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General Formalism for the Computation of Radiative Heat Transfer inside Buildings

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Abstract. Thermal performance of buildings is under permanent investigation since heating in buildings represents essential financial part of the operating cost. Heat loss is an unwanted phenomenon that increases demand for financial sources and thus the precise evaluations of heat losses are desired. A common procedure used for the evaluation of heat loss is based on heat conduction through walls. The interior and exterior temperatures along with the total thermal resistance of the wall are crucial parameters for determining heat loss. The temperatures in the vicinity of walls are considerably influenced by the thermal radiation emitted by the heating system installed in the room. In order to take into account this influence, it is necessary to develop a computational formalism for radiant heat transfer in inner spaces of buildings. In the present contribution such a computational formalism is proposed. It employed the matrix of view factors characteristic for the given room and the system of algebraic equations for radiosities and heat densities.

1. Introduction

Heat and moisture transport processes in building envelopes [1] - [3] have been the subject of long lasting interest of building physicists and technologists. Especially, heat losses, i.e. ventilation, infiltration, exfiltration or heat conduction through envelope materials are under the permanent investigation. The common calculations of heat losses due to conduction through layered materials rely on temperature differences between interior and exterior and the *total* thermal resistance (incorporating surface resistances) of the envelope. Alternatively, the heat losses may be calculated on the basis of the *surface* temperatures and material resistances of the envelope. Taking into account the area of the envelope, such calculations together with ventilation losses results in heat loss expressed in Watts and this quantity is considered as the necessary energy output of the heating system installed in the building. In fact, it is possible to develop another model for calculating thermal performance of an interior space. Starting the calculations from the heater, all the three kinds of heat transport processes (conduction, convection and radiation) directed from the heater to the walls can be accounted for in the thermal analysis. The heat flow has to be in equilibrium with heat conduction through the envelope whose surface temperatures may be optimized within the system of equations taking into account convective and radiant losses at the external side of the envelope. Although such a model may seem to be complicated, it offers optimizing not only the surface temperatures but also the surface (interfaces) thermal resistances. The model consists of three main parts: (i) Theoretical solution of radiant heat transfer in the interior, (ii) Theoretical solution of convective heat transfer in the interior and (iii) Solution of the system of transcendent equations representing thermal steady state between



interior and exterior. The present paper describes *the first part of this complex model*, namely, the theoretical solution of radiant heat transfers in the interior. It is based on the concept of view factors, the system of radiosities and radiant heat densities. It provides a complete solution of *radiative heat transfer* in inner spaces of buildings.

2. View factors

The view factor F reduces the total energy irradiated by a body to that part of energy that reaches the neighboring body (see Figure 1). This factor is dimensionless, assumes only positive values, which are restricted to the interval $F \in (0,1)$. The view factor is defined as follows [4] - [6]

$$F_{ij} = \frac{1}{S_i} \int_{(S_i)} \left[\int_{(S_j)} \frac{\cos \varphi_i \cos \varphi_j}{\pi R^2} dS_j \right] dS_i \quad (1)$$

$$F_{ji} = \frac{1}{S_j} \int_{(S_j)} \left[\int_{(S_i)} \frac{\cos \varphi_j \cos \varphi_i}{\pi R^2} dS_i \right] dS_j \quad (2)$$

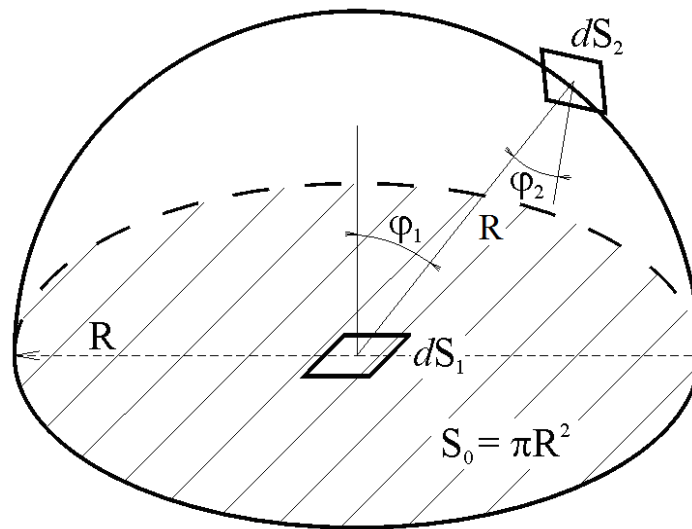


Figure 1. Heat exchange by radiation between two small surface elements dS_1 and dS_2 .

From Equations (1) and (2), the symmetry relation between F_{ij} and F_{ji} can be immediately deduced:

$$S_i \cdot F_{ij} = S_j \cdot F_{ji} \quad (3)$$

This *symmetry rule* holds quite *generally* regardless of the types of surfaces and their geometrical positions.

There is another important property of view factors. If a surface is not capable of irradiating itself, its view factor is zero:

$$F_{ii} = 0 \quad (4)$$

For example, the perfect planes or the external surfaces of spheres belong to this class of surfaces. It should be highlighted that the *zero rule* (4) does not hold generally but it is restricted to special surfaces.

The third property concerns the *closed envelopes* that consist of more different surfaces numbered as 1, 2, 3, ... n :

$$\sum_{j=1}^n F_{ij} = 1 \quad (5)$$

Relation (5) may be called the *summation rule*. It should be stressed that its validity is restricted solely to the closed envelopes. These may be, for example, inner spaces of buildings.

The view factors can be ordered into a matrix whose elements have to fulfil the basic rules (3) - (5):

$$\begin{pmatrix} F_{11} & F_{12} & \dots & F_{1n} \\ F_{21} & F_{22} & \dots & F_{2n} \\ \dots & \dots & \dots & \dots \\ F_{n1} & F_{n2} & \dots & F_{nn} \end{pmatrix} \quad (6)$$

According to summation rule (5), the sum of elements in each row of matrix (6) has to be 1. According to zero rule (4), *some* of the elements in the main diagonal of matrix (6) may be zero. Matrix (6) is quasi-symmetric according to symmetry rule (3). None of matrix elements is larger than 1. All elements are positive.

Due to properties (3) - (5), each room in a building has its own characteristic matrix (6). It is a squared matrix of order n , i.e. the matrix has n^2 elements. To construct this matrix, a certain minimum number of view factors has to be known prior to starting the construction. This minimum number is given as follows $(n^2 - n)/2 - m$ where m is the number of zero elements in the main diagonal (see rule (4)).

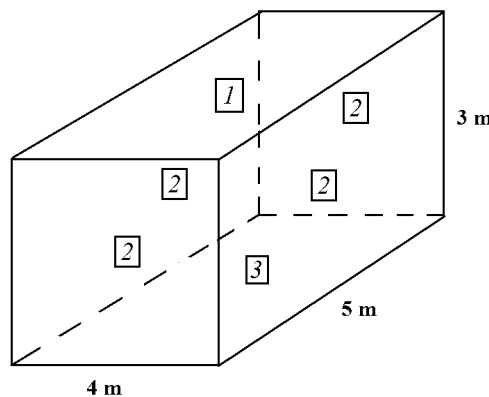


Figure 2. A simple room as a prototype for calculation of view factors.

To illustrate the construction of a view matrix, a simple room will be investigated (Figure 2). The dimensions of the room are 5m x 4m x 3m. The inner surfaces of the room will be numbered as follows. No. 1 is assigned to the ceiling ($S_1 = 20\text{m}^2$). Let us suppose that the sidewalls of the room have the same emissivity and temperatures and, thus, they can be considered as one surface no. 2 ($S_2 = 54\text{m}^2$). The floor has number 3 ($S_3 = 20\text{m}^2$). There will be two zero view factors in the main diagonal, namely $F_{11} = 0$ and $F_{33} = 0$ since the ceiling and the floor may be considered as perfect planes. Thus, $(n^2 - n)/2 - m = (3^2 - 3)/2 - 2 = 1$, which means that only one view factor must be

known as the necessary input element. Other factors will be determined by rules (3) - (5). Let us suppose that the known view factor is $F_{12} = 0.67$:

$$\begin{pmatrix} 0 & 0.67 & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & 0 \end{pmatrix} \quad (7)$$

The summation rule may be applied to the first row of the matrix, i.e. $\sum_{j=1}^3 F_{1j} = 1$:

$$\begin{pmatrix} 0 & 0.67 & 0.33 \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & 0 \end{pmatrix}$$

(8)

By applying symmetry rule (3), the elements F_{21} and F_{31} will be specified, i.e. $S_2 \cdot F_{21} = S_1 \cdot F_{12} \Rightarrow F_{21} = (S_1 / S_2) \cdot F_{12}$ and similarly $F_{31} = (S_1 / S_3) \cdot F_{13}$:

$$\begin{pmatrix} 0 & 0.67 & 0.33 \\ 0.248 & F_{22} & F_{23} \\ 0.33 & F_{32} & 0 \end{pmatrix} \quad (9)$$

Summation rule (5) will assist to complete the last row of the matrix:

$$\begin{pmatrix} 0 & 0.67 & 0.33 \\ 0.248 & F_{22} & F_{23} \\ 0.33 & 0.67 & 0 \end{pmatrix} \quad (10)$$

Symmetry rule (3) will lead us to the factor F_{23} , i.e. $F_{23} = (S_3 / S_2) \cdot F_{32}$:

$$\begin{pmatrix} 0 & 0.67 & 0.33 \\ 0.248 & F_{22} & 0.248 \\ 0.33 & 0.67 & 0 \end{pmatrix} \quad (11)$$

Finally, summation rule (5) will complete the second row of the matrix:

$$\begin{pmatrix} 0 & 0.67 & 0.33 \\ 0.248 & 0.504 & 0.248 \\ 0.33 & 0.67 & 0 \end{pmatrix} \quad (12)$$

This is a final characteristic matrix of the investigated simple room shown in Figure 2. Such matrices will serve for calculating radiative heat transfer in rooms. This transfer will be discussed in the next section.

3. Theory of radiative heat transfer between grey surfaces

The emissivity ε of grey surfaces assumes the value lying between zero and one. The radiative heat transfer between such surfaces is based on two laws, namely the Stefan-Boltzmann law and the

Kirchhoff law [2 - 6]. The grey bodies represent the majority of real bodies in nature. However, it should be recalled that the term 'grey' has little to do with real colours of bodies. It is only symbolic term for surfaces that fulfil two conditions:

- (i) Transmittance is zero $\tau = 0$.
- (ii) The sum of reflectance and absorbance (emissivity) is one, i.e. $\rho + \varepsilon = 1$.

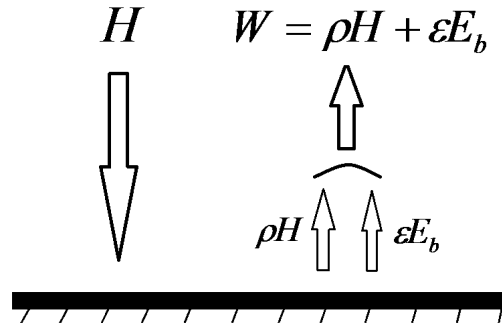


Figure 3. Surface energy exchange.

Let us derive equations describing radiative heat exchange between grey surfaces. In Figure 3, there is a scheme of energy exchange occurring on the surface of a radiative body. The symbol H represents the total radiative heat flux (Watt / m^2) coming from all neighbouring surfaces whereas the symbol W represents **radiosity**, which is the total heat flux emitted from the surface (Watt / m^2). Radiosity is the sum of the Stefan-Boltzmann radiation ($\varepsilon E_b = \varepsilon \sigma T^4$, $\sigma = 5.67 \cdot 10^{-8} \text{Watt} / (\text{m}^2 \text{K}^{-4})$) and the reflected heat flux (ρH), i.e.

$$W = \varepsilon E_b + \rho H \quad (13)$$

$$H = \frac{W - \varepsilon E_b}{\rho} \quad (14)$$

The resulted radiative heat flux (q) concerning the investigated surface may be determined as the difference between the emitted flux (W) and the coming flux (H):

$$q = W - H \quad (15)$$

By replacing H in Eq. (15) by the fraction from Eq. (14), we can obtain:

$$q = W - \frac{W - \varepsilon E_b}{\rho} = \frac{(\rho - 1)W + \varepsilon E_b}{\rho} = \frac{-\varepsilon W + \varepsilon E_b}{\rho} = \frac{\varepsilon}{\rho} (E_b - W) \quad (16)$$

For the heat flux q_i of the i -the surface, it is possible to write:

$$\boxed{q_i = \frac{\varepsilon_i}{\rho_i} (E_{bi} - W_i)} \quad (\text{Watt} / \text{m}^2) \quad (17)$$

It is desirable to derive some equations that would enable to quantify radiosity W_i . The i -the surface of area S_i is supplied by energy H_i coming from all the n neighbouring surfaces (including the i -th surface):

$$S_i H_i = \sum_{j=1}^n S_j F_{ji} W_j \quad (\text{Watt}) \quad (18)$$

Taking into account the rule of symmetry $S_j \cdot F_{ji} = S_i \cdot F_{ij}$, the following equations may be obtained

$$S_i H_i = \sum_{j=1}^n S_i F_{ij} W_j \quad (\text{Watt}) \quad (19)$$

$$H_i = \sum_{j=1}^n F_{ij} W_j \quad (\text{Watt} / \text{m}^2) \quad (20)$$

By considering Equations (20) and (13), an expression for radiosity emerges:

$$W_i = \varepsilon_i E_{bi} + \rho_i \sum_{j=1}^n F_{ij} W_j \quad i, j = 1, 2, 3 \dots n \quad (\text{Watt} / \text{m}^2). \quad (21)$$

Expression (21) is in fact a system of n algebraic equations. The solution of this system provides us with the n values of radiosities $\{W_i\}_{i=1}^n$. These radiosities inserted into Equations (17) will provide us with the total radiative heat fluxes q_i for each of the n surfaces. If the heat flux q_i is determined, it is easy to calculate the heat flow Φ_i corresponding to i -the surface:

$$\Phi_i = S_i q_i \quad (\text{Watt}) \quad (22)$$

It should be recalled that the quantities q_i and Φ_i may be either positive or negative (see Equation (15)). Their positive values mean that the corresponding surface emits heat energy whereas the negative values mean that the corresponding surface absorbs energy.

Equations (17), (21) and (22) are the basic relations that determine radiative heat transfer between grey surfaces arranged both within the closed and open envelopes. Since the vast majority of surfaces in practice are grey surfaces, equations (17), (21) and (22) have a real potential to be frequently used in the thermal technology of inner spaces of buildings.

4. Special envelope consisting of two surfaces

The simplest model of a room is shown in Figure 4. It consists of the heated circular floor and the cupola that is built above the floor. The envelope of such a room contains only two surfaces. No. 1 is the floor and no. 2 is the cupola. By using the derived system of Equations (17), (21) and (22), one may find that the radiative heat flows associated with the floor Φ_{12} and with the cupola Φ_{21} may be calculated directly by means of simple formulae based on the coefficients of mutual emissivities C_{12} and C_{21} as follows:

$$\Phi_{12} = S_1 C_{12} \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right] = S_1 \frac{C_b}{\frac{1}{\varepsilon_1} + \frac{S_1}{S_2} \left(\frac{1}{\varepsilon_2} - 1 \right)} \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right] \quad (\text{Watt}) \quad (23)$$

$$\Phi_{21} = S_2 C_{21} \left[\left(\frac{T_2}{100} \right)^4 - \left(\frac{T_1}{100} \right)^4 \right] = S_2 \frac{C_b}{\frac{1}{\varepsilon_1} + \left(\frac{S_1}{S_2} \cdot \frac{1}{\varepsilon_2} \right) - 1} \left[\left(\frac{T_2}{100} \right)^4 - \left(\frac{T_1}{100} \right)^4 \right] \quad (\text{Watt}) \quad (24)$$

However, these formulae are only applicable to two-surface envelope. When a room has more than two surfaces, formulae (23) and (24) cannot be used and the system of equations (17), (21) and (22)

have to be solved. Practical examples of both these computational procedures (using radiosities or mutual emissivity's) will be presented in our separate conference contribution.

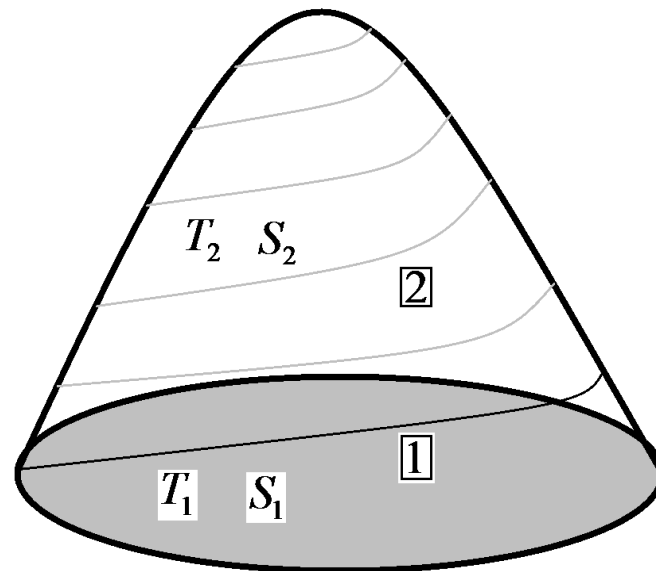


Figure 4. Two-surface envelope.

5. Conclusions

The classical thermal building technology is preferably focused on the phenomena closely related to heat conduction but the radiative heat phenomena remain practically untouched by this discipline. Nevertheless, it is the radiative heat that dominates the heat conduction *within the inner spaces of buildings*. Simple calculations of radiative and conductive heat transfers from heated floor to the cold ceiling show that it is the heat radiation that transfers about 99.8% of heat energy between these two surfaces.

In this paper, the computational procedure has been developed that enables to evaluate heat flow going to/from each surface of the envelope. The procedure is based on view factors and the system of algebraic equations providing radiosities that are used for determining heat fluxes and heat flows associated with each surface of the envelope. For two-surface envelopes, the model offers besides the computational procedure utilizing radiosities also an alternative procedure employing the concept of mutual emissivity. Practical examples of both these computational procedures are presented in our separate conference contribution called "Computations of radiative heat transfer inside buildings".

The developed computational procedure of radiative heat transfer inside rooms is only a first part of a more general model focused on heat losses of building envelopes. The second part of this general model will solve the convective heat transfer in interiors and in the third part of the model, the system of transcendent equations will be formulated representing thermal steady state between the interior and exterior.

Acknowledgment

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