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The Investigation on Adhesive Contact Problem of Gradient Piezoelectric Coating under Insulating Punch

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Abstract. This paper considers the two-dimensional adhesive contact problem for a rigid insulating cylindrical punch acting on a gradient piezoelectric coating bonded to half-space.It is assumed that the material parameters of functionally gradient piezoelectric materials vary exponentially along the thickness direction.The adhesion between the punch and the contact surface of the coating is described according to the adhesion Model given by Maugis. By using the Fourier integral transform, the adhesive contact problem of gradient piezoelectric coatings is reduced to singular integral equations. the numerical method is applied to solve the singular integral equations with Cauthy kernel.The numerical results show that both non-homogeneous parameters of gradient piezoelectric material and adhesion effect have significiant impact on the adhesive contact behavior of gradient piezoelectric coating.

1. Introduction

With the development of science and technology, functionally graded materials (FGM) [1], and functionally graded piezoelectric materials (FGPM) [2] have been received extensive attention from researchers. By using Fourier integral transform, Guler et al [3]. proved that appropriate gradient of shear modulus can obviously change the surface contact force distribution. By using transfer matrix method and Fourier integral transform technology, Ke et al [4]. analyzed the sliding frictional contact problem of a layered half-plane made of functionally graded piezoelectric materials (FGPMs) in the plane strain state. Liu et al [5]. analyzed the effects of the conductivity of the indenter on the distribution of the contact stress and the electric charge. When the microscopic problems are considered, the effect of adhesion has a great impact. Johnson et al [6] proposed the JKR model and measured a larger contact area than Hertz. Chen et al [7] [8] established an adhesion contact model for gradient materials with material parameters vary as power function and obtained a series of closed analytical solutions. Jin et al [9] solved the axisymmetric adhesive contact problem between rigid spheres and functionally graded materials half-space. To the best of author’s knowledge, there was no literatures on adhesive contact problem for functionally graded piezoelectric materials.

In this paper, the two-dimensional adhesive contact problem of a functionally gradient piezoelectric coating indented by a rigid insulating cylindrical punch is considered.

2. Formulation of the Problem and Solution

As shown in figure 1, the cylindrical insulating punch is pressed into the half-space coated by FGPM under the action of the line load. Hertz contact area $(-a,a)$, and the adhesive area $(-c,c)$ are formed on the surface of piezoelectric coating. It is assumed that the material parameters of the functional gradient piezoelectric coating change exponentially along the thickness direction as follows:

$$\left\{c_{kl}\left(z\right),e_{kl}\left(z\right),\varepsilon_{kk}\left(z\right)\right\}=\left\{c_{kl0},e_{kl0},\varepsilon_{kk0}\right\}e^{\beta z} \tag{1}$$

where, $c_{kl}(z)$, $e_{kl}(z)$, $\varepsilon_{kl}(z)$ is respectively the elastic constant, piezoelectric constant, dielectric constant, c_{kl0}, e_{kl0}, ε_{kl0} is the material parameter of homogeneous half space, β is the material gradient parameter of FGPM coating.

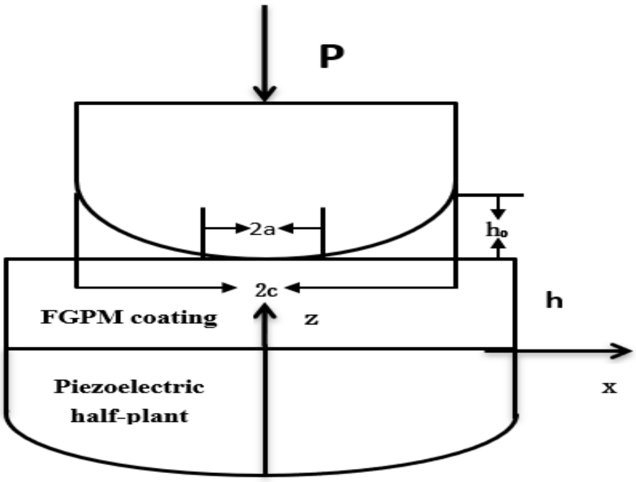

Figure 1. The rigid insulating cylindrical indenter acting on the functional gradient piezoelectric coated half-space

According to the linear piezoelectric constitutive relationships, static equilibrium equation and Maxwell's equation, we have

$$\begin{aligned}&c110\frac{\partial u_{x1}^2}{\partial x^2}+c440\frac{\partial u_{x1}^2}{\partial z^2}+\left(c130+c440\right)\frac{\partial u_{z1}^2}{\partial z\partial x}+\left(e130+e150\right)\frac{\partial \phi_1^2}{\partial z\partial x}\\&+\beta\left[c440\left(\frac{\partial u_{x1}}{\partial x}+\frac{\partial u_{z1}}{\partial z}\right)+e150\frac{\partial \phi}{\partial x}\right]=0\end{aligned} \tag{2a}$$

$$\begin{aligned}&c440\frac{\partial u_{z1}^2}{\partial x^2}+c330\frac{\partial u_{z1}^2}{\partial z^2}+\left(c130+c440\right)\frac{\partial u_{x1}^2}{\partial z\partial x}+e150\frac{\partial \phi_1^2}{\partial x^2}+e330\frac{\partial \phi_1^2}{\partial z^2}\\&+\beta\left[c130\frac{\partial u_{x1}}{\partial x}+c330\frac{\partial u_{z1}}{\partial z}+e330\frac{\partial \phi_1}{\partial z}\right]=0\end{aligned} \tag{2b}$$

$$\begin{aligned}&e150\frac{\partial u_{z1}^2}{\partial x^2}+e330\frac{\partial u_{z1}^2}{\partial z^2}+\left(e150+e310\right)\frac{\partial u_{x1}^2}{\partial z\partial x}-\varepsilon110\frac{\partial \phi_1^2}{\partial x^2}-\varepsilon330\frac{\partial \phi_1^2}{\partial z^2}\\&+\beta\left[e310\frac{\partial u_{x1}}{\partial x}+e330\frac{\partial u_{z1}}{\partial z}-\varepsilon330\frac{\partial \phi_1}{\partial z}\right]=0\end{aligned} \tag{2c}$$

According to the Maugis' theory, the normal stress includes two parts: Hertz contact stress σ_1 and adhesive stress σ_0:

$$p(t)=\sigma_1(t)-\sigma_0,\left|t\right|\le c \tag{3}$$

where the values of normal stress are positive. It is noted that the positive value represents the direction of stress along the coordinate system direction and negative value represents the direction of stress opposites to the coordinate system direction.

Because the contact area is far less than the punch radius, the shape of the punch can be expressed as:

$$u_{z1}(x)=\delta_0-\frac{x^2}{2R},\qquad \frac{\partial u_{z1}(x)}{\partial x}=-\frac{x}{R} \tag{4}$$

where, δ_0 is the maximum indentation depth.

Considering the asymptotic behavior of $M_{21}(s,h)$, one may prove

$$\lim_{s\to\pm\infty} sM_{21}(s,h)=\pm M_{21}^{\infty}$$

Then, extracting the normal displacement component, can be expressed as

$$u_{z1}(x,h)=\frac{-M_{21}^{\infty}}{\pi}\int_{-a}^{a}\ln|x-t|\,p(t)dt+\frac{1}{\pi}\int_{-a}^{a}Q(x,t)p(t)dt \tag{5}$$

where $Q(x,t)=\int_0^{\infty}[M_{21}(s,h)-\frac{M_{21}^{\infty}}{s}]\times\cos[s(x-t)]ds$

The derivative of equation (4) with respect to x, we can obtain

$$\frac{\partial u_{z1}(x,h)}{\partial x}=\frac{M_{21}^{\infty}}{\pi}\int_{-a}^{a}\frac{\sigma_1}{t-x}dt-\frac{M_{21}^{\infty}}{\pi}\int_{-c}^{c}\frac{\sigma_0}{t-x}dt+\frac{1}{\pi}\int_{-a}^{a}Q_1(x,t)\sigma_1 dt-\frac{1}{\pi}\int_{-c}^{c}Q_1(x,t)\sigma_0 dt \tag{6}$$

where $Q_1(x,t)=-\int_0^{\infty}s[M_{21}(s,h)-\frac{M_{21}^{\infty}}{s}]\sin[s(x-t)]ds$

According to the adhesive contact theory given by Maugis [10], the displacement in the adhesive contact area is as follows:

$$\begin{aligned}u_{z1}(\mathrm{a},0)-u_{z1}(c,0)&=\frac{-M_{21}^{\infty}}{\pi}\int_{-a}^{a}\ln\left|\frac{a-t}{c-t}\right|\sigma_1 dt+\frac{M_{21}^{\infty}}{\pi}\int_{-c}^{c}\ln\left|\frac{a-t}{c-t}\right|\sigma_0 dt\\&+\frac{1}{\pi}\int_{-a}^{a}(Q(a,t)-Q(c,t))\sigma_1 dt-\frac{1}{\pi}\int_{-c}^{c}(Q(a,t)-Q(c,t))\sigma_0 dt\end{aligned} \tag{7}$$

According to the equilibrium equation, the contact pressure satisfies:

$$\int_{-a}^{a}\sigma_1 dt-\int_{-c}^{c}\sigma_0 dt=P \tag{8}$$

By introducing the following variables:

$$t=a\eta,\ x=a\varsigma,\ -a\le(t,x)\le a,\ -1\le(\eta,\varsigma)\le 1,\ c=ma \tag{9}$$

the contact stress can be expressed as:

$$\sigma_1(\eta)=f(\eta)\sqrt{1-\eta^2} \tag{10}$$

Then, equation (5), (6) and (7) are reduced to

$$-\frac{a\varsigma_r}{R}=\sum_{l=1}^{N}\frac{(1-\eta_l^2)f(\eta_l)}{(N+1)}\left[\frac{M_{21}^{\infty}}{\eta_l-\varsigma_r}+aQ_1(\varsigma_r,\eta_l)\right]-\frac{\sigma_0 M_{21}^{\infty}}{\pi}\ln(\frac{m-\varsigma_r}{m+\varsigma_r})-\frac{ma\sigma_0}{\pi}\int_{-1}^{1}Q_1(a\varsigma_r,a\eta_l)d\eta \tag{11}$$

$$\frac{(m^2-1)a^2}{2R}-h_0=\sum_{l=1}^{N}\frac{(1-\eta_l^2)f(\eta_l)}{(N+1)}\left\{a\left[Q(a,a\eta_l)-Q(\mathrm{m}a,a\eta_l)\right]-aM_{21}^{\infty}\ln\left|\frac{1-\eta_l}{m-\eta_l}\right|\right\}$$
$$+\frac{ma\sigma_0M_{21}^{\infty}}{\pi}\int_{-1}^{1}\ln\left|\frac{1-\eta_l}{m-\eta_l}\right|d\eta_l-\frac{ma\sigma_0}{\pi}\int_{-1}^{1}[Q(a,a\eta_l)-Q(ma,a\eta_l)]d\eta_l \qquad (12)$$

$$\sum_{l=1}^{N}\frac{(1-\eta_l^2)f(\eta_l)}{(N+1)}(\pi a)=P+2ma\sigma_0 \qquad (13)$$

where $\eta_l=\cos[l\pi/(N+1)]$, $\varsigma_{\mathrm{r}}=\cos[\pi(2r-1)/2(N+1)]$, $r=[1,N+1]$

After given the Hertz contact area a and the ratio of the adhesive area and contact area m, the stress distribution $f(\eta_l)$ can be obtained from equation (11). If m and $f(\eta_l)$ satisfied equation (12). We can get the contact stress $\sigma_1(\eta)$ and applied force P. Otherwise. Another value of m is chose to solve equation (11) until m and $f(\eta_l)$ satisfied equation (12).

3. Numerical Results and Discussion

In this paper, the adhesive contact problem of functional gradient piezoelectric coating is analyzed by making use of the commercially available pzt-4 piezoelectric material [11]. It is assumed that the thickness of the gradient coating h equals to $0.01m$, the radius of the cylindrical punch R is $0.08m$. Figure 2 show the effects of adhesion parameter σ_0 on the relation between the contact area a and the applied linear loads P (a), the ratio of the adhesive area and contact area m (b), and the distribution of the contact stress $p(x)$ (c). It can be seen from the figure 2a that the contact radius increases with the increase of the adhesion σ_0 for the same applied force P. As the adhesive parameter σ_0 gradually decreases, the applied force P decreases when $P<0$. Figure 2b show that m decreases and gradually becomes flat with the increasing of the contact radius. For the same Hertz contact radius, the m value decreases as the adhesive force σ_0 increases. Figure 2c shows that the maximum contact stress gradually increases with the increase of adhesive force σ_0 when external loads $P=1000\,N/m$.Figure 3 show the effects of gradient index βh on Hertz contact area a (a), the ratio of the adhesive area and contact area m (b) and the distribution of the contact stress $p(x)$ (c) when the adhesion stress $\sigma_0=5\times10^6$.It can be seen from the figure 3a that the Hertz contact radius decreases as the gradient index increases when the applied force P are the same. Figure 3b show that with the increase of contact radius, the m value decreases and gradually becomes flat. when Hertz contact radius is the same, the m value decreases with the gradient index increases. Figure 3c shows that the maximum contact stress gradually increases with the increase of gradient index when applied force $P=1000\,N/m$.

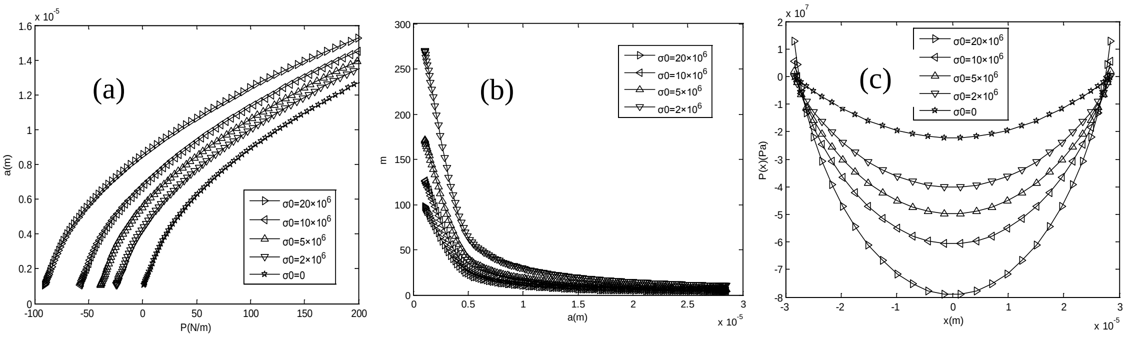

Figure 2. When βh=0.2 , the effect of adhesive stress σ_0 on the contact radius a (a) , length ratio of adhesive area to contact area m (b) and stress $p(x)$ (c)

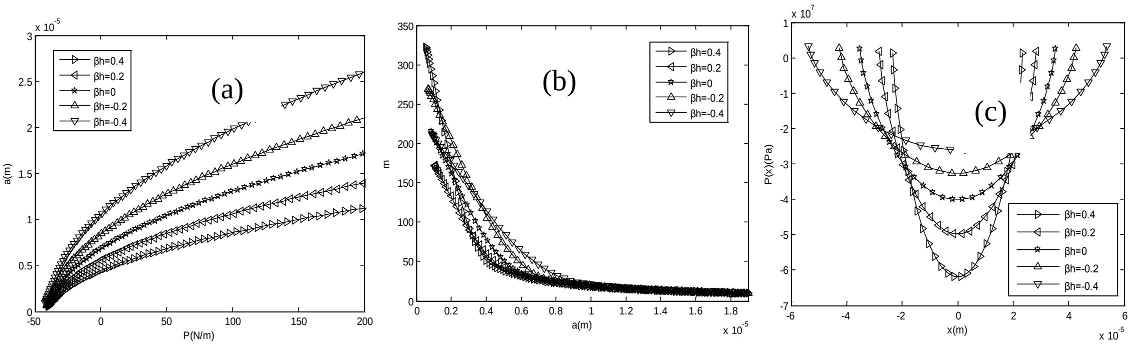

Figure 3. When $\sigma_0 = 5\times10^6$, the influence of gradient index (βh) on the contact radius (a) m value (b) and stress (c)

4. Conclusion

The gradient index of FGPM coating and the adhesion stress has obvious influence on the relation between applied force and the Hertz contact area, the adhesive contact area and the distribution of the contact stress. The adhesive contact behavior can be changed by adjusted the gradient index of FGPM coating.

Acknowledgments

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