IOP Conference Series: Materials Science and Engineering



# Shape Transformations of Plane Folded Sheets for Shell Roofing

To cite this article: Aleksandra Prokopska and Jacek Abramczyk 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **471** 082064

IOP Conf. Series: Materials Science and Engineering 471 (2019) 082064 doi:10.1088/1757-899X/471/8/082064

# Shape Transformations of Plane Folded Sheets for Shell Roofing

**Aleksandra Prokopska[1], Jacek Abramczyk[1]**

[1] Al. Powstańców Warszawy 12, bud. V, pok. D37, 35-959, Reszow University of Technology podkarpackie, Poland

jacabram@prz.edu.pl

**Abstract**. The major purpose of the paper is to point out that big shape transformations of nominally flat, thin-walled folded sheets having open profiles are possible. Principal boundary conditions can be defined for the transformed sheets, by means of the number, position and curvature of roof directrices, including intermediate members. The present research results indicate the necessity to perform further research in a much wider scope, i.e. experimental research, theoretical analyses, numerical models configured appropriately for diversified shapes of various profiles. The research should also aim at optimizing shape transformations, strength, stability, support, and stiffness of the shell as well as co-operation of shell folds in a roof. These results are systematically included in the innovative method of shaping of free forms and constructions of roofs and entire buildings. However, the method is not involved in the submitted paper.

## 1. Introduction

It is relatively easy to change the general shapes of nominally plane thin-walled steel sheets folded in one direction and connected along their longitudinal edges into a continuous plane corrugated strip into corrugated roof shells, figure 1. However, load has to act perpendicularly to the planes of the fold's flanges [1, 2]. The transformed strip can be obtained as a result of spreading on roof directrices during assembly so that a freedom of the transversal width and height increments of its folds should be ensured. The freedom depends on the relevant direction and position of the applied load and the supporting conditions of each fold in a shell. As a result of the free shape transformations all shell folds adapt their shapes in relation to the supporting conditions in a relatively wide range. It is worth stressing that the supporting conditions change for subsequent folds in the shell because the roof directrices are skew lines and have to be calculated at the initial step of the design process [3].

Achievement of various innovative attractive roof shell forms is conditioned by preserving relatively tough restrictions of shape transformations resulting from orthotropic geometrical and mechanical properties of the transformed sheets because initial stresses and strains as well as bending and twist deformations of sheets' flanges and webs are an essentially negative effect of these transformations indeed [4, 5], figure 2.

High values of the above features can be observed if they are not taken into account in detail during the designing and assembling process.

**Figure 1.** Corrugated shell sheets spread on straight and curved directrices.

Employing traditional design methods makes it impossible to assemble the designed shell sheeting into skew roof directrices because of the high values of the internal forces and even plasticity caused by the need to adjust the variable widths of subsequent folds to the designed spacing of the supporting points along each directrix.

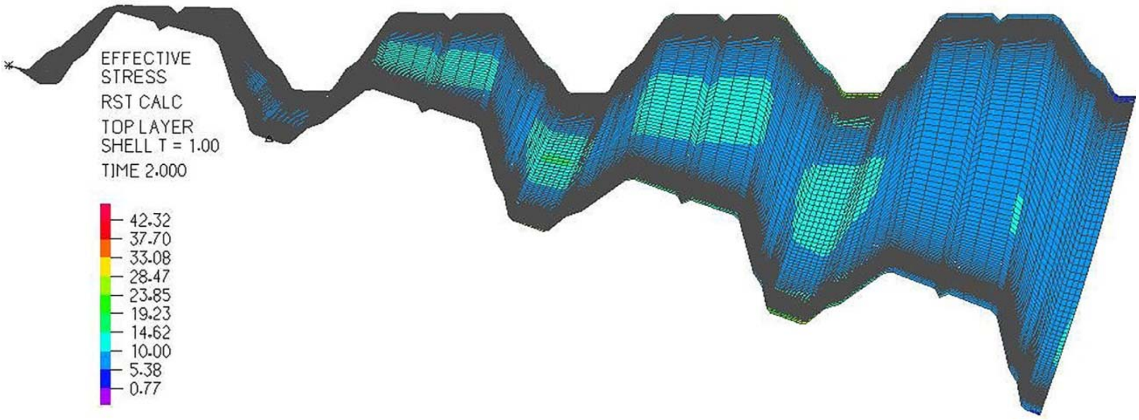
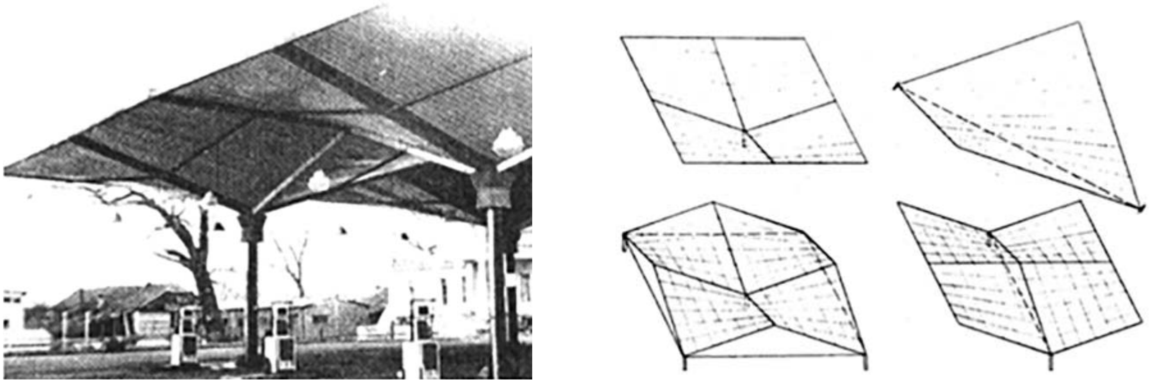



**Figure 2.** The computational model of an effectively transformed sheet and the graphical expression of its "effective" stresses in MPa.

## 2. Critical analysis of the present knowledge

Hyperbolic paraboloids are ruled quadrics [6], so they have two family of rulings passing orthogonally to each other. The simplest shell forms obtained as a result of these kinds of transformations are the so-called hypars [7, 8]. Straight hyperbolic paraboloids being specific type of hyperbolic paraboloids were used by Gergley, Edger and Parker [9]. Straight hyperbolic paraboloid is characterized by two specific rulings belonging to the above different families of the rulings [10]. Each of these two rulings is perpendicular to all rulings of the other family. Thus, it is a very important restriction in searching for diversified shell roof forms made up of such transformed one- or two-layered corrugated sheeting [10, 11].

Fisher, Resinger and Egger designed individual hyperbolic paraboloids and their structures, figure 3, [10, 12]. They shaped central sectors or one-fourth of the central sectors of hyperbolic paraboloids in various configurations. An improvement in shaping diversified innovative shells transformed elastically was reached by Gioncu and Petcu, [13].



(a) (b)

**Figure 3.** Hypar structures: (a) erected shed, (b) various configurations of hypar shell models

Because of the orthotropic properties of profiled sheet, the deformed shape of each transformed fold in a shell has to be optimized in relation to the real supporting conditions. At present, there is no such optimization in the above methods, so each fold is subjected to an additional, ineffective transformation adjusting its width to selected rulings of the designed hyperbolic paraboloid. Such forced transformations have to cause an important increase in stresses and strains of shell folds.

Reichhart [2] invented an innovative simple method for shaping free form corrugated roof shells transformed elastically, which involves using geometrical and mechanical properties of nominally plane folded steel sheets to shape rational transformed shell roofs. On the basis of his experimental tests on sheet transformations, it was defined the relations and restrictions that have to be preserved. The relations and restrictions were named a structural condition. Reichhart's concept consists in simplified calculating the widths of subsequent folds in a shell, figure. 4, the spacing of folds' supporting points along the directrices, by means of the structural condition and on the basis of the supporting conditions referring to the mutual position of two adopted directrices as well as the position of rulings $t_i$ towards these directrices. His simplified model of each subsequently considered shell fold is shaped as a shell sector restricted by two longitudinal rulings $t_i$ and $t_{i+1}$. We should remember that Reichhart's experimental tests were very simplified because they concerned really specific twist transformations of individual sheets, where the twist axis of each experimental sheet transformed was its axis of symmetry [2, 14]. Therefore, all smooth models were restricted to specified shapes that is central sectors of various right ruled paraboloids.

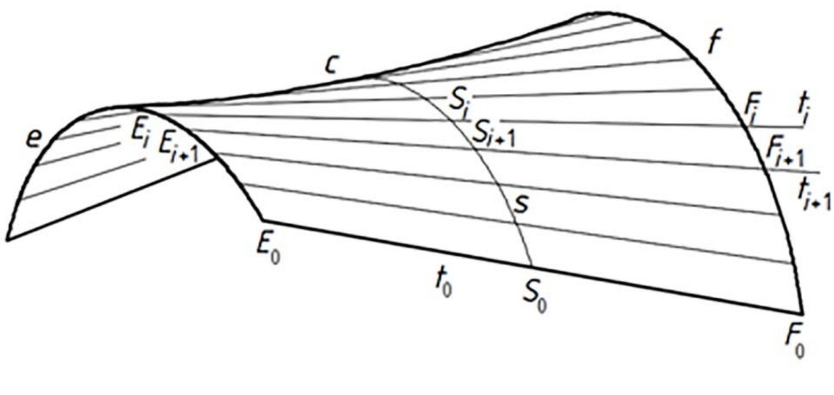



**Figure 4.** Smooth models for subsequent folds in shell as regular sectors limited by two $t_{i-1}$ and $t_i$ longitudinal rulings with the $s$ line of striction

Abramczyk's works [1, 4] has proved that the simplifications made by Reichhart cause very significant errors in roof shell shaping, because they lead to forced shape transformations inducing unnecessarily high stresses and even plastic deformations of shell fold's walls. The number of independent variables proposed by Reichhart is far too small and describing shape changes of transformed folds is highly insufficient. He has also presented the possibility of shaping innovative free forms for corrugated shell roofs [15] by means of lines of striction of warped surfaces including quadrics and helicoids [1, 16, 17] employing his experimental tests and analyses conducted on innovative computational models [4]. He has extended the potential possibility of defining smooth shell models of corrugated transformed shell sheeting for models dedicated to the bending-twist and additional shearing shape transformations, figure 4. Mathematical equations of a few selected surfaces, which are correct models for elastically transformed corrugated shell roofs are proposed by him in [1].

The experimental tests on shape transformations of individual sheets supported linearly realized by Kiełbasa [18] allow him to create an accurate folded computational model of the examined sheets. Wasilewski used folded computational models of transformed sheets to observe their mechanical characteristics under a characteristic load [19]. Aleksandra Prokopska and Abramczyk conducted interdisciplinary analyses of consistent architectural free forms roofed with transformed sheets [20]. As a result of these analyses she presented a wide range of theoretical solutions and their application in the architectural practice [21]. The solutions involve many interdisciplinary topics needed to develop experience in shaping attractive different architectural free forms and their visualizations [22].

## 3. The aims

The main aim is to present shape and mechanical changes of nominally plane folded sheets, which are the effect of deliberate effective transformations, that affect significantly the work of their folds in a shell. The paper proposes how these changes should be taken into account during the designing process of the shell.

The analyses were divided into two steps. In the first step, sheets' shape and mechanical changes caused by shape transformations during the assembly of the sheets into roof directrices were investigated. The second step involves the considerations on the shape and mechanical properties of the changes of shell folds fastened to roof directrices, and next subjected to the action of characteristic load distributed uniformly. In order to observe the influence of initial shape transformations on selected mechanical properties of a plane folded strip, two configurations of the strip were loaded uniformly: 1) a plane configuration before the shape transformation, 2) a shell configuration after the transformation.

## 4. The concept and scope of the work

Achieving various innovative free form shell roofs is relatively easy because thin-walled folded sheets having open cross-sections are very sensitive to changes of essential boundary conditions like geometrical supporting conditions, and natural boundary conditions like the ones resulting from assembling the shell folds to roof directrices and the manner of loading. Thus, big deformations of shell fold's walls and considerable mutual displacements of adjacent folds in a shell allowed under certain conditions make it possible to use them in shaping warped roof shells characterized by relatively big curvature leading to their great innovativeness and attractiveness. However, the deformations and displacements may aggravate some mechanical properties of the shell folds affecting their ability to carry the characteristic load.

There are no unambiguous and coherent experimental tests and accurate computational models, which results from diversified, major difficulties in their execution and elaboration of their results as well. In fact, the only attempt to comprehensively identify the issues related to shape transformations of plane folded sheets, and next loading them is the findings by Wasilewski [19], under professor Reichhart's supervision. It is very difficult to obtain stability and unambiguous results in terms of strength and stability changes of transformed sheets. It is much easier to detect variations in the width of shell folds during transformations. We assumed that steel sheets are of the same profile T85 x 0.75, figure. 11, and 5.00 m length. The modulus of elasticity E = 2.05 GPa.

Changes in the fold's width, stresses and strains in the span and at the transverse ends, in flanges and webs as well as in their shared edges are examined. The assembly of the sheets to the directrices starts with stiffening of the longitudinal edge of a shell, so the border fold is the first to be spread on the directrices and stiffened. Stabilization of this first fold, located to the left side of the shell, has been introduced for both cases of: an individual shell sheet, see figure 2, and a strip composed of two sheets connected to each other. All numerical models presented below relate to shape transformations limited to the unit twist angle of 5,5° to obtain the sufficient capacity for bearing the uniformly distributed characteristic load of up to 1 kN/m$^2$.

It was assumed that the shell folds are fastened with their lower flanges to the directrices. The constitutive relation between stresses and strains was also examined in the range considerably lower than critical load. The numerical models and their shape and mechanical changes have been executed with the ADINA numerical program [23, 24]. Structural systems proposed by Obrębski can be used to support such shell roofs [25].

Simplified smooth geometrical models for the transformed sheets and their strips are taken as sectors of warped surfaces limited by spatial quadrangles having two curvilinear directrices, figure 4. Rulings $t_i$ distinguished in the presented sector correspond to the longitudinal borders of the subsequent shell folds and pass through the supporting points of the folds to *e* and *f* directrices. The spacing of the points results from the changes in the fold's widths predicted according to the supporting conditions and calculated for each shell fold at the initial stage of the designing process. Therefore the widths of the subsequent shell folds are not adapted to the locations of the selected rulings on the adopted warped surface. Instead they have to be calculated using the condition that the freedom of the width increments is provided for the subsequent folds in the shell, and the warped surface can be approximated on the basis of these rulings.

## 5. Properties of folded sheeting transformed effectively

A thin-walled folded computational model of a nominally plane folded sheet transformed effectively into various warped shell shapes was created and configured by the author. The analyses presented in this section relate to this model. The nonlinear relation between the relative width increments $b_{wr}$ of the inner shell fold of the transformed sheet at its crosswise end and the measure of the unit twist angle $\alpha_j$ is shown in figure 5.

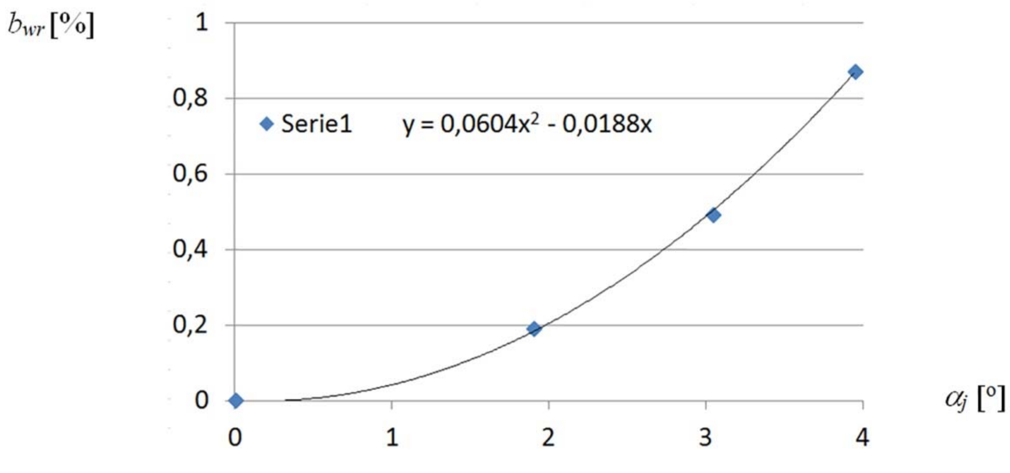



**Figure 5.** Nonlinear relation between $b_{wr}$ relative width increments of fold's crosswise ends and $\alpha_j$ unit twist angle of shell fold

The unit twist angle $\alpha_j$ is defined as the total twist angle divided by the length of the fold or sheet measured in meters. The relative width increments of the shell folds are defined as the quotient of their absolute width after transformation to the width before the transformation. Since the considered shell folds are transformed effectively, the highest normal stresses $\sigma_{yy}$ are assigned to the crosswise fold's ends and directions passing transversally to the longitudinal axes of the shell folds, figure 6.

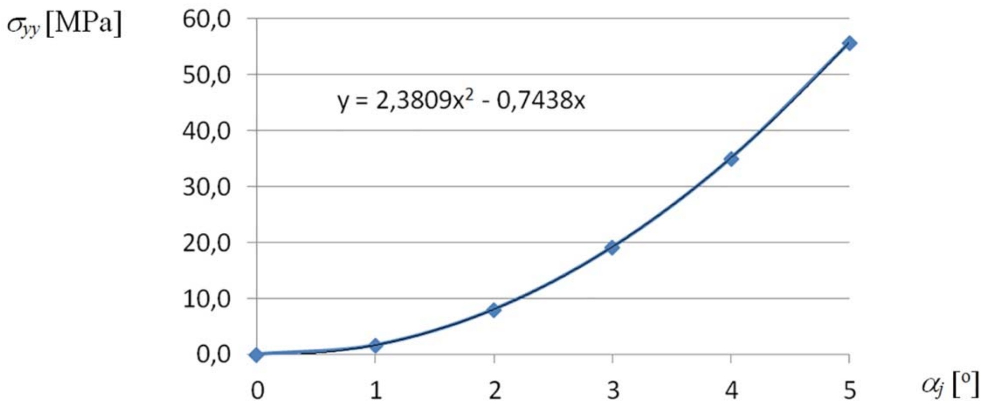



**Figure 6.** Nonlinear relation between $\sigma_{yy}$ stresses in the fold's midsurface in the directions orthogonal to the longitudinal axis of a shell fold at its crosswise end and $\alpha_j$ unit twist angle for the investigated computational model TR85 x 0.75 x 5,0 m ()

Because the constitutive relation between normal stresses $\sigma_{yy}$ and corresponding strains $\varepsilon_{yy}$ appearing in the areas of the biggest volumes in the direction orthogonal to the longitudinal axis of shell fold, that is parallel to the crosswise fold's edges, is linear, see figure 7, the methods using small strains and big deformations may be used for strength shaping in the tested range of the shape transformation degree. The other constitutive relations including the normal stresses in orthogonal directions and shear stresses are also linear.

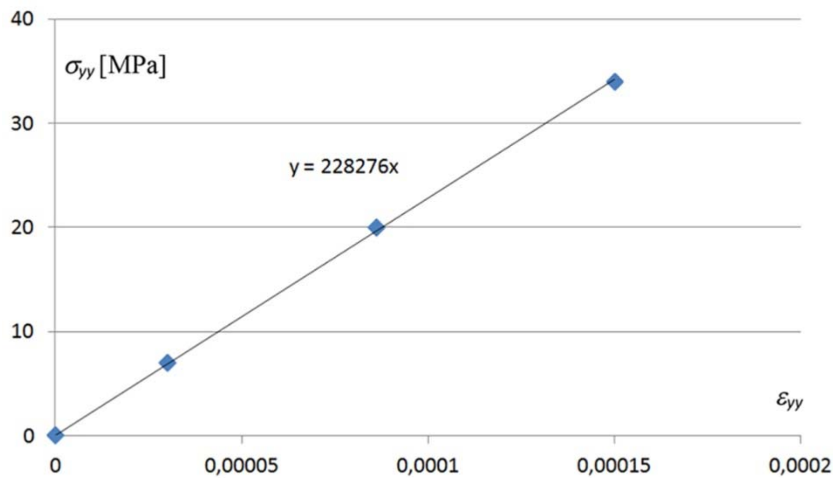



**Figure 7.** Linear relation between $\sigma_{yy}$ normal stresses and $\varepsilon_{yy}$ corresponding strains for elements located at the crosswise ends of a shell fold achieved for profile TR85 x 0.75 x 5,0 m

## 6. Properties of nominally plane sheeting transformed and loaded uniformly

In the present section, a uniformly distributed characteristic load is applied to the effectively transformed shell sheeting. In addition, the weight of the sheeting is taken into account, in contrast to the shape and mechanical changes of a thin-walled folded computational model of a nominally plane corrugated sheeting transformed initially only, and analyzed in the previous section.

The unit twisted angle of the investigated shell sheeting equal to 5,59$^{o}$ causes the important $\sigma_{yy}$ initial stresses of 55 MPa, figure 6. The volume of the characteristic load employed changes with constant increments of up to 1kN/m$^2$, which is a relatively big volume in relation to the distance between two supporting directrices of 4,5m. A graphical representation of a folded computational model of the sheeting loaded and a map of its "effective" stresses is presented in figure 8.

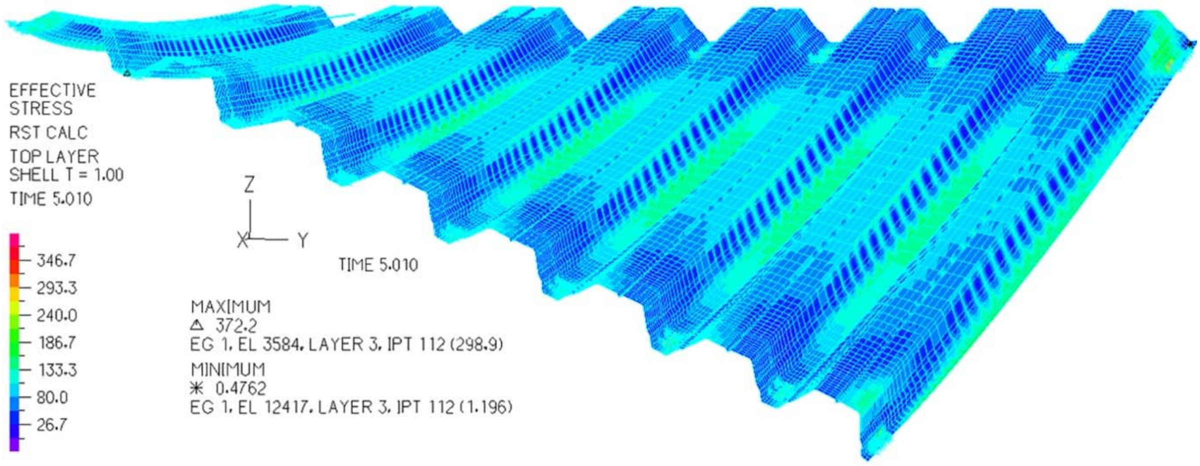



**Figure 8.** Computational model of an effectively transformed and uniformly loaded sheeting and the graphical expression of its "effective" stresses in MPa

The diagram presented in figure 9 shows Series 1 relation between the changes in stresses and in characteristic load appearing in the span of a shell fold. The initial values of $\sigma_{xx}$ stresses were generated by the effective shape transformation corresponding to the unit twist angle of 5,59$^{o}$ and $\sigma_{xx}$ act in the fold's span along the fold's longitudinal axis. It is worth stressing that the presented interdependence is related to the edges between lower flanges and oblique webs of the fold's crosswise end. In the diagram two additional curves named Series 2 and Series 3. Series2 can be seen. they express an analogous dependence related to the nominally plane folded sheeting subjected only to the uniformly distributed characteristic loading but without the initial shape transformation.

The both lines Series 1 and Series 2 converge, so that the effect of the initial effective shape transformations on the form of a uniformly loaded sheeting can be neglected in the fold's span. Series 3 was created on the results obtained by Wasilewski. Its divergence from Series 1 and Series 2 points at the aforementioned fact that the shape transformations were not effective.

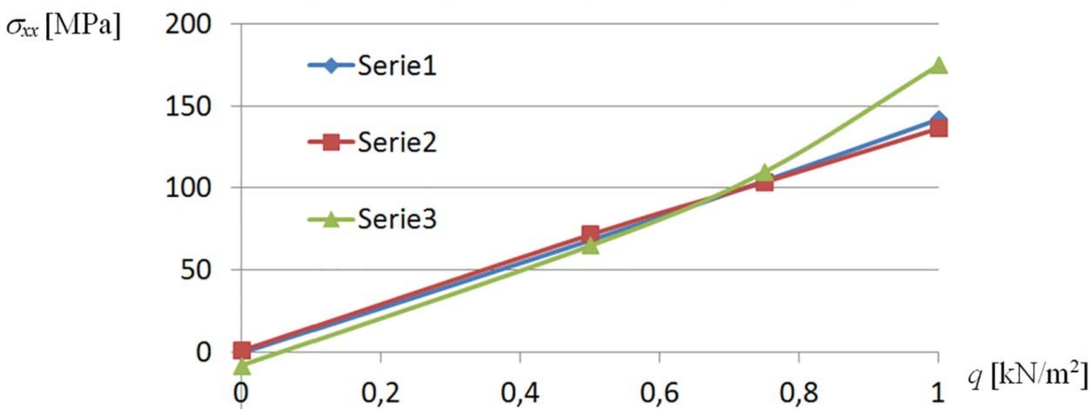



**Figure 9.** Relations between the changes in $\sigma_{xx}$ normal stresses and in $q$ characteristic load appearing in the span of: a shell fold - Series 1, a flat fold before shape transformation- Series 2, a shell fold transformed by Wasilewski – Series 3

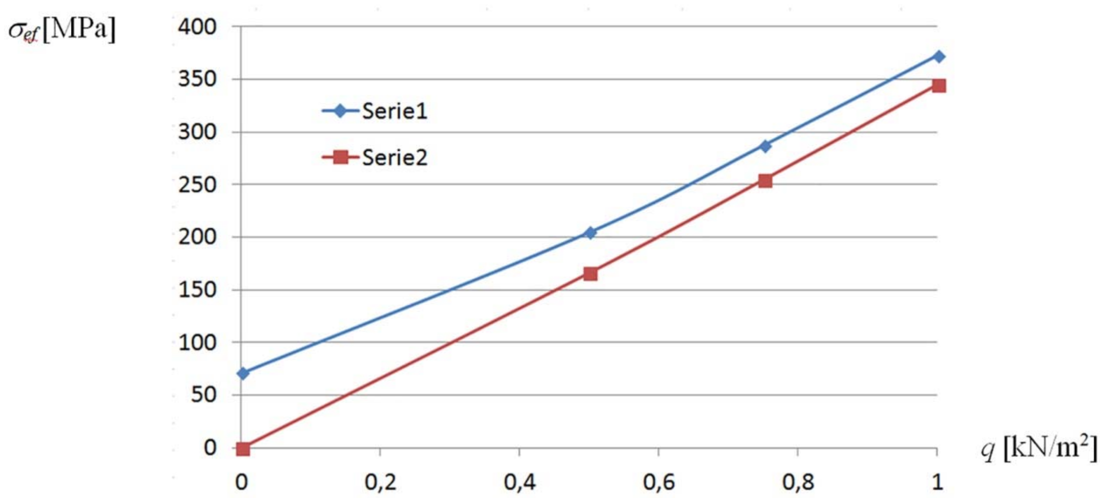



**Figure 10.** Linear relation between $\sigma_{ef}$ "effective" stresses and corresponding $\varepsilon_{yy}$ strains for elements located at the supporting areas of crosswise ends of a shell fold by the directrix: Series 2 – before and Series 1 after shape transformation

In the figure 10, the relation between the "effective" stresses in these areas and the volume of the characteristic load is presented. It can be seen an important impact of shape transformations on the stresses in the lower flanges from the above relation because of the significant distance between the mutual position of Series 1 and Series 2 curves.

## 7. Conclusions

Important geometrical and mechanical changes of nominally plane steel sheets folded in one direction resulting from their major elastic transformations allow for creating free form shell roofs, however, they indicate some difficulties that have to be taken into account during the process of shaping shell roof sheeting. The achieved curvature of such formed shell sheeting is determined by the unit twist degrees of its transformed folds. The range of fold's twist degree examined was up to 5,5°. The shell folds' changes generate significant limitations.

The variable spacing of the straight rulings belonging to the smooth shell sector modelling variously transformed subsequent shell folds should be calculated. The effective transformations generate significant values of normal stresses at most in the directions transverse to folds' longitudinal axes in the areas located at fold's transverse ends. The twist angle of 5° produces a decrease of the fold's bearing capacity by about one-eight.

Thin-walled flanges and webs allow all shell folds to adapt their cross-sections to the supporting conditions resulting from the mutual position and shape of skew directrices. In addition, the cross-sections change along the fold's longitudinal axis. However, the fold's initial shape transformations also cause a great difficulty in stabilizing the entire corrugated shell sheeting loaded with characteristic load. The initial shape transformation causes an increase in $\sigma_{yy}$ normal stresses of the border stabilized shell fold about two times. The effect of such transformation on the increase in the effective stresses appearing on the supporting areas is much smaller. The search for effective solutions in this field is needed due to high sensitivity of thin-walled profiles to boundary conditions and load.

The conclusions indicate the necessity and directions of further comprehensive, accurate experimental research and the creation of appropriate thin-walled computational models that will allow one to accurately examine and present the important orthotropic geometrical and mechanical properties of transformed folded shells, especially in places difficult to access for measurements using traditional methods, for example extensometers glued to shell fold's walls.

**Acknowledgments**
The authors would like to thank Piotr Koszelnik and Bartosz Miller – Deans of the Civil Engineering Environment and Architecture Faculty at Rzeszow University of Technology for their support, help and funding for the scientific research and postdoctoral grant of Jacek Abramczyk No. DS./H.BP.17.001 resulting in this work.

**References**

[1] J. Abramczyk, "Shell free forms of buildings roofed with transformed corrugated sheeting", Monograph, Publishing House of Rzeszow University of Technology, Rzeszów, 2017, ISBN 978-83-7934-154-2.
[2] A. Reichhart, "Specificity of shells made of elastic deformed profiled metal sheets", Monograph, Publishing House of Rzeszow University of Technology, Rzeszów, 2002, ISBN 83-7199-209-2, (in Polish).
[3] J. Abramczyk, "Building shells made up of bent and twisted flat folded sheets", *Proc. of the Int. Conf. on Engineering Sciences and Technologies, Advances and Trends in Engineering Sciences and Technologies*, pp. 233-238, 2015, DOI: 10.1201/b19249-42.
[4] J. Abramczyk, "Shape transformations of folded sheets providing shell free forms for roofing", *Proc. of the Int. Conf. 11th International Conference Shell Structures Theory and Applications, CRC Press Taylor and Francis Group*, vol. 4, pp. 409-412, 2017.
[5] J. Abramczyk, "Integrated building forms covered with effectively transformed folded sheets", *Procedia Engineering*, pp. 1545-1550, 2016, DOI: 10.1016/j.proeng.2016.08. 624.
[6] V. E. Nilson, "Testing a light gauge steel hyperbolic paraboloid shell", *Proc. ASCE, J. Struct. Div.*, vol. 88(STS), 1962.
[7] P. Gergeley, G. Winter, "Experimental investigation of thin-steel hyperbolic paraboloid structures", *Proc. ASCE, J. Struct. Div.*, vol. 98(ST10), 1972.
[8] V. Gergely, P. V. Banavalkar, J. E. Parker, "The analysis and behavior of thin-steel hyperbolic paraboloid shells", *A Research Project Sponsored by the America Iron and Steel Institute*, Report No. 338, New York, 1971.
[9] A. Grey, "Modern differential geometry of curves and surfaces with Mathematica", *4th edition, CRC Press: Champman & Hall, New York*, 2006, ISBN-10: 1584884487.
[10] J. Dźwierzyńska, J. Abramczyk, , "Engineering Descriptive Geometry", *Publishing House of Rzeszow University of Technology*, Rzeszów, 2015.

[11] H. Egger, M. Fischer, F. Resinger, "Hyperschale aus Profilblechen", *Der Stahlbau*, vol. 12, pp. 353–361, 1971.
[12] M. Fischer, "Versuche zur Ermittlung des Tragverhhaltens einer hyperbolischen Paraboloidschale aus einlagigen Trapezprofilblechen", *Stahlbau*, vol. 4,5, 1972.
[13] M. Fischer, Versuche zur Ermittlung des Tragverhhaltens einer hyperbolischen Paraboloidschale aus einlagigen Trapezprofilblechen, Stahlbau, v4,5, 1972.
[14] V. Petcu, D. Gioncu, "Corrugated hypar structures", *Proc. of the I Int. Conf. on Lightweight Structures in Civil Engineering*, Warsaw, pp. 637-644, 1995.
[15] A. Prokopska, "Creativity Method applied in Architectural Spatial (cubic) Form Case of the Ronchamp Chapel of Le Corbusier", *Journal of Transdisciplinary Systems Science*, vol. 12, pp. 49-57, 2007.
[16] A. Prokopska, "Methodological analysis of selected architectural solids", *Journal of Civil Engineering and Environment*, vol. 41, pp. 53-69, 2007.
[17] J. Dźwierzyńska and J. Abramczyk, "Reconstructing Architectural Environment from the Perspective Image," *Proceedia Engineering Elsevier* vol. 161, pp. 1445-1451, 2016.
[18] Z. Kiełbasa, "Computational shaping of the twisted folded sheet", *Proc. of the Conf. New challenges in constructions, Structural shaping, Thin-walled corrugated constructions, Publishing House PRz*, Rzeszów, 2000, (in Polish).
[19] A. Wasilewski, "Impact of the curvature of the corrugated shell sheeting on its stiffness", *Publishing House PRz, Rzeszów*, 2013, (in Polish).
[20] A. Prokopska, J. Abramczyk, "Parametric shaping of architectural free forms for building roofed with corrugated shells", *Proc. of the XXIII Int. Conf. on Lightweight Structures in Civil Engineering*, Bydgoszcz, pp. 8-13, 2017.
[21] A. Prokopska, J. Abramczyk, "Innovative systems of corrugated shells rationalizing the design and erection processes for free building forms", *Journal of Architecture Civil Engineering and Environment*, vol. 1, pp. 29-40, 2017.
[22] J. Dźwierzyńska, "Direct Construction of an Inverse Panorama from a Moving View Point," *Proceedia Engineering Elsevier* vol. 161, pp. 1608-1614, 2016.
[23] K.J. Bathe, "Finite Element Procedures", *Englewood Cliffs, N.J. Prentice Hall*, 1996.
[24] K.J. Bathe, Nonlinear Finite Element Analysis and ADINA, Proceedings of 12th Conference, Pergamon Press, J. Computers & Structures, v.72, 1999
[25] J.B. Obrębski, "Observations on Rational Designing of Space Structures", *Proc. of the Symposium Montpellier Shell and Spatial Structures for Models to Realization IASS*, Montpellier, pp. 24-25, 2004.