find the minimum value of the misfit function around the starting model m_0. In the framework of the Born approximation, we assume that the updated model m can be expressed as the sum of starting model m_0 and the model update Δm:

$$m = m_0 + \Delta m \quad (3)$$

Conducting Taylor-Lagrange expansion for Equation (3) at m_0, and calculating derivative with respect to m at both ends, we obtain a new equation expressed in matrix form:

$$\frac{\partial \mathrm{C}(m)}{\partial m}=\frac{\partial \mathrm{C}(m_0)}{\partial m}+\frac{\partial^2 \mathrm{C}(m_0)}{\partial m^2}\Delta m. \quad (4)$$

Equation (3) has the minimum value when $\frac{\partial \mathrm{C}(m)}{\partial m}=0$. Then the model update equals:

$$\Delta m=-[\frac{\partial^2 \mathrm{C}(m_0)}{\partial m^2}]^{-1}\frac{\partial \mathrm{C}(m_0)}{\partial m}, \quad (5)$$

where $\frac{\partial^2 \mathrm{C}(m_0)}{\partial m^2}$ is Hessian matrix, and $\frac{\partial \mathrm{C}(m_0)}{\partial m}$ is derivative of misfit function.

2.2. Multi-Scale Full Waveform Inversion

Recently Curvelet transform has been growing concerns because of its unique character. Curvelet transform was proposed by Emmanuel and David in 1999, developed from wavelet transform and overcome the inherent defects in the expression of image edge directional characteristics of wavelet transform. According to Curvelet transform theory, in signals containing noise, the larger Curvelet coefficients corresponds to valid signals, and whereas the noise. Therefore Curvelet transform can not only denoise the random noise, but also maintain valid signals. We can obtain Curvelet coefficients by the inner product of signal and the wavelet function:

$$C(j,l,k)=<f,\varphi_{j,l,k}>=\int_{R^2} f(x)\varphi_{j,l,k}(x)dx \quad (6)$$

Which j is scale, l is direction, k is displacement, f is signal, and $\varphi_{j,l,k}$ is wavelet function.

The Curvelet coefficients of an image can be divided into three different scale layers. The first layer is a matrix composed by low frequency coefficients, called coarse layer. The second layer is called detail layer, and is composed by middle frequency coefficients. And the third layer, fine layer, is composed by high frequency coefficients. With the increase of coefficient layer, the image is portrayed more and more sophisticated.

Multi-scale strategy is commonly used to improve the stability of FWI. In order to reduce the dependence on the initial model of FWI, this paper proposed a new multi-scale FWI method combining with multi-scale characteristic of Curvelet transform. FWI goes from low frequency to high frequency to update the velocity. And at the different stages of FWI, select different scales of observed seismic data involved in the inversion according to the use of frequency. That is, first extract large scale data of observed seismic data involved in inversion at the beginning stage of FWI, and gradually add smaller scale data with the increase of frequency and accuracy, as is shown in figure 2. Thus simulated data can be better matched with observed data, and reduce the dependence on initial model of FWI.

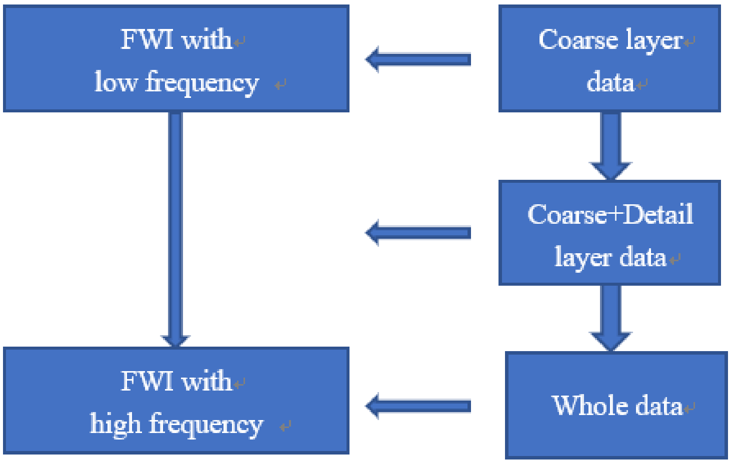
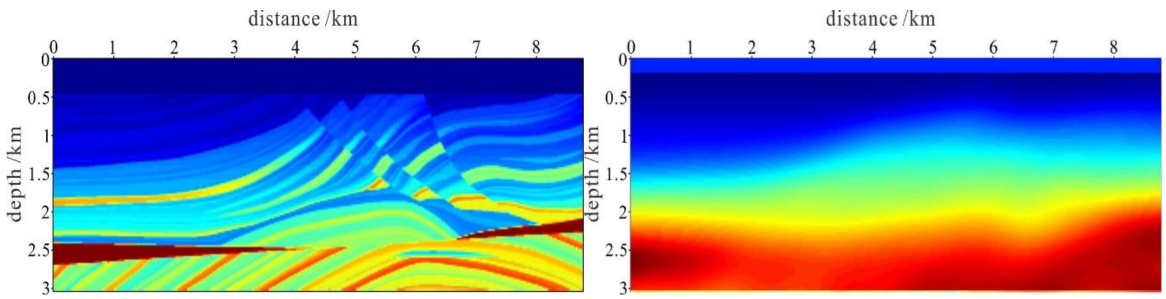


Figure 1. Schematic diagram of Curvelet transform multi-scale FWI.

3. Numerical Test

We test the multi-scale FWI method on a modified 2D Marmousi P-wave velocity model. We add a water layer above and don't update the water layer during the nonlinear procedure. Figure 3(a) and 3(b) are the true model and the starting model, respectively. The starting velocity model is a smoothed version of the true model. We use a Ricker wavelet with peak frequency of 12Hz. The source is located on the surface with the interval 24m. The grid size is 24m and sampling rate is 4ms. All the grid points on the surface acts as receivers. To illustrate the effectiveness of the new method, we reduce the accuracy of the starting model, as shown in figure 3(b). First extract coarse layer data of observed seismic data involved in inversion at the beginning stage of FWI, and gradually add detail layer data and fine layer data with the increase of frequency and accuracy. Figure 3(c) is the result of conventional FWI method, and figure 3(d), 3(e), 3(f) is the results of multi-scale FWI method. As we start the inversion with a relatively bad starting model, conventional FWI method fails to obtain an accurate result due to cycle skipping, while our new method obtains a more accurate result. By selecting different scale of seismic data involved in the inversion, simulated data can be better matched with observed data, thus the new method can reduce the dependence on starting model of FWI, and get a more accurate result. Figure 4 is the Kirchhoff migration results using the velocity results obtained by two FWI methods, respectively. From two migrated figures, we can see that more detail information of subsurface structure is recovered in multi-scale FWI result than that in conventional FWI result.



(a) (b)

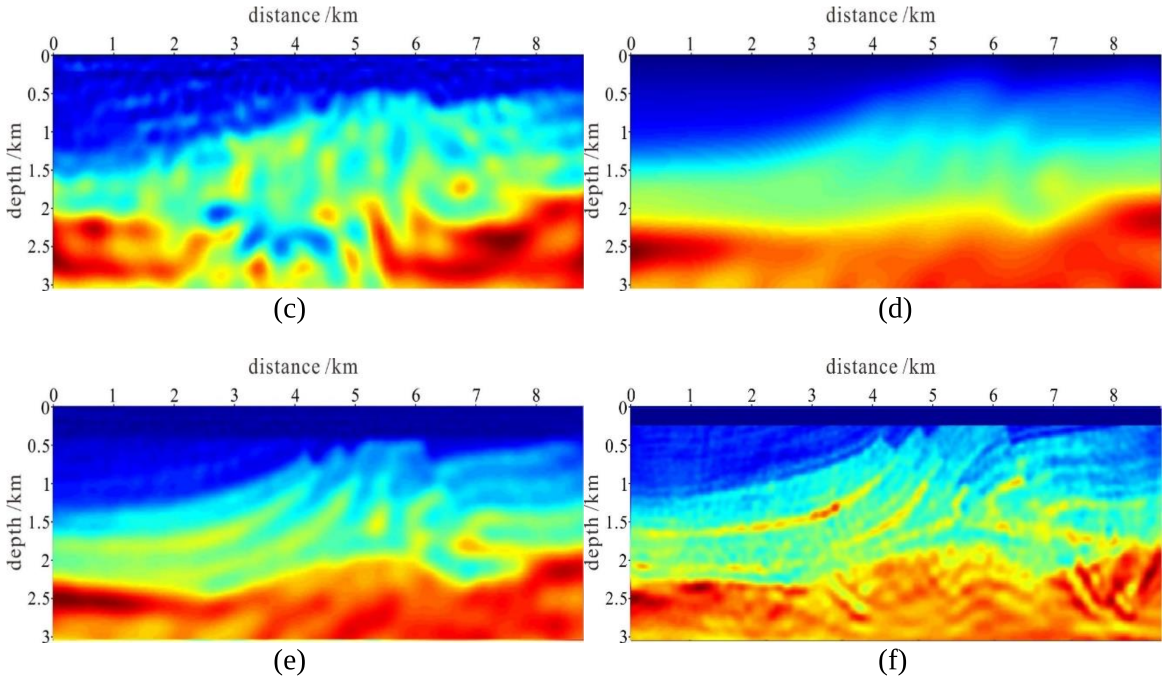


Figure 2. (a) True model, (b) starting model, FWI results of (c) traditional FWI method, (d) coarse scale data, (e) coarse + detail scale data, (f) whole data.

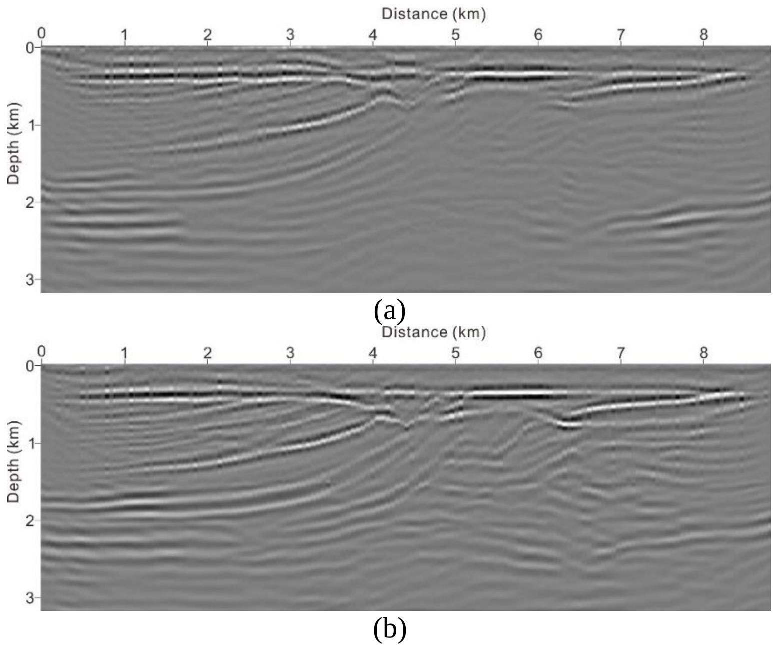


Figure 3. Migration results using the velocity models of (a) Conventional FWI; (b) multi-scale FWI method.

4. Conclusion

Cycle skipping is one of the main problems hindering FWI from practical application. To reduce the dependence on the starting model, we proposed a new multi-scale FWI method combining with multi-scale characteristic of Curvelet transform. FWI goes from low frequency to high frequency to update the velocity. And at the different stages of FWI, select different scales of observed seismic data involved in the inversion according to the use of frequency. Thus simulated data can be better matched with observed data, and reduce the dependence on starting model of FWI. Numerical examples on the

Marmousi model show that the new multi-scale method can mitigate the effect of cycle skipping problem and obtain a better result with a bad starting model than the conventional FWI method.

Acknowledgments
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