

PAPER • OPEN ACCESS

Planar Grillages Made of Short Steel Reciprocal Beams

To cite this article: Maciej Piekarski 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **471** 052079

View the [article online](#) for updates and enhancements.

Planar Grillages Made of Short Steel Reciprocal Beams

Maciej Piekarski ¹

¹ Rzeszow University of Technology, Faculty of Civil and Environmental Engineering and Architecture, al. Powstancow Warszawy 12, 35-959 Rzeszow, Poland

mgpiekar@prz.edu.pl

Abstract. The paper presents the concept of planar grillages made of the beams of only one length, assembled according to the principle specific for reciprocal frames. The joints of beams are located along their lengths, in the distance freely adopted from the end of the beam, what enables constructing from identical beams the grillages of different dimensions and density of beams, so also the carrying capacity. The analysis is inspired by research of Jose Sanchez and his collaborators (Sanchez, Escrig, Rodriguez 2010, Sanchez, Escrig 2011), especially in terms of selection of beams, which have been assumed same in the form of steel rectangular pipes. The eight grillages made of identical beams, but differing in pattern of layout and of density of beams and thus dimensions of the whole grillage, have been taken into consideration. It has been assumed that each the grillage is subjected to influence of identical external load. The joints have been considered alternatively, both as stiff or articulated ones. The aim of the analysis was to compare the maximum bending moments and deflections in grillages. The calculations have been made with the use of Robot Structural Analysis Professional software. The results show that along with the increase of the distance between the joints of the beams, the difference between the parameter values measured for grillages of the same beams arrangement, but differing in the use of rigid or articulated joints, decreases. It is possible to obtain the same carrying capacity in the grillage spatially arranged in the way specific for reciprocal frame, but with the application of articulated joints, as in the grillage classically arranged, with the use of stiff joints. The relative ease of making articulated joints in relation to rigid joints is the essential advantage. The grillages of bigger sizes can be constructed from identical beams as the smaller ones, and their carrying capacity is limited only to a small extent. The paper presents direct comparison of the values of analysed parameters, as well as the comparison of relative load and beam efficiency coefficients, given in the form of graphs. Static analysis is supplemented with the description of the possibility of constructing grillages made of steel beams in a manner ensuring simple assembly and cooperation with the floor slab, in particular through the use of slim floor technology.

1. Introduction

The subject taken up in the paper concerns reciprocal frames that means bar structures consisting of equivalent beams, of which each supports at least one other beam, itself being supported by at least one additional beam [1]. The history of the reciprocal frames is long. In the majority of studies, sketches drawn by Villardo de Honnecourt, Leonardo da Vinci and Sebastiano Serlio (figure 1) are referred to as prototype designs of reciprocal structures. It is not known whether any of them has ever been directly implemented, but they have inspired researchers and designers active over the span from the 16th to the 19th century. The construction of floor structures from beams shorter than their spans was used in practice, because, in addition to description given in [2], two floors have preserved to the



present time, one in the Kelmscott Manor court and the other in the Wollaton Hall palace, both of them in England. The creation of Kelmscott Manor and Wollaton Hall is dated back to 1570 [3] and 1580-88 [4]. The information about their history questions the above-mentioned years as the dates of making the both floors. The part of Kelmscott Manor covering the location of the floor was added in 1670. As for Wollaton Hall, it is known that in 1642 it was consumed by fire.

The structural material used at that time was wood. According to a common opinion, the motive for constructing floor structures of beams shorter than the distance between the supports was a limited availability of tree trunks fit for making beams which, by their length and cross-sectional dimensions, could make a support on the opposite walls. The analysis of old drawings and descriptions, as well as the history of the Wollaton Hall and Kelmscott Manner facilities, shed a different light on the motives of the builders. A decisive factor could have been the need for building a floor between the existing walls, carrying long and heavy beams up the stairs and then installing them. Then the designers' awareness of the grillage advantages that allowed the use of less structural material compared to structures supported on beams arrange in parallel to each other, might not be without significance, either.

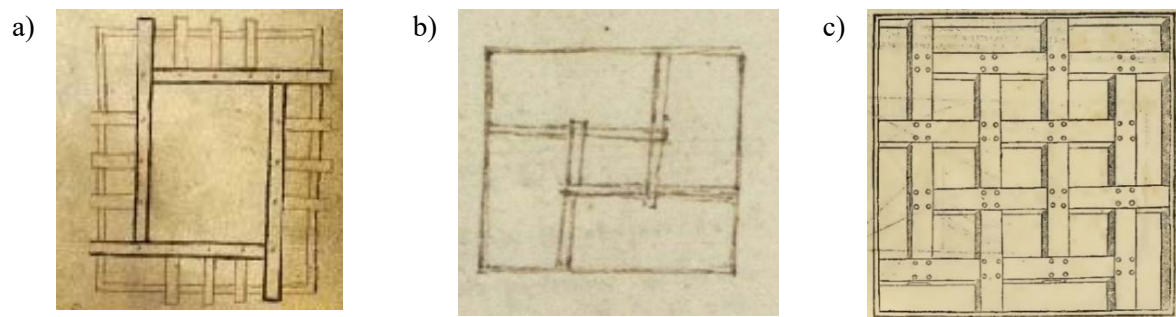


Figure 1. Pioneering designs of reciprocal structures made by: a) Villard de Honnecourt, b) Leonardo da Vinci, c) Sebastiano Serlio.

2. Geometry of reciprocal grillages

An attribute of the reciprocal structure, which is responsible for the variety of grids formed of elements with the same length L , is the freedom of choosing the location of joints along the length of the beam. The homogeneity postulate requires that the value of parameter e (figure 3b) should be constant for the entire grid; nevertheless, uniform and not identical grids may be created for different values of e . Orthogonal grids play a key role from the reason that in architecture perpendicularity of walls dominates, and despite their simplicity, they may provide a basis for the creation of grillages of various sizes and capacities. The spectrum of orthogonal grillage schemes includes, in addition to the three characteristic configurations (figure 3b, c, d.), also a grid with the geometry of the classic grillage (Fig.3a) which, in a sense, is a special variation of each of the previous configurations.

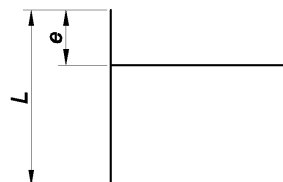


Figure 2. The distance of the joint from the end of the beam as a factor differentiating homogeneous grillages built of reciprocal beams.

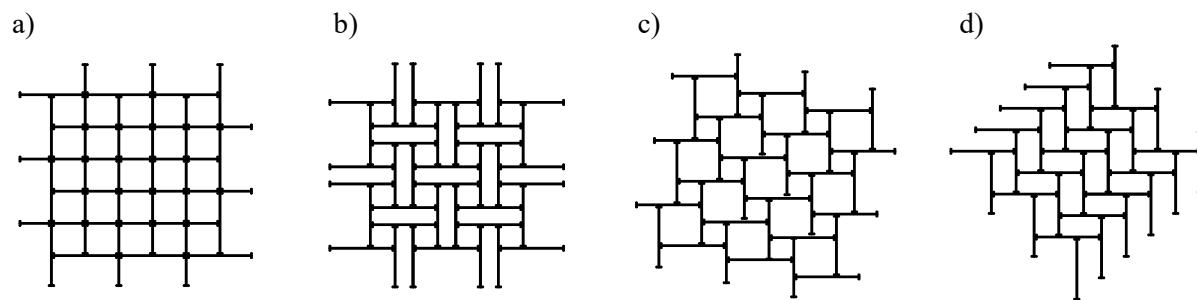


Figure 3. Diagrams of homogeneous orthogonal grids built of reciprocal beams.

3. Static analysis

A factor affecting the magnitude of internal forces in grillages is the type of joints. In grillages made from steel sections, the use of both articulated and rigid joints may be considered, although the formation of rigid joints requires more work. A negative consequence of rigid joints may be also torsional moments in beams, especially when using beams from closed profiles.

A series of numerical tests aimed at comparing the values of bending moments and deflections in grillages differing in geometrical configuration and the type of joints was carried out by Jose Sanchez and his co-workers [5][6]. The results obtained by them have shown that the effect of the type of joints is the greatest for grillages made from beams joining halfway up, when the use of articulated joints instead of rigid ones results in an increase in the value of bending moments by 130%, and deflections by 67%. For other configurations of beams, the differences were less significant and amounted to 10-37% for bending moment values and 10-22% for deflections. Torsional moments in grillages with rigid joints ranged from 17% to 25% of the maximum bending moment values.

The author carried out a numerical analysis of grillages aimed at determining the rationality of joining identical beams into different configurations, supporting floors with the same area, as well as of grillages maintaining the same configuration but differing in the size of the supported floor, due to a change in parameter e . Steel rectangular pipes 3 m long and 300 x 200 x 5 mm in cross-section were used as beams. The analysis, carried out with the application of the Robot Structural Analysis Professional software, involved 16 grillages built according to eight different schemes (figure 4), for which stiff and articulated joints of beams were considered. It has been assumed that each grillage is supported at all ends of beams, which are located on the perimeter of the grillage. In principle, the role of each of the grillages was to support a square slab subjected to an external load of 2 kN/m², which is the sum of the slab's own weight and the operational load. The slab was not modelled, and the load from the slab was transferred to the grillage through a flat panel that did not have a structural function. The own weight of the beams was also taking into account. The sides of the square slabs were assumed to have dimensions of 10.5 m, 12 m and 13.5 m, due to the fact that each of them could be supported with a grillage made from beams connected in the middle and also with the grillage according to the scheme in figure 3b. In addition, to support the first two slabs, a grillage with the configuration according to figure 3c could be used.

The results of the analysis are summarized in table 1. They confirm the significant influence of the type of joints in the case of grillages made from beams joined in the mid-length; however, the value of the ratio of maximum bending moments for structures with articulated and rigid joints is the higher, the more the grillage is extensive. For grillages modelled in accordance with schemes B, E and H (figure 4), the difference in values for articulated and rigid joints is negligible and decreases with the expansion of the grillage. The results for the grillages according to schemes C and F (figure 4) are difficult to compare with each other.

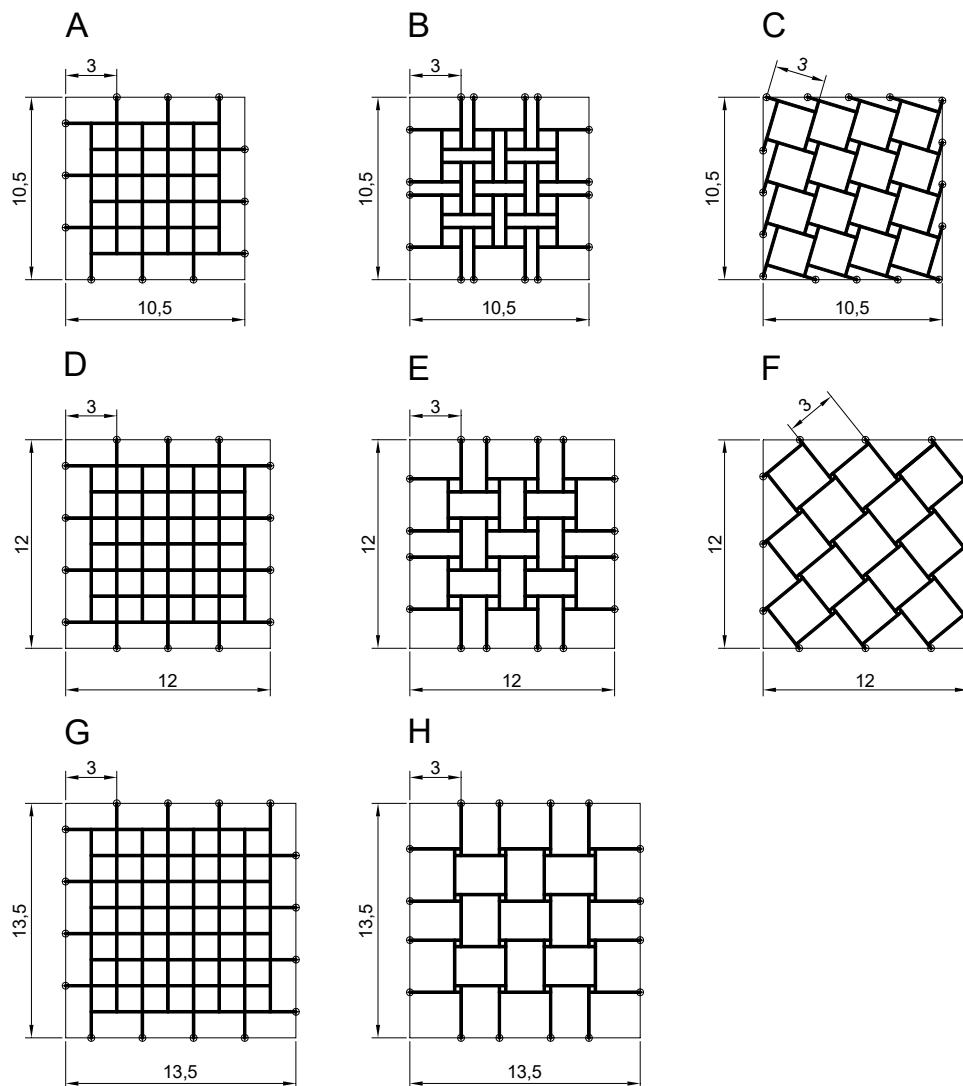


Figure 4. Schemes of grillages subjected to static analysis.

Table 1. The results of a comparative static analysis of grillages built using reciprocal beams.

Grillage scheme	A		B		C		D		E		F		G		H	
Number of beams	36		40		40		49		40		36		64		40	
Joint type	stiff	articulated	stiff	articulated	stiff	articulated	stiff	articulated	stiff	articulated	stiff	articulated	stiff	articulated	stiff	articulated
Maximum bending moment M_b [kNm]	42	61	37	42	33	47	50	82	54	59	62	67	57	100	76	81
Maximum torsional moment M_t [kNm]	8	-	7	-	11	-	11	-	10	-	15	-	15	-	14	-
Maximum transverse force T [kN]	31	42	33	38	40	69	37	56	66	78	132	232	41	68	177	210
Maximum deflection d [mm]	27	38	30	33	28	39	46	67	59	63	56	77	69	104	108	114

All of the analysed grillages are made from identical beams. Since with the same value of unit load q_0 for each grillage, the maximum values of bending moments M_b are different, it can be inferred that each of the grillages can transfer the relative load q_w , as expressed by formula (1), representing a part of load q_0 which will generate in the reference grillage the maximum value of the bending moment M_{max} . It is assumed that this grillage is grillage A with rigid joints, and the maximum moment value for this grillage is the maximum permissible value of M_{max} . Because the grillages differ in beam configuration, area A , and the number of beams n , it is worth assessing the efficiency of a single beam in the grillage. This is recognized to be directly proportional to the grillage area and inversely proportional to the number of beams. It is proposed that the efficiency index e_b be expressed with formula (2), in which A is the area of the grillage, and n , the number of beams in the grillage. Comparison of the values of coefficients f_q for individual grillages and node types is shown in figure 5, while those of coefficients f_{be} - in figure 6.

The results show that grillages made from reciprocal beams can be more effective than classic ones or, with the same effectiveness, they can gain an advantage thanks to the use of articulated joints instead of stiff joints (grillage A with rigid joints and grillage B with articulated joints). The results transformed into coefficients f_q and f_{be} show that using the same beams in grillages differing in size and subjected to a load with varying magnitudes may be reasonable, when the quick assemble of the structure or the re-use of the structural elements is the priority.

$$q_w = q_0 \cdot \frac{M_{max}}{M_i} = f_q \cdot q_0 \quad (1)$$

$$f_{be} = f_q \cdot \frac{A}{A_{ref}} \cdot \frac{n_{ref}}{n} \quad (2)$$

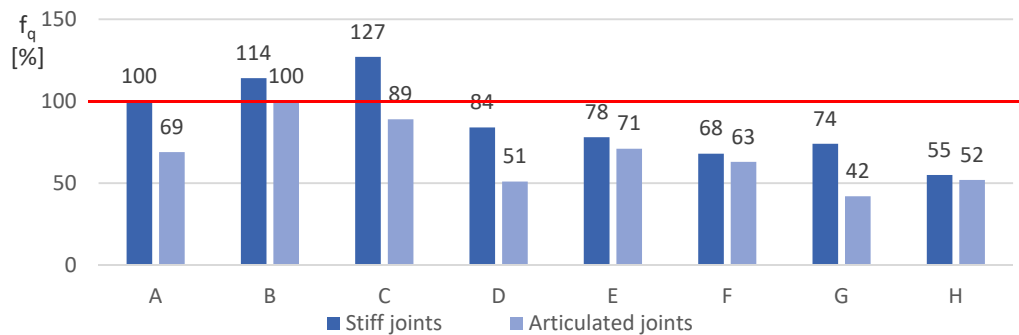


Figure 5. Comparison of relative load coefficients f_q .

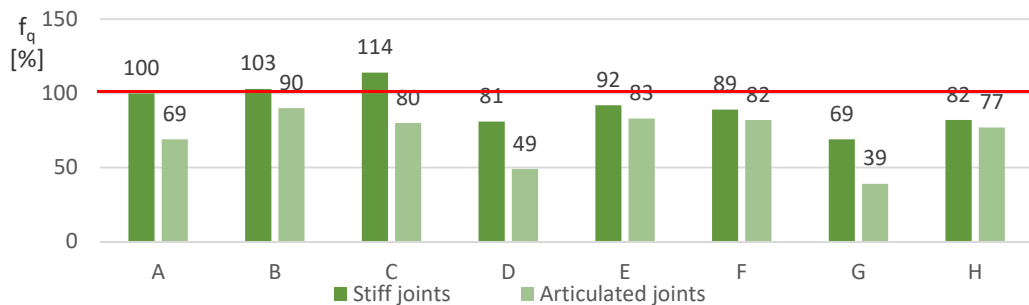


Figure 6. Comparison of beam efficiency coefficients f_{be} .

Table 2. Graphs of bending moments in selected grillages being a subject of the analysis

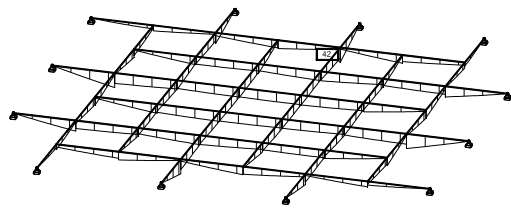
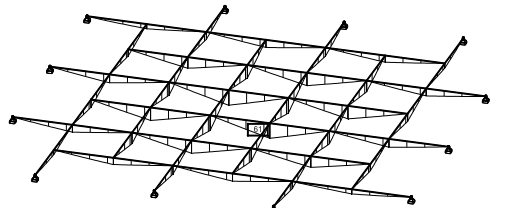
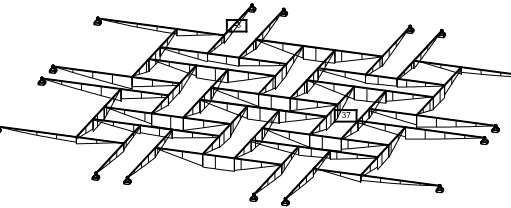
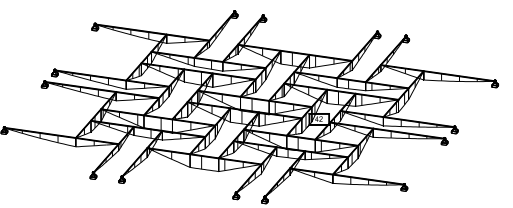
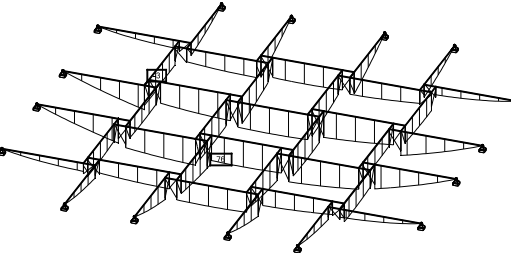
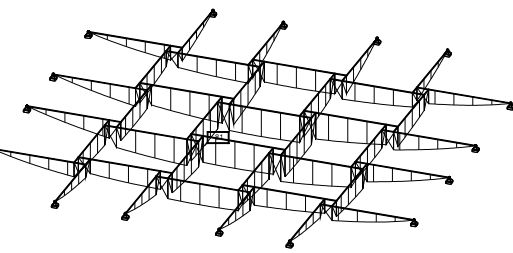
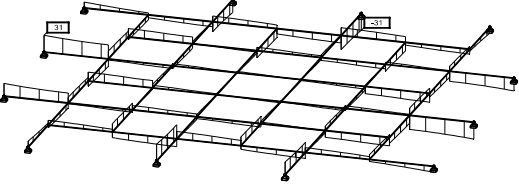
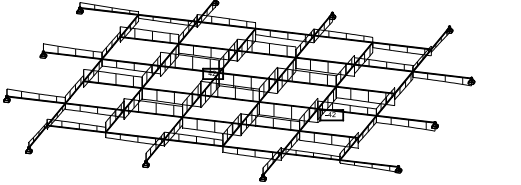
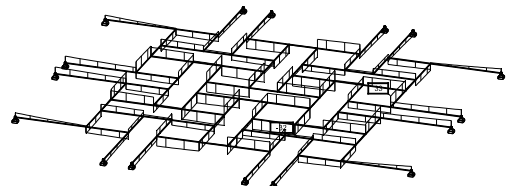
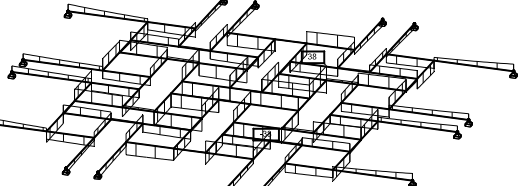
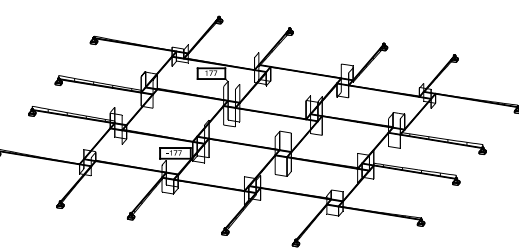
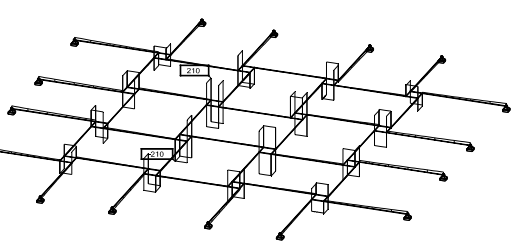
Grillage type	Stiff joints	Articulated joints
A		
B		
H		

Table 2 summarizes charts of bending moments in grillages constructed in accordance with schemes A, B and H, alternatively, with either rigid or articulated joints. The selection of grillages creates conditions for comparing structures differing in joint type, beam configuration and dimensions. The graphs show that, from a static point of view, the grillages are sets of single-span beams, for which the concentrated forces from adjacent beams make the predominant load. As two other beams are attached to the majority of grillage members along their lengths, the spacing between the attached beams has the greatest influence on the bending moments. The maximal value of the moment increases as the joints are moved closer to the middle of the beam, and it is the largest when the beams are attached exactly in the middle of the span. Characteristic feature of grillages with rigid nodes is discontinuity of bending moments, caused by torsional moments, which arise in the structure due to the eccentric application of force from the beam supported to the supporting beam, as the consequence of the use of closed profiles in the role of beams. The static equilibrium of nodes in such grates is achieved due to the combined balancing of the values of bending and torsional moments.

The complete assessment of the solution, which relies on the use of beams with a single length for building different grillages, requires a broader analysis, because apart from bending moments, other internal forces are also induced in the grillages which, though less important, have values that differ more than those of the bending moments. In addition, the maximum values of lateral forces increase significantly as the beam layout becomes less dense (e.g. grids F and H). The lateral forces assume extreme values on the intervals from the joints to the beam ends, which may make the forming of joints difficult, but it allows the beam webs in the middle of the span to be weakened, which is necessary, e.g., for making technological openings. The diagrams of lateral forces for selected grillages are presented in table 3.

Table 3. Graphs of transversal forces in selected grillages analysed

Grillage type	Stiff joints	Articulated joints
A		
B		
H		

4. Construction

There are two considerations that determine the design of a grillage: whether the joints are to transfer bending moments, and where the floor slab is to be located. In grillages made from steel sections, the ability of a joint to transfer the bending moment depends on a connection between the flanges. Two locations of the slab are possible: on the beams or between them. The disadvantage of the first solution is a greater floor thickness, while the drawback of the second one is a larger weight, because of the thicker slab. While it is easy to make either a monolithic or prefabricated slab on the upper flanges of the beams, it is difficult to arrange prefabricated elements on the bottom flanges. A way to solve this problem is the *slim floor* technology [7] consisting in the use of sections with the bottom flange wider than the upper one, which allows the prefabricates to be inserted into the spaces between the beams from the top and to be supported on the lower flanges (figure 7). The cross-sections of beams can be I-shaped or rectangular, made from halves of I-sections and a flat bars, or in the form of a plate girder, or made by connecting an additional flat bar to a typical I-section or rectangular profile (figure 8).

The implementation of the *slim floor* technology in grillages is possible, it only requires the use of sections in which the flat bar, which widens the bottom flange, is at either end shorter than the main section of the beam by half the difference between the widths of the flat bar and the section (figure 9). Such cross-sections enable the ends of one beam to be supported on the widened bottom flanges of the other beam (figure 10). Since the grillage is first loaded with its own weight, next with the weight of the slab and then with the live load, the connection may be reasonable at least during the assembly. Figure 11 shows a beam made from a rectangular tube with an added flat bar that widens the base and plates that increase the beam support surface area. The choice of the cross-section is justified by the creation of conditions for supporting the elements that fill the grillage boxes. The holes facilitate the attachment of adjoining beams in alternate places. The beams according to figure 11 make it possible to construct grillages with a free-form configuration based on a design grid with module m (figure 12).

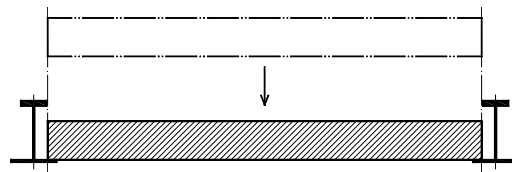


Figure 7. The idea of slim floor technology.

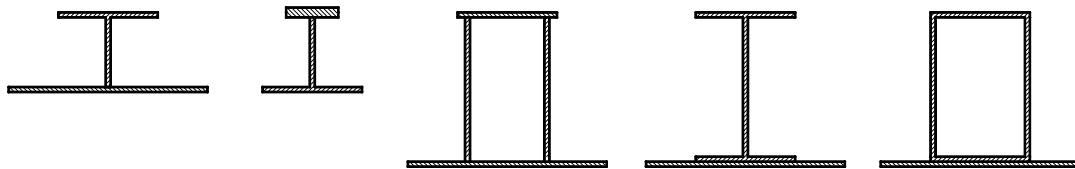


Figure 8. Sections of beams for use in the slim floor technology.

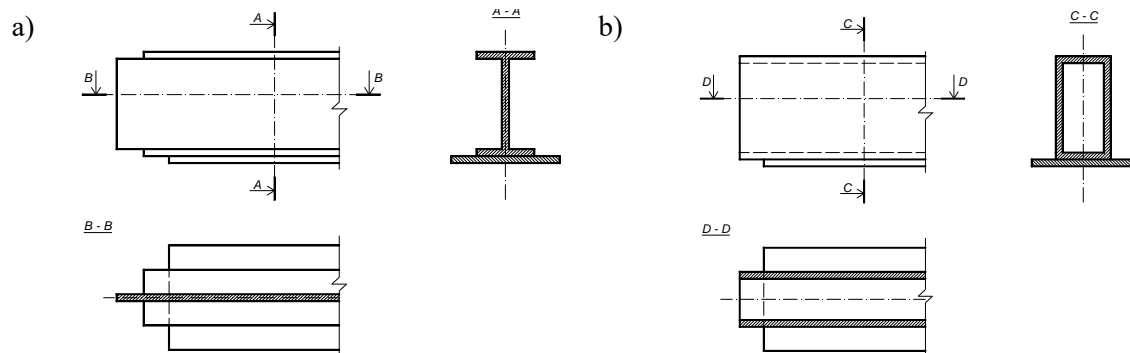


Figure 9. Beams for use in grillages in slim floor technology: a) made from I-section, b) made from rectangular pipe.

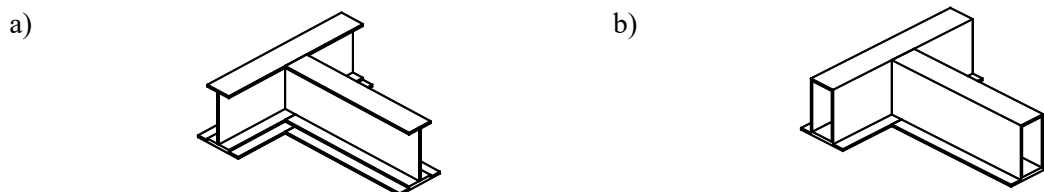


Figure 10. Connections of beams: a) I-beam, b) rectangular.

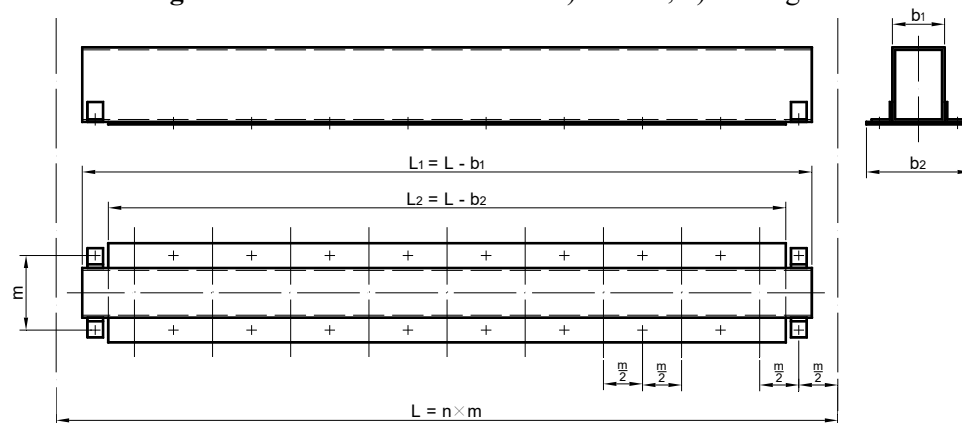


Figure 11. Beam adapted to the construction of grillages based on a modular grid.

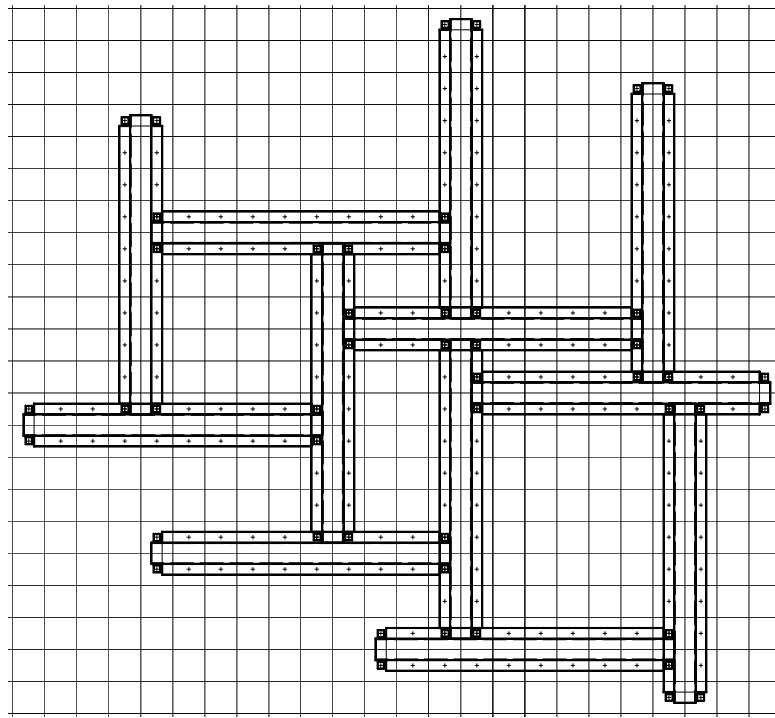


Figure 12. An example of a free-form grillage made from the identical beams.

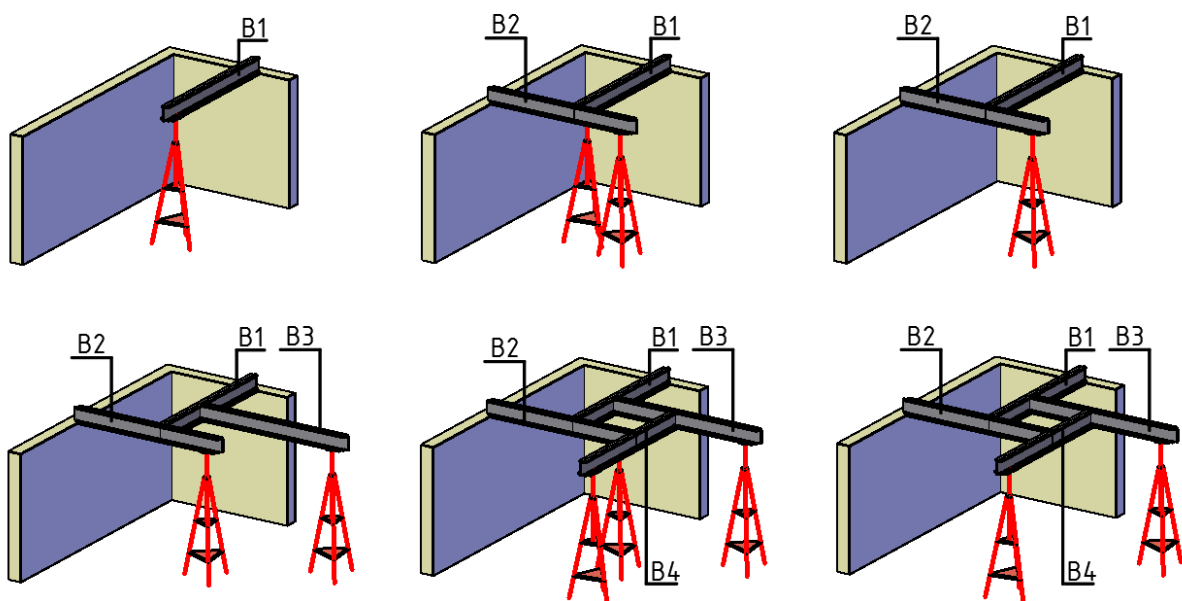


Figure 13. The use of temporary support structures and assembly sequence of reciprocal grillage.

An issue requiring a separate analysis is the method of making the grillage from short beams, including the order of assembly of individual elements and the solution of supporting the structure for the time of assembly (figure 13). Since none of the beams is supported solely on the walls or columns, each requires a temporary support. The assembly should be started from the beam B1, which, in one end, rests on the target support, which allows the use of only one temporary support supporting the other end of the beam, and also allows control of the beam position in a horizontal plane of an

established ordinate. As next in order, the beam B2 should be mounted, also supported on a fixed support and supporting the first beam. After stabilizing the connection of both beams, the support supporting the first beam B1 can be removed. The next beams can be mounted in accordance with the same logic, i.e. as the next one should always be assembled a beam (e.g.B3) forming a connection with a beam being already in the target position (B2), while the other end must be supported on the temporary support, and after stabilization of connection with the previous beam, temporary support of this beam (B2) can be removed.

The assembly of the grillage requires the use of a construction crane, but due to the shortness of the beams, its lifting capacity does not have to be significant. The construction of temporary supports should provide them with the possibility of adjusting the height. The assembly should be carried out in such a way that as soon as possible the largest number of beams has been supported on the permanent supports.

5. Conclusions

The assumptions of the slab-grillage floor system are geared to taking advantage of the possibility of supporting the structure along the perimeter, which translates into two-way static work and thus a reduction of the values of bending moments and the structure's own weight. Basing the concept on the reciprocity of components makes it possible to achieve additional benefits in the form of a reduction of the dimensions of individual elements and the ability to use them in various geometric configurations. Indirectly, this means the possibility of adapting the structure to the value of the acting load, as well as constructing different floor structures from the same components.

The possibility of making floors from small-size elements simplifies the conditions of assembly which can be performed by less qualified personnel and without the use of specialized equipment, as well as under difficult access conditions. A circumstance which lessens these advantages is the need for using a formwork for the time of carrying out the job. The concept of grillages made from short steel reciprocal beams creates the possibility of the use of not complicated assortment of structural elements for construction from them of rapid assembled and easy demountable structures with parameters adapted to current needs, which can be useful in military service or civil defense.

References

- [1] O. Popovic Larsen "Reciprocal Frame Architecture", Architectural Press, Oxford 2008.
- [2] J. Wallis "Opera Mathematica", E Theatro Sheldaniano, Oxford 1695.
- [3] D. W. Insall "Kelmscott Manor and Its Repair for the Society of Antiquaries", International Council of Monuments and Sites, London 1972.
- [4] E. Morton "Wollaton Hall: Prospect Room Floor", *Journal of Architectural Conservation*, vol. 17/3 pp. 23-41, London 2011.
- [5] J. Sanchez, F. Escrig, M. T. Rodriguez, "Una aproximacion analitica a las mallas reciprocas disenadas por Leonardo", *Informes de la Construcion*, vol. 62, no. 318, pp. 5-14, 2010.
- [6] J. Sanchez, F. Escrig, "Frames Designed by Leonardo with Short Pieces. An Analytical Approach" *International Journal of Space Structures*, vol. 26. no. 4. pp. 289-302, 2011.
- [7] D. L. Mullett, R. M. Lawson "Slim Floor Construction using Deep Decking", The Steel Construction Institute, Ascot 1993.