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# Dynamic Modelling of the Effects of Combined Horizontal and Vertical Curves on Side Friction Factor and Lateral Acceleration

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**Abstract.** In this study, multiple simulations were conducted using CarSim and TruckSim dynamic simulation software, along with SPSS software for statistical analysis. The simulations were conducted on different types of vehicles: a sedan, SUV, and truck. Speed, delta, grade, and position (location of horizontal and vertical curves relative to each other) were the parameters considered in the simulations. According to the results, a sedan and SUV experience more side friction than a truck in the beginning and at the quarter of the vertical curve. The side friction factors in critical situations are higher than the values in AASHTO 2011. A truck is more likely to roll over because of its high lateral acceleration. The statistical analysis showed that the grade and position did not affect the side friction factor for the sedan and SUV, and could thus be neglected. The results of this study could influence the design of safer roadways.

## 1. Introduction

Ensuring the safety of vehicles is one of the most fundamental principles of the Traffic Engineering and Transportation Planning Department. Lack of safety consideration in design of roadway geometry has caused significant damage in recent years. Vehicle engineering and roadway design are two important factors that influence highway safety [1]. Because of population growth and an increasing number of vehicles, traffic accidents have become one of the main problems in the transportation sector. To solve this problem, the main priority of relevant organizations, such as the Ministry of Transportation and traffic police, must be recognizing the primary causes of accidents. Curves are essential components of road safety. Many factors could be considered in designing road curves, such as the type of road, topography, climate, and speed. On an inconsistently designed road, where parts of the road do not meet the driver's expectations, the driver is required to drive more carefully. Under such circumstances, the driver may make mistakes that increase the probability of an accident. Thus, identifying accident-prone curves is necessary to decrease the probability of accidents. Horizontal curve design requires careful consideration of many factors including safety, stability, topography, forecast traffic volumes, and weather conditions. Horizontal and vertical curves complement each



other and should therefore not be designed independently. When horizontal and vertical curves occur simultaneously, the safety of the road decreases. Thus, having a proper balance between horizontal curves and vertical curves is desirable. Skidding and rolling over may occur more frequently with the combination of horizontal and vertical curves [2, 3]. Rollover threshold is not a significant consideration for passenger cars, but it is important for trucks for which the rollover threshold surpasses the side friction [4].

AASHTO guidelines, one of the most commonly used road design guidelines, does not provide a quantitative methodology for modifying the geometry of horizontal curves in consideration of vertical curves [5]. AASHTO 2011 considers a point-mass model, which is one of the primary models and has the following disadvantages:

- The effect of the longitudinal grade on the side friction factor is neglected. However, longitudinal grades can alter the distribution and performance of vehicles. Side friction occurs under the influence of upgrades and downgrades. The required side friction on downgrades (due to braking force) and upgrades (due to tensional force) increases [6].
- The forces that act inside and outside of the tire are not the same [7]. Consequently, the side friction varies among the tires while the vehicle corners through horizontal curves [8].
- The coincidence of the horizontal and vertical crest curves is clearly not planar, but the simple point mass model considers that the vehicles are on a planar surface [9].
- This model does take into consideration the vehicle dimensional characteristics [10].

Because of these reasons, the results of AASHTO 2011 are different from other research studies that use distinct models. Torbic et al. indicated that the side friction factors obtained from coincidental horizontal and vertical curves were drastically different from the results found in the AASHTO Green Book [9]. The side friction demand while cornering along horizontal curves is 15% greater than that predicted by the Green Book [8]. Consequently, more complex models should be used for analysing vehicle cornering behaviour [11, 12]. Several models, such as the transient bicycle, steady-state bicycle, and multi-body simulation models have provided better results than the point-mass model. Varunjikar showed that the side friction obtained from the transient bicycle model better matched the results from simulation software, such as CarSim, during braking [7].

In this study, CarSim and TruckSim vehicle dynamic simulation software were used. Two-axle passenger cars and two-axle trailers are supported by the CarSim software. The cornering behaviour of vehicles with two, three, or four axles can be simulated by the TruckSim software. The objective of this study was to analyze the influence of coinciding horizontal and vertical crest curves on the side friction factor and lateral acceleration, considering parameters such as speed, delta, grade, and position; the results of this study could be a reference for roadway designers.

## 2. Literature review

The friction factor varies with changing slip. The maximum value of the friction coefficient occurs at between 10% and 20% of slip (Hall, 2009). Pavement surface friction is one of the most important issues to be considered in traffic safety. Echaveguren et al. investigated friction and obtained the equation for the intersection point of supplied friction with demanded friction, as follows.

$$F_{60} \times e^{60-S/S_p} - k_2 P = S^2 / k_1 R \quad (1)$$

The maximum safe speed ( $S_{lim}$ ) and the maximum friction ( $f_{lim}$ ) can be obtained from the intersection point [14].

The friction factor between the tires and the pavement surface must be sufficient to prevent the vehicle from skidding off the road. The limit of the friction factor should be equal to or greater than  $f_{lim} = t/2h$  at the center of gravity height ( $h$ ) and the track width ( $t$ ) [15].

Krempel presented that pavement friction is distributed in the longitudinal and radial directions and derived the following equation [16].

$$\left[ F_T / F_{T,max} \right]^2 + \left[ F_R / F_{R,max} \right] = n^2 < 1 \quad (2)$$

where  $F_T$  = longitudinal friction demand;  $F_{T,max}$  = maximum longitudinal friction factor;  $F_R$  = side friction demand;  $F_{R,max}$  = maximum side friction factor; and  $N$  = total amount of friction supplied.

The first research work on the confluence of horizontal and vertical curves was performed by Dunlap et al. [17]. They derived an equation to estimate the side friction factor, which was based on the point mass model.

$$f = \frac{v^2}{gR} \cos(0.01e) - \sin(0.01e) - \sin(0.01e) \cos(G) \quad (3)$$

where  $v$ ,  $g$ ,  $R$ ,  $e$ , and  $G$  are speed, acceleration of gravity, radius, superelevation, and longitudinal grade, respectively [17].

The primary side friction factor for horizontal curves was obtained from the AASHTO Green Book.

$$f = \frac{VDS^2}{g \cdot R} - 0.01 \cdot e \quad (4)$$

where  $f$  is the side friction demand, and  $VDS$  is the road design speed [5].

The West Virginia Department of Transportation conducted a case study in a mountainous region to derive the superelevation equation for sharp curves combined with steep slopes. The following equation is used to calculate the superelevation for passenger vehicles (Eck and French, 2002):

$$e + f = \frac{V^2}{15R} + G(\sin\theta) \quad (5)$$

New equations, which were based on sprung vehicles, were proposed by Chang [19] for determining the minimum horizontal curve radius. These equations are exclusive because they concern the gravity movement during cornering [19].

$$\text{Passenger cars: } R = \frac{v^2}{121(0.5e+f)} \quad (6)$$

$$\text{Truck: } R = \frac{v^2}{122.5(122.5(0.75+f))} \quad (7)$$

Bonneson, in other research, indicated that the AASHTO Green Book ignored the side friction demand for horizontal curves with sharp curves [6].

Abdi et al. studied the effect of the combination of horizontal curvature and longitudinal grade on side friction factors. They used CarSim and TruckSim multimode simulation software with consideration of two states, with and without braking, on downgrades. They found that a greater side friction factor occurred on downgrades and that the most critical state was for the rear axle of a sedan [20]. Considering that the load is transferred from the rear axle to the front axle on downgrades [21], this means that, for the same curve, the vehicles on the downgrade are safer and more comfortable than are the vehicles on the upgrade.

Mehrara et al. showed that the most accident-prone part of a combined vertical and horizontal curve occurs at the beginning of the horizontal curve, because of the changes in the steering angle [22]. At the beginning of a horizontal curve, the susceptibility to rolling over and skidding is 8.5% greater than at any other part of the curve. Sedans and SUVs are more likely to skid than trucks because of the greater side friction demand they produce, whereas trucks are more likely to roll over [20].

Abdi et al., in other research, derived new equations for calculating side friction factors for a sedan, SUV, and truck [24].

$$\text{For Sedan: } f = 3.769 - 3.108 \ln v - 0.003g \quad (8)$$

$$\text{For SUV: } f = 0.663 - 0.12 \ln v + 7.479E - 7g \quad (9)$$

$$\text{For Truck: } f = 0.827 - 0.155 \ln v - 0.001g \quad (10)$$

Alexander et al. presented the results of a comprehensive project intended to identify roadway design practices that maximize the margin of safety between the friction supplied and friction demanded. The results illustrated that the point mass model used by AASHTO is suitable for analysing the frictional margin in situations without lane changes. However, a transient model is appropriate for lane changes and severe braking [25].

### 3. Methodology

This study used vehicle dynamics simulation software to investigate vehicle manoeuvring through combinations of horizontal and vertical crest curves. First, the software input data was defined and, after several tests, the results were extracted as plots and animations for analysis.

#### 3.1. Input data

Input data consisted of the vehicle type, driver's behaviour (based on the rotation of the steering wheel and the driver's speed in various conditions), type of road (horizontal, vertical, or combined vertical and horizontal curve), and road level. Data for each of these variables are as follows:

**Type of vehicle:** Three types of vehicles were set up, including an E-class sedan, SUV, and truck. In the software, all default vehicle specifications were maintained.

**Driver's behaviour:** The driver's behaviour was based on the tendency to rotate the steering wheel to be a specific distance from the edge of the road. The vehicle's speed in different conditions was 80, 100, and 120 km/h.

**Horizontal direction:** Horizontal direction consisted of the horizontal curve radius and superelevation. The superelevation rate was 8%. The horizontal curve radius was calculated based on the different design speeds.

**Vertical direction:** Vertical crest curves were considered in this study; they consisted of three different combinations of entering and exiting grades ( $G = (2, -2), (4, -6), \text{ and } (6, -4)$ ).

One goal of this study was to evaluate the effects of the positions of the horizontal and vertical curve with respect to each other. In the simulation, the positions of these curves were considered as one of nine options:

- SH, MV: the start of the horizontal curve was in the middle of the vertical crest curve;
- MH, MV: the middle of the horizontal curve was in the middle of the vertical crest curve;
- QH, MV: the quarter of the horizontal curve was in the middle of the vertical crest curve;
- SH, SV: the start of the horizontal curve was at the start of the vertical crest curve;
- SH, QV: the start of the horizontal curve was at the quarter of the vertical crest curve;
- QH, SV: the quarter of the horizontal curve on the start of the vertical crest curve;
- QH, QV: the quarter of the horizontal curve on the quarter of the vertical crest curve;
- MH, SV: the middle of the horizontal curve on the start of the vertical crest curve; and
- MH, QV: the middle of the horizontal curve on the quarter of the vertical crest curve.

**Maximum friction factor:** Based on Fertado et al., the maximum friction factor was assumed to be 0.9 for dry pavement [26].

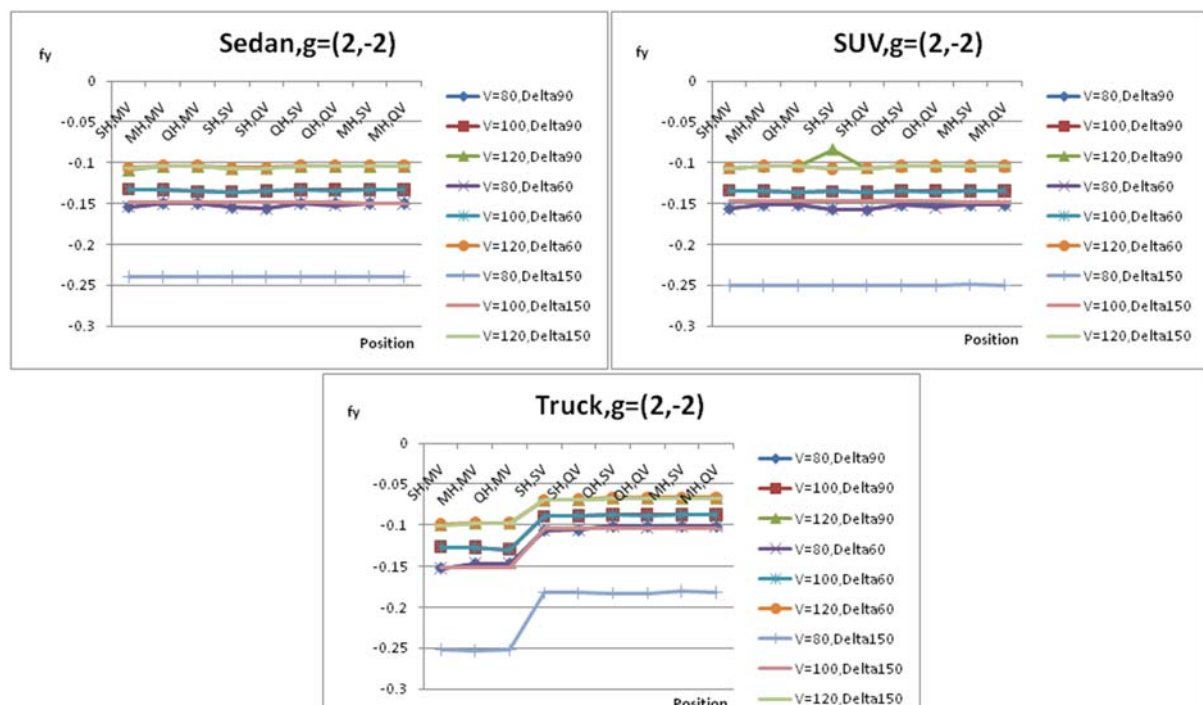
**Delta:** Delta can be used to calculate the length of a horizontal curve  $= R \times \Delta$ . Three different deltas of 60°, 90°, and 150° were used in this study.

The main variables used in this study were the horizontal curve radius, design speed, longitudinal grade, vertical crest curve length, ideal steering angle, angle between the front and rear of the vehicle, vehicle center of gravity height, delta, displacement of the center of gravity during rotation, demanded side friction for rotation, supplied side friction for rotation, longitudinal friction, dynamic forces acting on the vehicle during the rotation, and super elevation rate.

#### 4. Simulation outcomes

##### 4.1. Side friction factors at different positions and grades

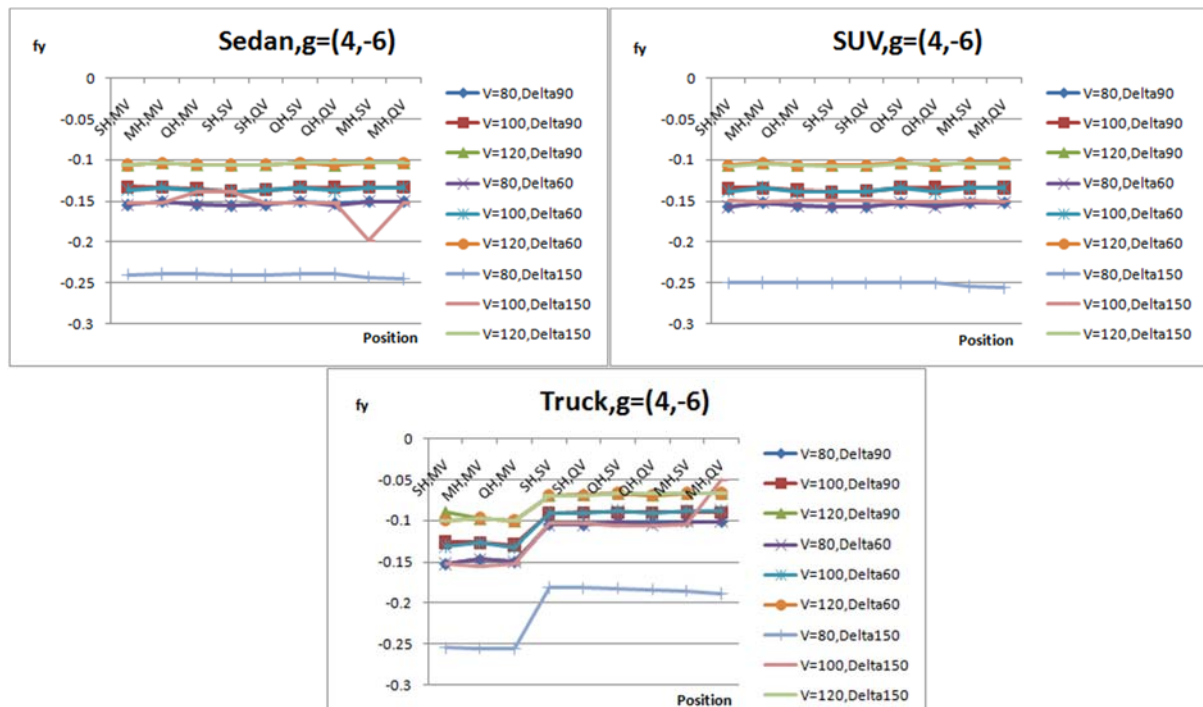
Figure 1 shows the side friction factors of a sedan, SUV, and truck on a grade of (2, -2). According to Figure 1, vehicles at a constant speed of 80 km/h and delta of 150° experienced the greatest side friction factors. Vehicles at a speed of 120 km/h had the lowest side friction factors. These results were confirmed for a sedan, SUV, and truck.



**Figure 1.** Deviation of Side friction factor on a grade of (2, -2)

The maximum side friction force exerted by the truck was in the middle of the vertical crest curves. The location of the vertical and horizontal curves relative to each other did not have a significant effect on the side friction factors of the sedan or SUV. Based on Figure 1, the side friction factors of a truck in the beginning and at the quarter of the vertical curve were less than those of a sedan and SUV.

Figure 2 shows the side friction factors of vehicles on a grade of (4, -6). The results show that the maximum side friction forces of the sedan and SUV occurred in the middle of the horizontal curves. A speed of 80 km/h and delta of 150° was the most critical condition for all types of vehicles, meaning that, under these conditions, the vehicles had a greater risk of skidding.

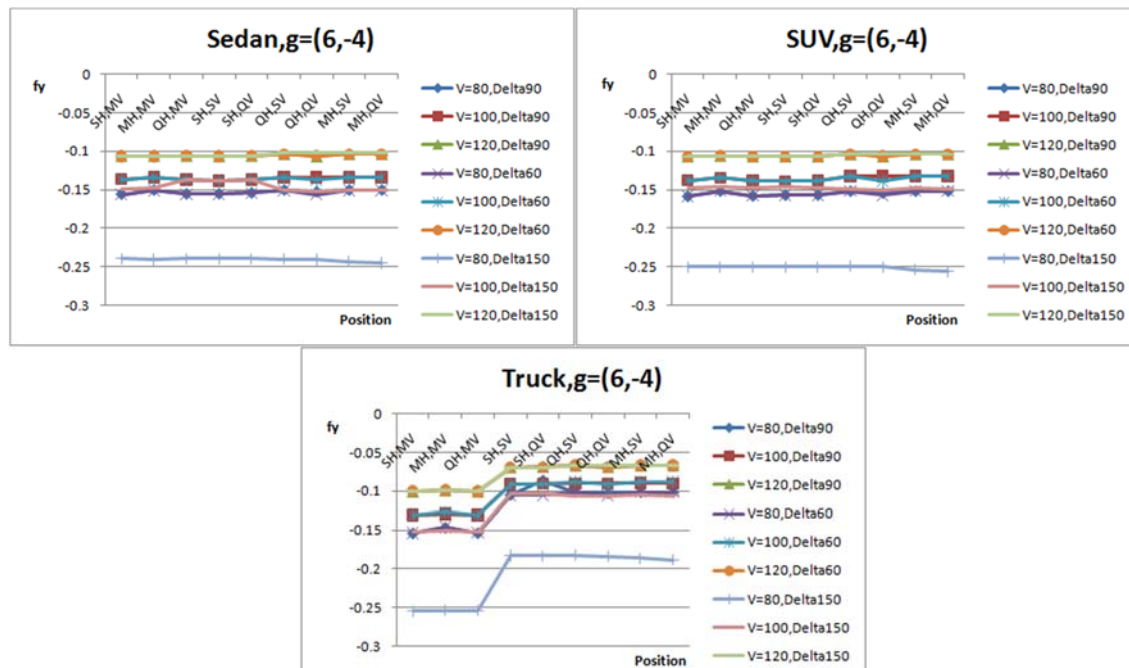


**Figure 2.** Deviation of Side friction factor on a grade of (4, -6)

Figure 3 shows the side friction of vehicles on a grade of (6, -4). The results show the same trend as the (2, -2) and (4, -6) grades. The grades of the vertical curves did not have a considerable effect on the side friction factors of the vehicles during cornering through the horizontal curves. According to the figures, due to their greater side friction factors, the sedan and SUV at the beginning of and at the quarter of the way through the vertical curves were more at risk of skidding than was the truck. The maximum side friction factors for all grades exceeded the recommended values in AASHTO 2011. For example, the maximum side friction factor for all vehicles (sedan, SUV, and truck) for a grade of (4, -6) was 0.25 (for 80 km/h and delta of 150°), which is greater than the value recommended by AASHTO 2011 (0.14 for 80 km/h).

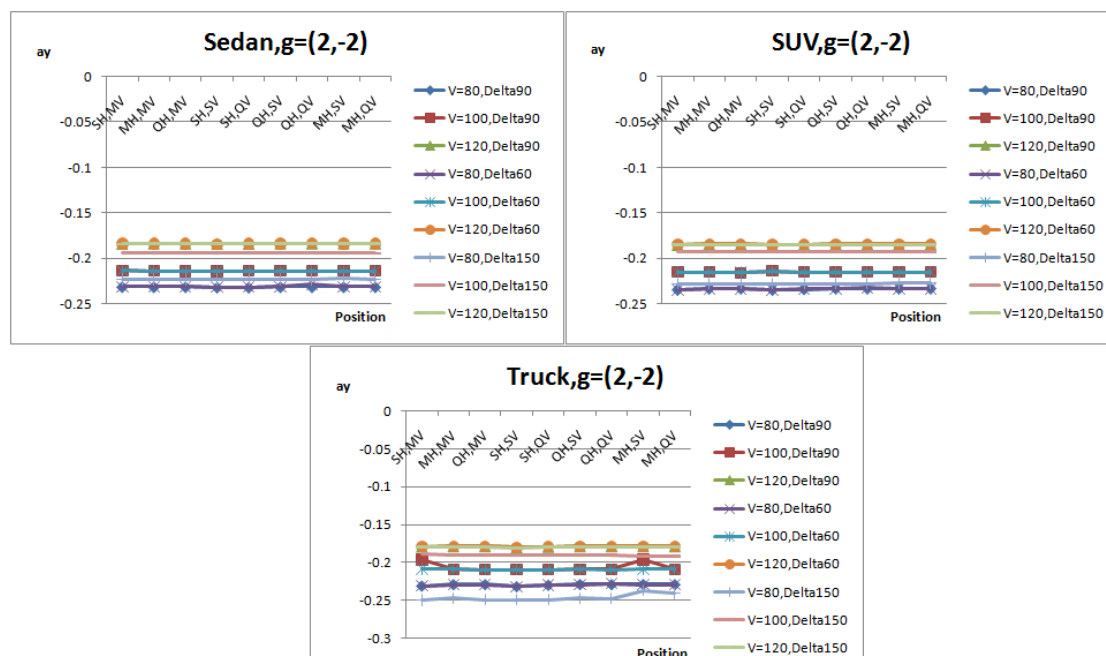
#### 4.2. Lateral acceleration in different position and grades

Lateral acceleration in horizontal curves is another parameter regarding the safety of vehicles, especially their resistance to rollover. Overall, there are two terms in lateral acceleration: centrifugal acceleration and centripetal acceleration. The acceleration resulting from surface friction and super elevation is called centripetal acceleration, [27]. Centrifugal acceleration is a virtual acceleration that occurs when the vehicle moves in a circular manner and which acts opposite to the direction of centripetal acceleration.



**Figure 3.** Deviation of Side friction factor on a grade of (6, -4)

Figure 4 shows the lateral acceleration of vehicles on a grade of (2, -2). The maximum lateral acceleration ( $a_y$ ) of a truck occurs at 80 km/h and a delta of 150°, while the maximum value of  $a_y$  for the SUV and sedan occur at 80 km/h and a delta of 90°. Generally, the speed of 80 km/h was the most critical parameter for lateral acceleration.



**Figure 4.** Deviation of Lateral acceleration on a grade of (2, -2)

Figs. 5 and 6 show the lateral acceleration of vehicles on grades of (4, -6) and (6, -4). The graphs have the same trend as on the grade of (2, -2). The location of the horizontal and vertical curves



relative to each other could be neglected for the sedan and SUV. The lateral acceleration of the truck was greater than the  $a_y$  of the SUV and sedan (except in one case, at 80 km/h and a delta of  $150^\circ$ ). Therefore, the truck was the most at-risk for rollover because of its higher center of mass in comparison with the other considered vehicles.

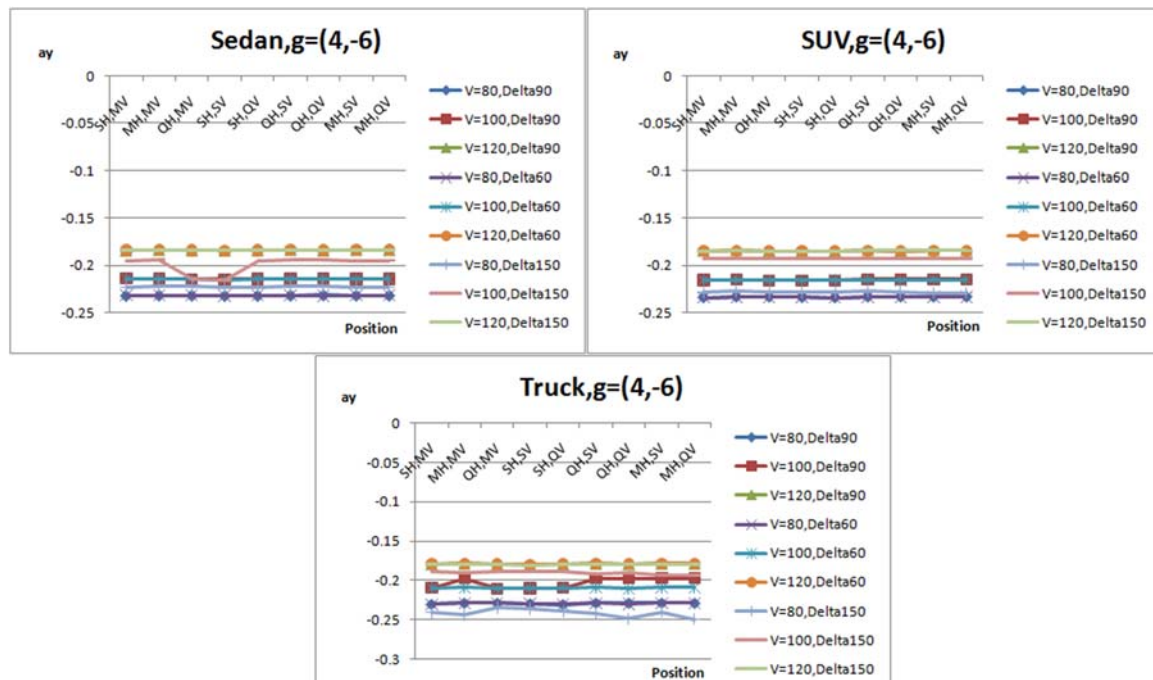


Figure 5. Deviation of lateral acceleration on a grade of (4, -6)

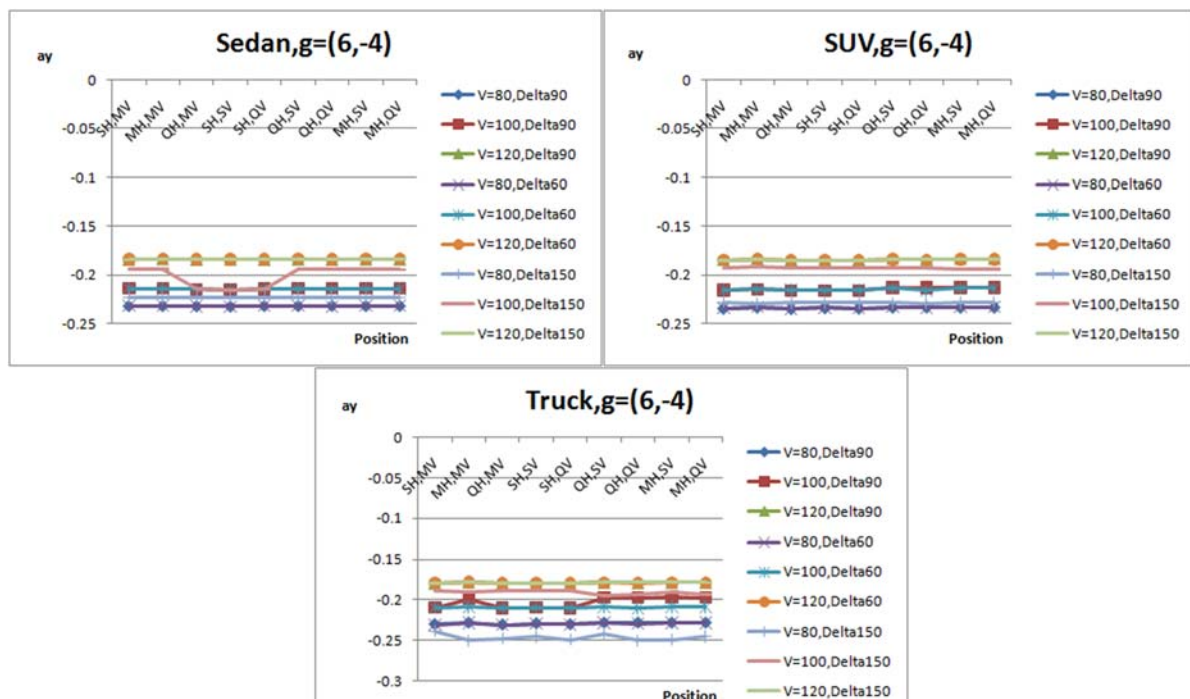


Figure 6. Deviation of lateral acceleration on a grade of (6, -4)

#### 4.3. Modelling of side friction factor

Tables 1-3 show the results of the statistical analysis of the side friction factors. The side friction factor ( $f$ ) was considered as a dependent variable and all required variables, such as  $D$  (delta),  $P$  (position),  $g$  (grade), and  $V$  (speed) were entered. Based on the information in Tables 1, 2, 3), new equations were defined for the sedan, SUV, and truck, as follows. Beta is a standardized regression coefficient that shows the importance of each variable. A variable that has a greater Beta value has a greater influence on the dependent variable. The histograms of the maximum side friction values of the sedan, SUV, and truck are shown in Figure 7.

**Table 1.** Result of Statistical Analysis for Sedan

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
<b>Constant</b>	-0.168	0.009		-19.033	0.000
<b>V</b>	0.036	0.002	0.674	16.108	0.000
<b>g</b>	-0.002	0.002	-0.040	-0.964	0.336
<b>P</b>	-0.001	0.001	-0.042	-1.014	0.312
<b>D</b>	-0.019	0.002	-0.353	-8.425	0.000

**Table 2.** Result of Statistical Analysis for SUV

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
<b>Constant</b>	-0.188	0.007		-27.560	0.000
<b>V</b>	0.041	0.002	0.778	23.159	0.000
<b>g</b>	0.000	0.002	-0.009	-0.257	0.797
<b>P</b>	0.000	0.001	0.012	0.346	0.730
<b>D</b>	0.018	0.002	-0.354	-10.528	0.000

**Table 3.** Result of Statistical Analysis for Truck

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
<b>Constant</b>	-0.178	0.007		-24.134	0.000
<b>V</b>	0.035	0.002	0.656	18.381	0.000
<b>g</b>	0.000	0.002	-0.008	-0.236	0.814
<b>P</b>	0.007	0.001	0.395	11.051	0.000
<b>D</b>	-0.018	0.002	-0.332	-9.299	0.000

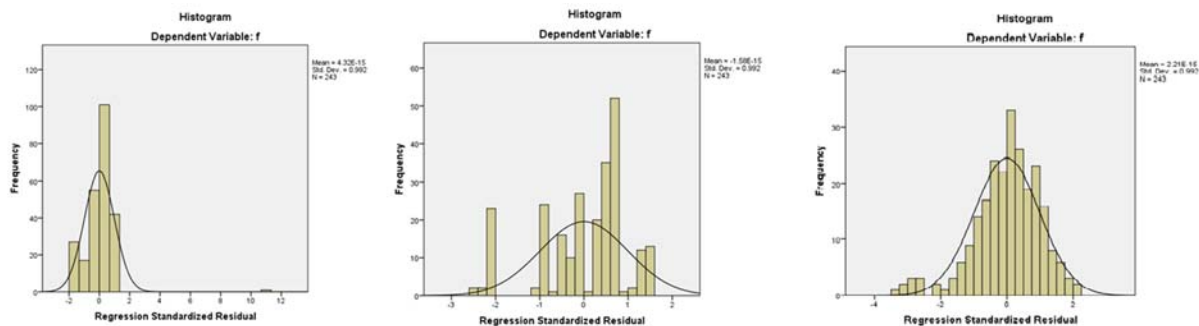
The model of the side friction factor for a sedan is Eq. (11). Based on the statistical analysis, speed, and delta were the most effective parameters on the side friction factor. The significance of the position and grade were found to be 0.312 and 0.336, respectively. Therefore, the position and grade parameters could be omitted from the equation. A similar analysis was conducted for the SUV, for which the results are shown in Eq. (12).

$$f_{\text{Sedan}} = 0.674V - 0.040g - 0.42p - 0.353D \quad (11)$$

$$f_{\text{SUV}} = 0.778V - 0.009g - 0.012p - 0.354D \quad (12)$$

The side friction model for the truck is given in Eq. (13). According to Table 3 and Eq. (13), the speed, position, and delta affected the side friction of the truck. Speed was the most effective parameter. The significance of grade was found to be 0.814; therefore, this parameter could be omitted from the equation.

$$f_{\text{Truck}} = 0.656V - 0.008g - 0.395p - 0.332 \quad (13)$$



**Figure 7.** Distribution of maximum friction for (L to R) a sedan, SUV, and truck

## 5. Conclusions

According to the results obtained from the software, the side friction factor of the truck in the beginning and at the quarter of the vertical curves was less than those of the sedan and SUV. Therefore, at these positions, the sedan and SUV were more exposed to the risk of skidding than the truck. In the middle of the vertical curves, the truck was more likely to skid, in comparison with other locations. Vehicles at a speed of 80 km/h and delta of 150° experienced the greatest side friction factors. The side friction factors in critical conditions were greater than the values in AASHTO 2011; therefore, it is necessary to reconsider the values suggested in AASHTO.

Generally, at a speed of 80 km/h, vehicles are more likely to roll over due to high lateral acceleration values. The lateral acceleration of a truck was greater than that of the SUV and sedan (except for one case). Thus, the truck was more exposed to the risk of rolling over than were the other vehicles. The location of the vertical and horizontal curves relative to each other and the values of the grades did not have significant effects on the lateral acceleration of the sedan and SUV. Thus, these two parameters could be neglected when a sedan or SUV is considered.

Based on the statistical analysis and recommended models for the side friction factor, speed, and delta were found to be the parameters that most greatly affected the side friction. Curve position affected the side friction of the truck; however, it did not significantly affect the sedan or SUV. Consequently, road designers should consider side friction factors according to the type(s) of vehicles expected to use the roadway. In addition, the parameters of speed, delta, and position of the curves (for trucks) should be considered in highway design.

## 6. References

- [1] Stine, J. S., Hamblin, B.C., Brennan, S.N., and Donnell, E.D. "Analyzing the influence of median cross-section design on highway safety using vehicle dynamics simulations." *J. Accid. Anal Prev.*, 42(6), 2010, pp. 1769-1777.
- [2] Awadallah, F. "Theoretical analysis for horizontal curves based on actual discomfort speed." *J. Transp. Eng.*, 131(11), 2005, pp. 843-850
- [3] You, K., Sun, L., and Gu, W. "Reliability-based risk analysis of roadway horizontal curves." *J. Transp. Eng.*, 138(8), (2012), pp. 1071-1081
- [4] Harwood, D.W., D.J. Torbic, K.R. Richard, W.D. Glauz, and L. Elefterdiadou. NCHRP Report 505. Review of Truck Characteristics as Factors in Roadway Design. Transportation Research Board of National Academies, Washington, D.S. 2003

- [5] AASHTO (American Association of State Highway and Transportation Officials). A Policy on Geometric Design of Highways and Streets. Washington D.C. (2011)
- [6] Bonneson, J. A. Superelevation Distribution Methods and Transition Designs. NCHRP Project 439. Washington D.C. Transportation Research Board. (2000).
- [7] Varunjikar, T. Design of Horizontal Curves with Downgrade using Low-order Vehicle Dynamics Models. The Pennsylvania State University. The Graduate School. Department of Mechanical Engineering. (2011).
- [8] Macadam, C. C., Fancher, P. S., and Segel, L. Report. No. UMTRI-72895. Side friction for superelevation on horizontal curves. Transportation Research Institute, Univ. of Michigan, Ann Arbor, Mich. (1985).
- [9] Torbic, D.J., Donnell, E.T., Brennan, S.N., Brown, A., Laughlin, M.K., Bauer, K.M. "superelevation design for sharp horizontal curves on steep grades." Transportation Research Board, Paper No: 14-4501. (2014).
- [10] Hassan, Y., Easa, S. M., and Abdel Halim, A. "State of the art of three-dimensional highway geometric design." Canadian Journal of Civil Engineering, 1998, 25(3): 500-511.
- [11] Bonneson, J. A. "A kinematic approach to horizontal curve transition design." Transportation Research Board, Paper No: 00-0590. (1999)
- [12] Kontaratos, M., Psarianos, B., and Yiotis, A. "Minimum horizontal curve radius as a function of grade incurred by vehicle motion in driving mode." J. Transp. Res. Rec., 1994, pp. 86-93.
- [13] Hall, J.W., Smith, K. L., Titus-Glover, Wambold, J. C., Yager, T. J. and Rado, Z. Guide for Pavement Friction. NCHRP Web-Only Document 108. National Cooperative Highway Research Program, Transportation Research Board. Washington, DC. February 2009.
- [14] Echaveguren, T., Bustos, M., DE Solminihaç, H. "A method to evaluate side friction in horizontal curves, using supply-demand concepts." 6th International Conference on Managing Pavements. (2004)
- [15] German Road and Transportation Research Association. Guidelines for the Design of Roads. RAS-L-1, Berlin, Federal Republic of Germany. (1984).
- [16] Psarianos, B., Kontaratos, M., Katsios, D. "Influence of Vehicle Parameters on Horizontal Curve Design of Rural Highways." Transportation Research Board. p 22:1-10. (1998).
- [17] Dunlap, D. F., Fancher, P. S., Scott, R. E., MacAdam, C. C., and Segal, L. Influence of combined highway grade and horizontal alignment on skidding. American Association of State Highway and Transportation Officials, Washington, D.C. (1978).
- [18] Eck, R.W., and French, L. J. Effective superelevation for large Trucks on sharp curves and steep grades. West Virginia University, Report 153. (2002).
- [19] Chang, T. H. "Effect of vehicles suspension on highway horizontal curve design." J. Transp. Eng., 127(1), 2001, pp. 89-91.
- [20] Abdi Kordani, A., and Mehrara Molan, A. "The effect of Combined Horizontal Curve and Longitudinal Grade on Side Friction Factors." Journal of Civil Engineering, Springer, Vol. 19, No 3, 2015.
- [21] Gillespie, T.D. Fundamental of vehicle dynamics, Society of Automotive Engineers. (1992)
- [22] Mehrara Molan, A. A., and Abdi Kordani, A. "Multi-Body simulation modeling of vehicle skidding and roll over for horizontal curves on longitudinal grades." TRB 2014 Annual Meeting.
- [23] Abdi Kordani, A., Tavassoli Kallebasti, M and Hossein Sabbaghian, M. "Analyzing the influence of coinciding horizontal curves and vertical sag curves on side friction and lateral acceleration using simulation modeling." Transportation Research Board 15-148. (2015).
- [24] Abdi Kordani, A., Mehrara Molan, A and Monajjem, S. "New formulas of side friction factor based on three dimensional model in horizontal Curves for various Vehicles." ASCE Congress. 2014
- [25] Alexander, B., and Sean, B. "On the required complexity of vehicle dynamic models for use in simulation-based highway design." Safety Research Journal 49 (2014) 105–112.2014.

- [26] Furtado, G., Easa, S. M., and Abd El Halim, A. O. Vehicle stability on combined horizontal and vertical alignments. Annual Conference of Canadian Society for Civil Engineering, Quebec, Canada, pp. 2027-2036. (2002)
- [27] Torbic, D. J., M. K. Laughlin, D. W. Harwood, K. M. Bauer, C. D. Bokenkroger, L. M. Lucas, J. R. Ronchetto, S. Brennan, E. Donnell, A. Brown and T. Varunjikar . National Cooperative Highway Research Program Report 774: Superelevation Criteria for Sharp Horizontal Curves on Steep Grades. Transportation Research Board, Washington, DC. (2014).