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Dynamic Systems: Passivity and Positivity Properties

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Abstract. This paper is concerned with the discussion of passivity results and the characterization of the regenerative versus passive systems counterparts in dynamic systems. In particular, the various concepts of passivity as standard passivity, strict input passivity, strict output passivity and very strict passivity (i.e. joint strict input and output passivity) are given and related to the existence of a storage function and a dissipation function. The obtained results are related to external positivity of systems and positivity or strict positivity of the transfer matrices and transfer functions in the time- invariant case.

1. Introduction

This paper discusses certain aspects of passivity results in dynamic systems as well as the characterization of the regenerative versus passive systems counterparts. In particular, the various concepts of passivity as standard passivity, strict input passivity, strict output passivity and very strict passivity (i.e. joint strict input and output passivity) are given and related to the existence of a storage function and a dissipation function. Basic previous background concepts on passivity and positivity have been given in [1-5], in [6-9], and also in [10-12] and in some related references therein. The obtained results are linked to the properties of external positivity of dynamic systems and the positivity and the strict positivity of the transfer matrices and transfer functions in the time- invariant case. The way of proceeding in the case of passivity failing or how to eventually increase the passivity effects via linear feedback is also discussed to the light of the synthesis of the appropriate feed-forward or feedback controllers or, simply, by adding a positive parallel direct input-output matrix interconnection gain having a minimum positive lower-bounding threshold gain which is also an useful idea for asymptotic hyperstability of parallel disposals of systems, [10]. Finally, the concept of passivity is discussed for switched systems which can have both passive and non-passive configurations which become active governed by switching functions. The passivity property is guaranteed by the switching law under a minimum residence time at passive active configurations provided that the first active configuration of the switched disposal is active and that there are no two consecutive active non-passive configurations in operation.

2. Notation

$\boldsymbol{R}_{0+} = \boldsymbol{R}_+ \cup \{0\}$, where $\boldsymbol{R}_+ = \{r \in \boldsymbol{R} : r > 0\}$, $\overline{p} = \{1, 2, \ldots, p\}$,

$\boldsymbol{Z}_{0+} = \boldsymbol{R}_+ \cup \{0\}$, where $\boldsymbol{Z}_+ = \{r \in \boldsymbol{R} : r > 0\}$,

$D \succ 0$ denotes that the real matrix D is positive definite while $D \succeq 0$ denotes that it is positive semidefinite, $\lambda_{min}(.)$ and $\lambda_{max}(.)$ denote, respectively, the minimum and maximum eigenvalues of the real symmetric $(.)$-matrix, $\hat{G} \in \{PR\}$ denotes that the transfer matrix $\hat{G}(s)$ of a linear time-invariant system is positive real, i.e. $Re\hat{G}(s)\geq 0$ for all $Re\ s>0$ and $\hat{G}\in\{SPR\}$ denotes that it is strictly positive real, i.e. $Re\hat{G}(s)>0$ for all $Re\ s\geq 0$.

A dynamic system is positive (respectively, externally positive) if all the state components (respectively, if all the output components) are non-negative for all time $t\geq 0$ for any given non-negative initial conditions and non-negative input , $\boldsymbol{i}=\sqrt{-1}$ is the complex unity, I_m is the m-th identity matrix, the superscript T stands for matrix transposition.

$\boldsymbol{H}_\infty$ is the Hardy space of all complex-valued functions $F(s)$ of a complex variable s which are analytic and bounded in the open right half-plane $Re\,s>0$ of norm $\| F\|_\infty = sup\{|F(s)|: Re\,s>0\} = sup\{|F(\boldsymbol{i}\omega)|:\omega\in\boldsymbol{R}\}$(by the maximum modulus theorem) and $\boldsymbol{RH}_\infty$ is the sub set of real-rational functions of $\boldsymbol{H}_\infty$.

3. Basic Concepts and Results

Consider a dynamic system $G:\mathsf{H}_e\to\mathsf{H}_e$ with state $x\in\boldsymbol{R}^n$, input $u\in\boldsymbol{R}^m$ and output $y\in\boldsymbol{R}^m$, where H_e is the extended space of the Hilbert space H endowed with the inner product $\langle .,.\rangle$ from $\mathsf{H}_e\times\mathsf{H}_e$ to $\boldsymbol{R}$ consisting of the truncated functions $u_t(\tau)=u(\tau)$ for $\tau\in[0,t]$ and $u_t(\tau)=0$; $\forall t,\tau(>t)\in\boldsymbol{R}_{0+}$ and $u:\boldsymbol{R}_{0+}\to\boldsymbol{R}^m$. If $u\in\mathsf{H}_e$ then $\forall t\geq 0$.

Definitions [2].The above dynamic system is: L_2 *-stable* if $u\in L_2^m$ implies $Gu\in L_2^m$.

Nonexpansive if $\exists\lambda$ and $\exists\gamma>0$ s. t. for all $u\in\mathsf{H}_e$

$\int_0^t (Gu)^T(\tau)u(\tau)d\tau\leq\lambda+\gamma^2\int_0^t u^T(\tau)u(\tau)d\tau$; $\forall t\geq 0$.

Passive if $\exists\varepsilon\geq 0$ such that $\int_0^t y^T(\tau)u(\tau)d\tau\geq-\varepsilon$; $\forall t\geq 0$.

Strictly- input passive if $\exists\varepsilon\geq 0$ and $\exists\varepsilon_u>0$ s. t.

$\int_0^t y^T(\tau)u(\tau)d\tau\geq-\varepsilon+\varepsilon_u\int_0^t y^T(\tau)u(\tau)d\tau$; $\forall t\geq 0$.

Strictly- output passive if $\exists\beta\geq 0$ and $\exists\varepsilon_y>0$ s. t.

$\int_0^t y^T(\tau)u(\tau)d\tau\geq-\varepsilon+\varepsilon_y\int_0^t y^T(\tau)y(\tau)d\tau$; $\forall t\geq 0$.

Strictly input/output passive (or *very strictly passive*) if $\exists\beta\geq 0$, $\exists\varepsilon_u>0$ and $\exists\varepsilon_y>0$ s. t. $\int_0^t y^T(\tau)u(\tau)d\tau\geq-\varepsilon+\varepsilon_u\int_0^t y^T(\tau)u(\tau)d\tau+\varepsilon_y\int_0^t y^T(\tau)y(\tau)d\tau$; $\forall t\geq 0$.

The constants ε, ε_u and ε_y are, respectively, referred to as the passivity, input passivity and output passivity constants.

Proposition 1. Consider a linear time-invariant SISO (i.e. m=1) system whose transfer function $\hat{G}\in\{PR\}$. Then, the following properties hold:

$\int_0^t y(\tau)u(\tau)d\tau\geq 0$ and $y(t)u(t)\geq 0$; $\forall t\geq 0$ and, furthermore, if $u\in L_2$ then $y\in L_2$ so that the system is passive.

Assume, in addition, that $\hat{G}\in\{SPR\}$. Then $\gamma\geq\int_0^t y(\tau)u(\tau)d\tau\geq\varepsilon_u\int_0^t|u(\tau)|^2d\tau-\varepsilon$ for any $t\in(0,\infty]$ and some $\gamma,\varepsilon\in\boldsymbol{R}_{0+}$.

If, furthermore, the system is externally positive in the sense that the output is non-negative for all time if the initial consditions are non-negative and the input is non-negative for all time, then $\gamma\geq\int_0^t y(\tau)u(\tau)d\tau>0$; $\forall t\geq 0$ for any given non-negative initial conditions and non-negative input.

Define $R_{\hat{G}} = \left\| \frac{1-\hat{G}(\boldsymbol{i}\omega)}{1+\hat{G}(\boldsymbol{i}\omega)} \right\|_\infty = \sup_{\omega \in \boldsymbol{R}_{0+}} \left| \frac{1-\hat{G}(\boldsymbol{i}\omega)}{1+\hat{G}(\boldsymbol{i}\omega)} \right|$ as the relative passivity index of the transfer function $\hat{G}(s) = \frac{\hat{N}(s)}{\hat{D}(s)} \in \boldsymbol{RH}_\infty$ ($\hat{N}(s)$ and $\hat{D}(s)$ being the numerator and denominator polynomials of $\hat{G}(s)$). Then, the constraint $a_G \le R_{\hat{G}} = \left\| \frac{\hat{D}_G(\boldsymbol{i}\omega) - \hat{N}_G(\boldsymbol{i}\omega)}{\hat{D}_G(\boldsymbol{i}\omega) + \hat{N}_G(\boldsymbol{i}\omega)} \right\|_\infty \le b_G$ is guaranteed for some $a_G, b_G(\ge a_G) \in \boldsymbol{R}_{0+}$ if

$$\frac{1-a_G^2}{2\left(1+a_G^2\right)}\left(Re^2\ \hat{D}_G(\boldsymbol{i}\omega) + Re^2\ \hat{N}_G(\boldsymbol{i}\omega)\right) \ge \frac{1-b_G^2}{2\left(1+b_G^2\right)}\left(Re^2\ \hat{D}_G(\boldsymbol{i}\omega) + Re^2\ \hat{N}_G(\boldsymbol{i}\omega)\right) \quad ; \quad \forall \omega \in \boldsymbol{R}_{0+}$$

.If $b_G \le 1$ (respectively, $b_G < 1$) then $\hat{G} \in \{PR\}$ (respectively, $\hat{G} \in \{SPR\}$).

Note that positivity is a very important property in some dynamic systems related to biological or epidemic-type models. See, for instance, [5-9] and references therein The generalization of Proposition 1 to the multi input multi-output (MIMO) case (i.e. $m > 1$) is direct by replacing the instantaneous power $y(t)u(t)$ by the scalar product $y^T(t)u(t)$ in the corresponding expressions. In particular, the subsequent two results discuss how the basic passivity property can become a stronger property as, for instance, strict-input passivity or very strict passivity, by incorporating to the input-output operator a suitable parallel static input-output interconnection structure.

4. Hyperstability and Passivity and Non-passivity of Dynamic Systems

It turns out that passive systems are intrinsically stable and either consume or dissipate energy for all time. However, unstable systems are essentially non-passive although some stable systems are also non-passive. Looking at Definition 3, we can give the next one:

Definition 7. A dynamic system is said to be *Non-passive* (or *Active* or, so-called, *Regenerative*) if $\int_0^{t_i} y^T(\tau)u(\tau)d\tau + \varepsilon_{t_i} < 0$ for some unbounded sequences $E = \{\varepsilon_{t_i}\} \subseteq \boldsymbol{R}_{0+}$, $T = \{t_i\} \subseteq \boldsymbol{R}_{0+}$ which satisfy the conditions:

$0 < \delta_{i-1} \le t_{i+1} - t_i \le \delta_i < \infty$; $\forall i \in \boldsymbol{Z}_{0+}$ for some positive bounded sequence $\Delta = \{\delta_i\}$,

$0 < \theta_{i-1} \le \tilde{\varepsilon}_{t_i} = \varepsilon_{t_{i+1}} - \varepsilon_{t_i} \le \theta_i < \infty$; $\forall i \in \boldsymbol{Z}_{0+}$ for some positive bounded sequence $\Theta = \{\delta_i\}$,

$\varepsilon_i, t_i \to +\infty$ as $i \to +\infty$.

The subsequent results follow:

Proposition 2. The following input output energy constraint is fulfilled by non-passive dynamic systems $\lim_{t\to\infty} \int_0^t y^T(\tau)u(\tau)d\tau = -\infty$.

Note that non-passive systems can reach an absolute infinity energy measure in finite time under certain atypical inputs as, for instance, a second-order impulsive Dirac input of appropriate component signs at some time instant $t_1 < \infty$ with $u(t) = 0$ for $t > t_1$. Then, $\int_0^{t_1} y^T(\tau)u(\tau)d\tau = \lim_{t\to\infty} \int_0^t y^T(\tau)u(\tau)d\tau = -\infty$.

Proposition 3. The following properties hold:

A passive system cannot be non-passive in any time sub-interval. A non-passive system in some time interval cannot be a passive system.

A passive system is always stable and also dissipative (i.e. the dissipative energy function takes non-negative values for all time) including the conservative particular case implying identically zero dissipation through time.

A non-passive system can be stable or unstable (so, stable systems are non- necessarily passive).

It is convenient to point out that, under certain rather weak assumptions about the switching operation, a switched system is passive for all time under a switching law $\sigma: \boldsymbol{R}_{0+} \times SW \to \{1, 2, \dots, p, p+1, \dots, q\}$, with at least one configuration being active, if:

a) The first active configuration of the switching law on $[t_0 = 0, t_1) \in SW_P$.

b) The switching law does not involve two consecutive active configurations being non-passive.

c) Each active passive configuration respects a minimum residence time, quantified in the proof, which can exceed a minimum common residence at such a configuration before next switching to another one to be marked as active.

A related property to passivity is that of the hyperstability. Such a property is a strong absolute stability property implying that the controlled system is closed-loop stable under a very general class of feedback controllers which satisfy a so-called Popov ́s inequality. An asymptotically hyperstable system has a non-negative bounded input- output energy for all time which is also (strictly) positive within any nonzero time interval Consider the subsequent dynamic system under eventually time-delayed disturbances, whose nominal feed-forward part is linear and time-invariant while the feedback controller is nonlinear and, eventually, time-varying belonging to a class to be characterized later on through a Popov ́s inequality:

$$\dot{x}(t) = Ax(t) + bu(t) + \eta(t) \; ; \; y(t) = c^T x(t) + du(t) + \eta_0(t) \tag{1}$$

$$u(t) = -\varphi(y(t), t) \tag{2}$$

where $x \in \boldsymbol{R}^n$ is the state vector and $u(t)$ and $y(t)$ are the scalar input and output, where A is real square matrices of order n, b and c are real n- vectors and d is a real scalar, which is subject to initial conditions $x(t) = \phi(t)$ for $t \in [-h(0), 0]$ where $h: \boldsymbol{R}_{0+} \to [h_m, h_M]$ is a, in general, bounded piecewise-continuous time- varying internal delay and $\phi : [-h(0), 0] \to \boldsymbol{R}^n$ is an absolutely continuous n-vector function with eventual finite isolated jump discontinuities, $x(0) = \phi(0)$, $\eta(t) = \eta(\bar{x}_h(t), u(t), t)$ and $\eta_0(t) = \eta(x(t), u(t), t)$ are, in general, unstructured real n-vector disturbance functions which take account of parametrical and unmodeled dynamics uncertainties and noise disturbances being eventually subjected to an internal delay $h(t) \ge h_m$, where $\bar{x}_h(t)$ is the strip $x : [t - h(t), t] \to \boldsymbol{R}^n$; $\forall t \in \boldsymbol{R}_0$.

It is assumed that the disturbance functions are subject to certain regularity conditions such as Lipschitz continuous with respect to all their arguments in order to ensure the existence and uniqueness of the solution for any bounded admissible initial conditions.

The nominal system is defined as the disturbance- free one, that is $\eta(t) = \eta(\bar{x}_h(t), u(t), t) \equiv 0$, $\eta_0(t) = \eta(x(t), u(t), t) \equiv 0$. Note that the nominal system is delay- free.

The function $\varphi : \boldsymbol{R} \times \boldsymbol{R}_{0+} \to \boldsymbol{R}$ defining the, in general, eventually non-linear and time-varying controller is any member of a class $\{\Phi\}$ satisfying a Popov ́s -type integral inequality of the form:

$$\Gamma(t_0, t) = \int_{t_0}^{t} y(\tau)\varphi(y(\tau), \tau) d\tau \ge -\gamma_0 \tag{3}$$

for some finite $\gamma_0 \in \boldsymbol{R}_+$; $\forall t_0 \in \boldsymbol{R}_{0+}, t(\ge t_0) \in \boldsymbol{R}_+$. Such a class $\{\Phi\}$ is said to be a hyperstable class of controllers and any $\varphi \in \{\Phi\}$ is said to be Φ-hyperstable.

If the closed-loop system is globally stable for any controller $\varphi \in \{\Phi\}$ with the forward- time invariant block having a positive real transfer function then it is said to be *Φ-hyperstable* so that the sate and the output are uniformly bounded for all time for any given finite initial conditions. If, in addition, the feed-forward block has a strictly positive real transfer function then the closed-loop system is *Φ-asymptotically hyperstable*, so that the state and output are uniformly bounded and converge asymptotically to zero as time tends to infinity, that is, it is globally asymptotically stable for any finite initial conditions and any controller of class $\{\Phi\}$. Thus:

1) the feed-forward controlled plant is *asymptotically hyperstable* (or *strictly positive*, or *strictly passive*) if its transfer function is strictly positive real,

2) the class Φ of feedback controllers is *hyperstable* if it satisfies the above Popov ́s type inequlity,

3) any closed-loop configuration of the controlled *Φ-asymptotically hyperstable* (respectively, *Φ-hyperstable*) plant with a controller of class $\{\Phi\}$ (i.e. with a Φ-hyperstable controller) is Φ-asymptotically hyperstable (respectively, Φ- hyperstable).

Now, define truncate functions and h -delay truncated functions for any $0 \le T \le \infty$, respectively, as

$$v_T(t)=\begin{cases} v(t) & \text{for } t\in[0\,,T] \\ 0 & \text{for } t>T \end{cases} \;;\; v_{hT}(t)=\begin{cases} v(t) & \text{for } t\in[-h(t)\,,T] \\ 0 & \text{for } t>T \end{cases} \tag{4}$$

It is assumed that $\eta : \boldsymbol{R}^{n+1}\times\boldsymbol{R}_{0+}\to\boldsymbol{R}^n$, $\eta : \boldsymbol{R}^{n+1}\times\boldsymbol{R}_{0+}\to\boldsymbol{R}$ and $\varphi : \boldsymbol{R}\times\boldsymbol{R}_{0+}\to\boldsymbol{R}$ satisfy standard regularity conditions (so as to guarantee the uniqueness of the state and output trajectory solutions for any given finite initial conditions on $[-h(0)\,,0\,]$. Such solutions can be expressed by the concourse of the truncated relevant signals of the system by assuming that $u(-t)=0$, $\eta(-t-h(t))=0$ and $\eta_0(-t-h(t))=0$; $\forall t\in\boldsymbol{R}_+$, as follows:

$$x(t)=0\,;\; t\in(-\infty\,,-h(0))\;;\; x(t)=\phi(t);\; t\in[-h(0)\,,0)$$
$$x(t)=e^{At}\left(x(0)+\int_0^\infty e^{-A\tau}\left(bu_t(\tau)+\eta_{ht}(\tau)\right)d\tau\right);\;\forall t\in\boldsymbol{R}_{0+}$$
$$y(t)=c^Te^{At}\left(x(0)+\int_0^\infty e^{-A\tau}\left(bu_t(\tau)+\eta_{ht}(\tau)\right)d\tau\right)+du(t)+\eta_0(t)$$
$$=y_f(t)+c^Te^{At}\left(x(0)+\int_{-\infty}^\infty e^{-A\tau}\eta_{ht}(\tau)\,d\tau\right)+\eta_0(t);\;\forall t\in\boldsymbol{R}_{0+}$$

where $y_f(t)$ is the forced output. It is well- known that the Fourier transform exist of any absolutely-integrable vector real function on $(-\infty,+\infty)$. Truncated functions with no escape time instants over finite time intervals always fulfil this property so that for any finite $t\in\boldsymbol{R}_{0+}$, the Fourier transforms $\hat{u}_t(\boldsymbol{i}\omega)$, $\hat{x}_t(\boldsymbol{i}\omega)$, $\hat{y}(\boldsymbol{i}\omega)$, $\hat{\eta}_{ht}(\boldsymbol{i}\omega)$ and $\hat{\eta}_0(\boldsymbol{i}\omega)$ exist for all $\omega\in\boldsymbol{R}$. From Parseval ́s theorem, it follows that the following input-output energy measure $E(t)$ on the time interval $[0\,,t]$ fulfils the subsequent associated relationships for any $t\in\boldsymbol{R}_{0+}$:

$$E(t)=\int_0^t y(\tau)u(\tau)\,d\tau=\int_{-\infty}^\infty y_t(\tau)\,u_t(\tau)\,d\tau$$

$$=\frac{1}{2\pi}\int_{-\infty}^\infty\hat{y}_t(\boldsymbol{i}\omega)\hat{u}_t(-\boldsymbol{i}\omega)d\omega=E_0(t)+\tilde{E}(t)\;;\;\forall t\in\boldsymbol{R}_{0+}$$

where the nominal value and its incremental term generated by the combination of the unforced output response and the contributions of the uncertainties to the output $E_0(t)$ and $\tilde{E}(t)$

The main relevant result whose proof is omitted follows.

Proposition 4. Assume that

a) the dynamic system is under feedback control generated by any controller $\varphi\in\{\Phi\}$, where the class $\{\Phi\}$ is defined by the Popov ́s -type inequality ,

b) the nominal transfer function, namely, that when $\eta(t)=\eta(\bar{x}_h(t)\,,u(t),t)\equiv 0$ and $\eta_0(t)=\eta(x(t),u(t)\,,t)\equiv 0$, satisfies $\hat{g}\in\{SSPR\}$ such that $\min_{\omega\in\boldsymbol{R}_{0+}}\hat{g}(\boldsymbol{i}\omega)\ge\lambda d$ for some real constant $\lambda\in(0\,,1]$, where the direct input-output interconnection gain d is positive,

c) $\tilde{E}(t)\le\tilde{\gamma}(t)\le+\infty$; $\forall t\in\boldsymbol{R}_{0+}$ for some real function $\tilde{\gamma}:\boldsymbol{R}_{0+}\to\boldsymbol{R}$ subject to the constraint:

$\tilde{\gamma}(t)\le\tilde{\varepsilon}_\gamma\int_0^t u^2(\tau)d\tau$; $\forall t\in\boldsymbol{R}_{0+}$

Thus, the following properties hold:

(i) $u\in L_2$ (the space of square-integrable functions on $[0\,,\infty])$, $u(t)\to 0$ as $t\to\infty$, $\operatorname{ess\,sup}_{t\in\boldsymbol{R}_{0+}}|u(t)|<+\infty$, $\sup_{t\in\boldsymbol{R}_{0+}}|\tilde{E}(t)|<+\infty$, $0<E(t)<+\infty$ and $0<E_0(t)<+\infty$; $\forall t\in\boldsymbol{R}_+$ for any non-identically zero control on a time interval $[0\,,t_0)$ of nonzero measure if $\tilde{\varepsilon}_\gamma<\lambda d$. Also, $x(t)$ and $y(t)$ are uniformly bounded for all time for any given admissible finite initial conditions.

(ii) If $\|\eta\|_2$ and η_0 are in $L_\infty \cup L_1 \cup L_2$ then the state $x:[0,+\infty]\times\left([-h(0),0]\times \boldsymbol{R}^n\right)\to \boldsymbol{R}^n$ and output $y:[0,+\infty]\times\left([-h(0),0]\times \boldsymbol{R}^n\right)\to \boldsymbol{R}$ are uniformly bounded for each given vector function of initial conditions. As a result, the closed-loop system is Φ - hyperstable for the class $\{\Phi\}$ of output feedback controllers defined by the Popov ́s inequality. Furthermore, if those unstructured functions describing the uncertainties are identically zero then the closed-loop system is Φ -asymptotically hyperstable for any finite initial conditions, i.e. globally asymptotically stable for any member of the class $\{\Phi\}$ of output feedback controllers being defined by the Popov ́s inequality.

Proposition 5. A switched system is passive for all time under a switching law with at least one of the inviolved configurations is active, if for $\{t_i\}$; $i\in \boldsymbol{Z}_{0+}$ being the sequence of switching time instants:

a) The first active configuration of the switching law on $[t_0=0,t_1)$ is passive

b) The switching law does not involve two consecutive active configurations being non-passive.

c) Each active passive configuration respects a minimum residence time, quantified in the proof, which can exceed a minimum common residence time which depends on the parameterizations and can be explicitly calculated.

5. Conclusions

Some relationships between positivity, passivity and hyperstability in dynamic systems, as well as and the characterization of the regenerative versus passive systems counterparts, have been described and mutually inter-linked in a simple way.

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